Aufgabe: $\frac{\sqrt{27}}{3^{0.5}} = \frac{\sqrt{27}}{\sqrt{3}} = \frac{\cancel{\sqrt{3}}\cdot\sqrt{9}}{\cancel{\sqrt{3}}} = \sqrt{9} = 3$

Indexverschiebung:

$$\sum_{k=m}^{n} a_k = a_m + a_{m+1} + \dots + a_{n-1} + a_n$$

$$\sum_{k=m+t}^{n+t} a_{k-t} = a_{m+t-t} + a_{m+t+1-t} + \dots + a_{n+t-1-t} + a_{n+t-t}$$

$$= a_m + a_{m+1} + \dots + a_{n-1} + a_n$$

Bsp: $\sum_{k=1}^{4} k^2 = 1^2 + 2^2 + 3^2 + 4^2 = \sum_{k=3}^{6} (k-2)^2$

$$\sum_{k=2}^{5} k + \sum_{k=3}^{6} k^2 = \sum_{k=3}^{6} (k-1) + \sum_{k=3}^{6} k^2 = \sum_{k=3}^{6} (k^2 + k - 1)$$

$$= \sum_{k=2}^{5} k + \sum_{k=2}^{5} (k+1)^2 = \sum_{k=2}^{5} \Big(k + \underbrace{(k+1)^2}_{k^2+2k+1}\Big) = \sum_{k=2}^{5} (k^2 + 3k + 1)$$

Arithmetische Summenformel:

$$\begin{array}{l} 1 + 2 + 3 + \dots + (n-1) + n \\ n + (n-1) + (n-2) + \dots + 2 + 1 \quad \oplus \\ \hline \underbrace{(n+1) + (n+1) + (n+1) + \dots + (n+1) + (n+1)}_{n \text{ Summanden}} \end{array} \Rightarrow 1 + 2 + \dots + n = \frac{n\cdot(n+1)}{2}$$

Geom. Summenformel:

$$\sum_{k=0}^{4} 2^k = 2^0 + 2^1 + 2^2 + 2^3 + 2^4 = \frac{1-2^5}{1-2} = \frac{1-32}{-1} = 31$$

Beweis der geom. Summenformel:

$$(1-q)\left(\sum_{k=0}^{n} q^k\right) = \sum_{k=0}^{n} q^k - q\cdot\sum_{k=0}^{n} q^k = \sum_{k=0}^{n} q^k - \sum_{k=0}^{n} \underbrace{q^{k+1}}_{q\cdot q^k}$$

$$= \sum_{k=0}^{n} q^k - \sum_{k=1}^{n+1} q^k = \underbrace{q^0}_{1} + \sum_{k=1}^{n} q^k - \left(\sum_{k=1}^{n} q^k + q^{n+1}\right)$$

$$= 1 + \cancel{\sum_{k=1}^{n} q^k} - \cancel{\sum_{k=1}^{n} q^k} - q^{n+1} = 1 - q^{n+1}$$

$$\Rightarrow \sum_{k=0}^{n} q^k = \frac{1-q^{n+1}}{1-q}$$

Bsp: $\sqrt{25} = \sqrt{5^2} \overset{5\geq 0}{=} 5$

$\sqrt{25} = \sqrt{(-5)^2} \overset{-5<0}{=} -(-5) = 5$

Bsp zu 3.10

$$8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$$

$$16^{\frac{-3}{4}} = \left(16^{\frac{1}{4}}\right)^{-3} = \left(\sqrt[4]{16}\right)^{-3} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

zu Satz 3.12

$$\underbrace{x^2}_{\hat{=}a^2} + \underbrace{px}_{2b\;a} + q = 0 \iff x^2 + px + \left(\frac{p}{2}\right)^2 - \left(\frac{p}{2}\right)^2 + q = 0$$

$$\iff x^2 + px + \left(\frac{p}{2}\right)^2 = \underbrace{\left(\frac{p}{2}\right)^2 - q}_{\frac{p^2}{4} - q = \frac{p^2 - 4q}{4}} \qquad p^2-4q = D$$

$$\iff \left(x + \frac{p}{2}\right)^2 = \frac{D}{4}$$

$$\iff \begin{cases} \text{keine Lsg } x, & \text{falls } D<0 \\ x + \frac{p}{2} = 0, & \text{falls } D=0 \\ x + \frac{p}{2} = \pm\frac{\sqrt{D}}{2}, & \text{falls } D>0 \end{cases}$$

Binomische Formel:

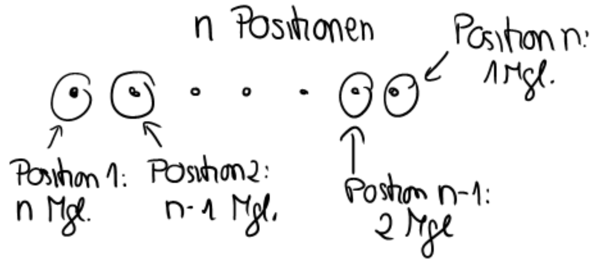
$$(a+b)^2 = a^2 + 2ab + b^2$$

Bsp: $x^2 \underbrace{-6}_{=p}x + \underbrace{9}_{q} = 0$. Dann ist $D = (-6)^2 - 4 \cdot 9 = 36 - 36 = 0$

$\Rightarrow$ Die Gleichung hat die eindeutige Lsg $x = -\left(\frac{-6}{2}\right) = 3$

Bsp zu 3.13:

$$5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$$



$$(n+1)! = \underbrace{1 \cdot 2 \cdot 3 \cdot \ldots \cdot n}_{n!} \cdot (n+1) = n! \cdot (n+1)$$

Bsp: $\binom{5}{2} = \frac{5!}{2!(5-2)!} = \frac{5!}{2! \cdot 3!} = \frac{1 \cdot \cancel{2} \cdot \cancel{3} \cdot 4 \cdot 5}{1 \cdot 2 \cdot 1 \cdot \cancel{2} \cdot \cancel{3}} = \frac{20}{2} = 10$

Additionsthm: $\binom{n+1}{k+1} = \binom{n}{k} + \binom{n}{k+1}$

Spezialfall zu $n=2$: (Satz 3.15)

$$(x+y)^2 = \sum_{k=0}^{2} \binom{2}{k} x^k y^{2-k} = \underbrace{\binom{2}{0}}_{1} \underbrace{x^0}_{=1} \cdot \underbrace{y^{2-0}}_{y^2} + \underbrace{\binom{2}{1}}_{2} \underbrace{x^1}_{x} \underbrace{y^{2-1}}_{y} + \underbrace{\binom{2}{2}}_{1} x^2 \underbrace{y^{2-2}}_{y^0 = 1}$$

$$= y^2 + 2xy + x^2$$