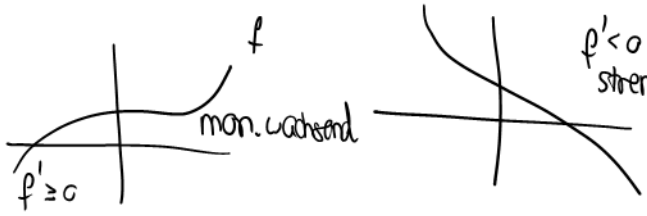
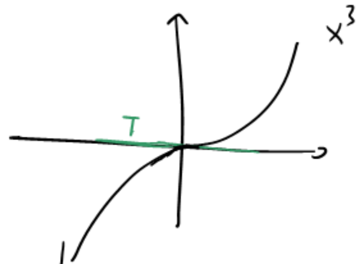
Aufgabe: $\dfrac{\sqrt{3}+\sqrt{12}}{\sqrt{3}-\sqrt{12}} = \dfrac{(\sqrt{3}+\sqrt{12})(\sqrt{3}+\sqrt{12})}{(\sqrt{3}-\sqrt{12})(\sqrt{3}+\sqrt{12})} = \dfrac{3+2\cdot\sqrt{3}\cdot\sqrt{12}+12}{3-12} = \dfrac{15+2\overbrace{\sqrt{36}}^{=6}}{-9} = \dfrac{27}{-9} = \dfrac{3}{-1} = -3$

Alternativ: $\dfrac{\sqrt{3}+\overbrace{\sqrt{12}}^{\sqrt{3}\cdot\sqrt{4}}}{\sqrt{3}-\underbrace{\sqrt{12}}_{\sqrt{3}\cdot\sqrt{4}}} = \dfrac{\cancel{\sqrt{3}}(1+\sqrt{4})}{\cancel{\sqrt{3}}(1-\sqrt{4})} = \dfrac{1+2}{1-2} = \dfrac{3}{-1} = -3$

zu 8.1



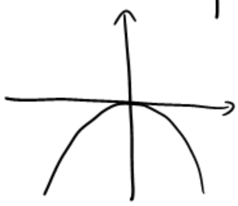
Bsp: $f: [0,1] \to \mathbb{R}$, $f(x) = x^2$ erfüllt $f'(x) = 2x > 0$ für alle $x \in (0,1)$
$\Rightarrow f$ ist auf $[0,1]$ streng mon. wachsend.



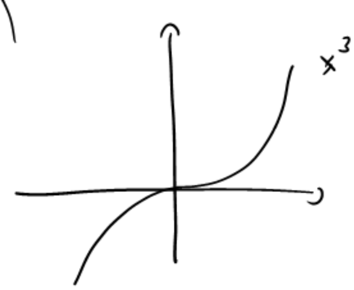
Bsp: $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^3$ ist streng mon. wachsend, obwohl $f'(x) = 3x^2$ und damit $f'(0) = 0$

zu 8.2: $f: \mathbb{R} \to \mathbb{R}$, $x \mapsto x^2$ ist linksgekrümmt, da $f'(x) = 2x$, $f''(x) = 2 > 0$ für alle $x \in \mathbb{R}$

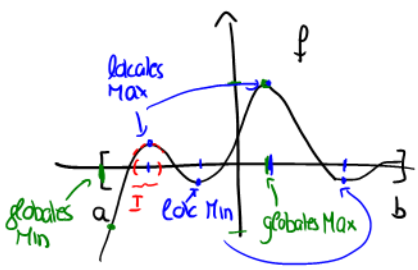
$f: \mathbb{R} \to \mathbb{R}$ $x \mapsto -x^2$ ist rechtsgekrümmt, da $f''(x) = -2 < 0$ für alle $x \in \mathbb{R}$



zu 8.3 $f: \mathbb{R} \to \mathbb{R}$, $x \mapsto x^3$ hat in $x_0 = 0$ eine Wendestelle, denn $f''(x) = 6x$ mit $f''(x) < 0$ für $x < 0$ und $f''(x) > 0$ für $x > 0$



zu 8.4



$D = [a,b]$

Bsp zu 8.5 $g: \mathbb{R} \to \mathbb{R}$, $x \mapsto x^2$ hat in $x_0 = 0$ ein lokales (und globales) Minimum
$\Rightarrow g'(0) = 0$. Tatsächlich: $g'(x) = 2x$, also $g'(0) = 0$.

Achtung: $f: \mathbb{R} \to \mathbb{R}$, $x \mapsto x^3$ hat in $x_0 = 0$ kein lokales Extremum, aber trotzdem gilt $f'(x) = 3x^2$, also $f'(0) = 0$.

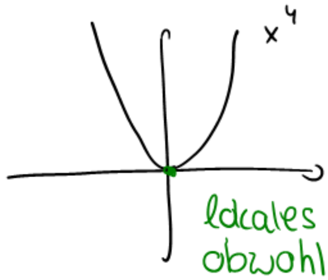
zu Satz 8.6

- $f: \mathbb{R} \to \mathbb{R} \quad x \mapsto x^4$

  Es gilt $f'(x) = 4x^3$, also $f'(x) = 0 \Leftrightarrow x = 0$

  $f''(x) = 12x^2$, also $f''(0) = 0$

  $f'''(x) = 24x$, also $f'''(0) = 0$



lokales Minimum, obwohl Satz 8.6 nicht anwendbar ist.

Bsp zu 9.1

$F: \mathbb{R} \to \mathbb{R}$, $F(x) = \frac{1}{3}x^3 + 4$

$f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$

. Dann ist F Stammfkt von f, denn

$$F'(x) = \tfrac{1}{3} 3x^2 + 0 = x^2 = f(x)$$

Bew zu 9.2 2.

Seien F, G Stammfkt zu f. Dann gilt

$$(G - F)'(x) = G'(x) - F'(x) \underset{\substack{\text{Stammfkt} \\ \text{zu } f}}{\overset{G,F}{=}} f(x) - f(x) = 0$$

$$\overset{\text{Satz 8.1}}{\Rightarrow} (G-F)(x) = C \Leftrightarrow G(x) - F(x) = C \Leftrightarrow G(x) = F(x) + C$$

Bsp zu 9.3

- Sei $n \in \mathbb{N}_0$ $\quad \int x^n dx = \frac{1}{n+1} x^{n+1} + C$
- $\int \frac{1}{\sqrt{x}} dx = 2 \cdot \sqrt{x} + C$
- $\int \sin x \, dx = -\cos x + C$

Bsp zu 9.4

$$\int \underbrace{a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0}_{\sum_{k=0}^{n} a_k x^k} dx = \frac{a_n}{n+1} x^{n+1} + \frac{a_{n-1}}{n} x^n + \ldots + \frac{a_1}{2} x^2 + a_0 x + C$$

$$= C + \sum_{k=0}^{n} \frac{a_k}{k+1} x^{k+1}$$

zu Satz 9.6:

$$(f \cdot g)'(x) = f'(x) \cdot g(x) + f(x) g'(x) \Rightarrow f(x) g'(x) = (f \cdot g)'(x) - f'(x) g(x)$$

$$\Rightarrow \int f(x) g'(x) dx = f(x) g(x) - \int f'(x) g(x) dx$$

Bsp

$$\int \underset{f(x)}{x} \cdot \underset{g'(x)}{(x^2 + x)} dx = x \cdot \left(\tfrac{1}{3}x^3 + \tfrac{1}{2}x^2\right) - \int 1 \cdot \left(\tfrac{1}{3}x^3 + \tfrac{1}{2}x^2\right) dx$$

$$= \tfrac{1}{3}x^4 + \tfrac{1}{2}x^3 - \tfrac{1}{12}x^4 - \tfrac{1}{6}x^3 + C$$

$$= \tfrac{1}{4}x^4 + \tfrac{1}{3}x^3 + C$$

Alternativ: $\int x(x^2 + x) dx = \int x^3 + x^2 dx = \frac{1}{4}x^4 + \frac{1}{3}x^3 + C$