Aufgabe: Finde $x, y \in \mathbb{R}$ mit $x<y$, so dass $\begin{array}{l} x+y=4 \quad (1) \\ x\cdot y = 3 \quad (2) \end{array}$ gilt

Lsg: $x\cdot y = 3 \Rightarrow x, y \neq 0$ und $y = \frac{3}{x}$

Einsetzen in (1) liefert: $x + \frac{3}{x} = 4 \iff x^2+3 = 4x \iff x^2-4x+3=0$

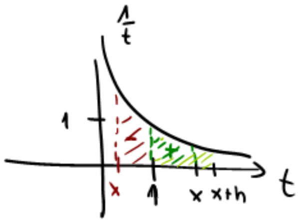
$$\iff \underbrace{x^2-4x+4}_{(x-2)^2} \underbrace{-4+3}_{=-1} = 0 \iff (x-2)^2 = 1$$

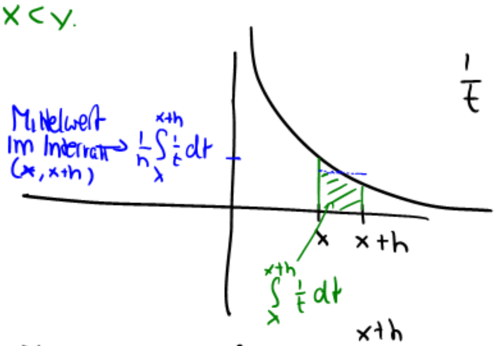
$$\iff x = 2+\sqrt{1} = 3 \lor x = 2-\sqrt{1} = 1$$

Für $x=3$ gilt $y = \frac{3}{3} = 1$

Für $x=1$ gilt $y = \frac{3}{1} = 3$ ← hier gilt $x<y$.

zu 10.1





zu 10.2 Differenzenquotient:

$$\lim_{h\to 0} \frac{\ln(x+h)-\ln x}{h} = \lim_{h\to 0} \frac{1}{h}\left(\int_1^{x+h} \frac{1}{t}\,dt - \int_1^{x} \frac{1}{t}\,dt\right) = \lim_{h\to 0} \frac{1}{h}\int_x^{x+h} \frac{1}{t}\,dt = \frac{1}{x}$$

zu 10.3

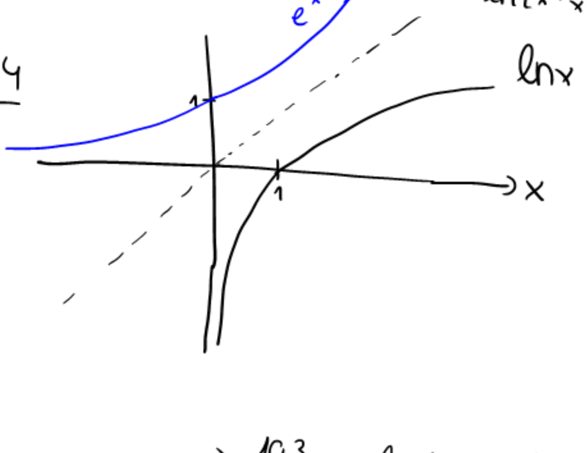
1. $\ln 1 = \int_1^1 \frac{1}{t}\,dt = 0$

3. $\ln(x\cdot y) = \int_1^{x\cdot y} \frac{1}{t}\,dt = \underbrace{\int_1^{x} \frac{1}{t}\,dt}_{\ln x} + \underbrace{\int_x^{x\cdot y} \frac{1}{t}\,dt}_{(*) = \ln y} = \ln x + \ln y$

$$(*) \quad \int_x^{x\cdot y} \frac{1}{t}\,dt \underset{\frac{ds}{dt} = \frac{1}{x} \iff dt = x\,ds}{\overset{s = \frac{t}{x}}{=}} \int_{\frac{x}{x}}^{\frac{x\cdot y}{x}} \frac{1}{\underbrace{(s\cdot x)}_{t}} \cdot \underbrace{x\,ds}_{dt} = \int_1^{y} \frac{1}{s}\,ds = \ln y$$

2. $\ln\left(x\cdot\frac{1}{x}\right) = \ln(1) = 0 \overset{3.}{\Rightarrow} \underbrace{\ln x + \ln\left(\frac{1}{x}\right)}_{\ln\left(x\cdot\frac{1}{x}\right)} = 0 \Rightarrow \ln\left(\frac{1}{x}\right) = -\ln x$

zu Def 10.4



zu 10.5

5. $\ln(\exp x \cdot \exp y) \overset{10.3}{\underset{3.}{=}} \ln(\exp x) + \ln(\exp y) = x + y = \ln(\exp(x+y))$

$\Rightarrow \exp x \cdot \exp y = \exp(x+y)$

6. $\exp'(x) = \frac{1}{\ln'(\exp(x))} = \frac{1}{\frac{1}{\exp(x)}} = \exp(x)$

Idee von Def 10.7:

$$"a^b = \exp(\ln(a^b)) = \exp(b \ln a)"$$

Bsp: • $2^{\sqrt{3}} = \exp(\sqrt{3} \cdot \ln 2) \approx 3.322$

• Für $x > 0$ gilt $x^x = \exp(x \ln x)$

$$\Rightarrow (x^x)' = (\exp(x \ln x))' = \exp'(x \cdot \ln x) \cdot (x \ln x)'$$
$$= \exp(x \ln x) \cdot (\underbrace{1}_{(x)'} \cdot \ln x + x \cdot \underbrace{\tfrac{1}{x}}_{(\ln x)'})$$
$$= x^x \cdot (\ln x + 1)$$

• Für $x > 0$ und $b \in \mathbb{R}$ gilt für $f: (0,\infty) \to \mathbb{R}$, $x \mapsto x^b$:

$$(x^b)' = (\exp(b \ln x))' = \underbrace{\exp(b \ln x)}_{= x^b} \cdot b \cdot \frac{1}{x} = b \cdot x^b \cdot x^{-1} = b x^{b-1}$$

zu 10.8

2. 1. Fall: $x > 0$. Wegen $(\ln x)' = \frac{1}{x}$ gilt $\int \frac{1}{x} dx = \ln x + C = \ln|x| + C$

2. Fall $x < 0$. Dann gilt $\int \frac{1}{x} dx = -\int \frac{1}{(-x)} dx = -(-\ln(-x)) + C$
$$= \ln(-x) + C \overset{x<0}{=} \ln|x| + C$$

Bsp: $\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx = -\int \frac{-\sin x}{\cos x} dx = -\ln|\cos x| + C$

Mit $f(x) := \cos x$, $f'(x) = -\sin x$

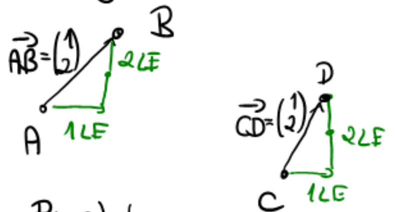
Bsp: $\int \ln x \, dx = \int 1 \cdot \ln x \, dx \overset{\text{Partielle Integration}}{=} x \cdot \ln x - \int x \cdot (\ln x)' dx$
$$= x \cdot \ln x - \int \cancel{x} \cdot \frac{1}{\cancel{x}} dx = x \ln x - \int 1 dx$$
$$= x \ln x - x + C = x(\ln x - 1) + C$$

Bsp zu Def 11.2

$$\begin{pmatrix}1\\2\end{pmatrix} + \begin{pmatrix}3\\4\end{pmatrix} = \begin{pmatrix}1+3\\2+4\end{pmatrix} = \begin{pmatrix}4\\6\end{pmatrix} \qquad (-2) \cdot \begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}-2\\-4\end{pmatrix}$$

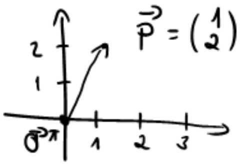
2 Mgl. zur Interpretation eines Vektors

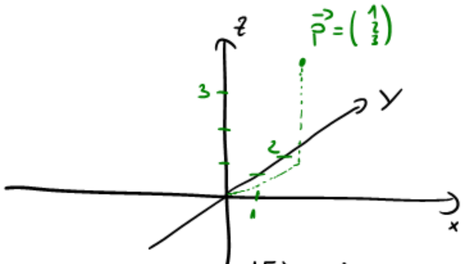
1. als gerichtete Strecke zwischen zwei Punkten



$\vec{AB} = \vec{CD}$ unabhängig von der Lage der Punkte.

2. als Punkte im Raum





Bsp zu 11.4: $2 \cdot \begin{pmatrix}1\\2\end{pmatrix} + 1 \cdot \begin{pmatrix}3\\4\end{pmatrix} = \begin{pmatrix}5\\8\end{pmatrix}$ ← Der Vektor $\begin{pmatrix}5\\8\end{pmatrix}$ ist eine Linearkombination von $\begin{pmatrix}1\\2\end{pmatrix}, \begin{pmatrix}3\\4\end{pmatrix}$