Scientific Computing on GPUs Examples in Image Processing

Robert Strzodka

caesar research center
Bonn, Germany

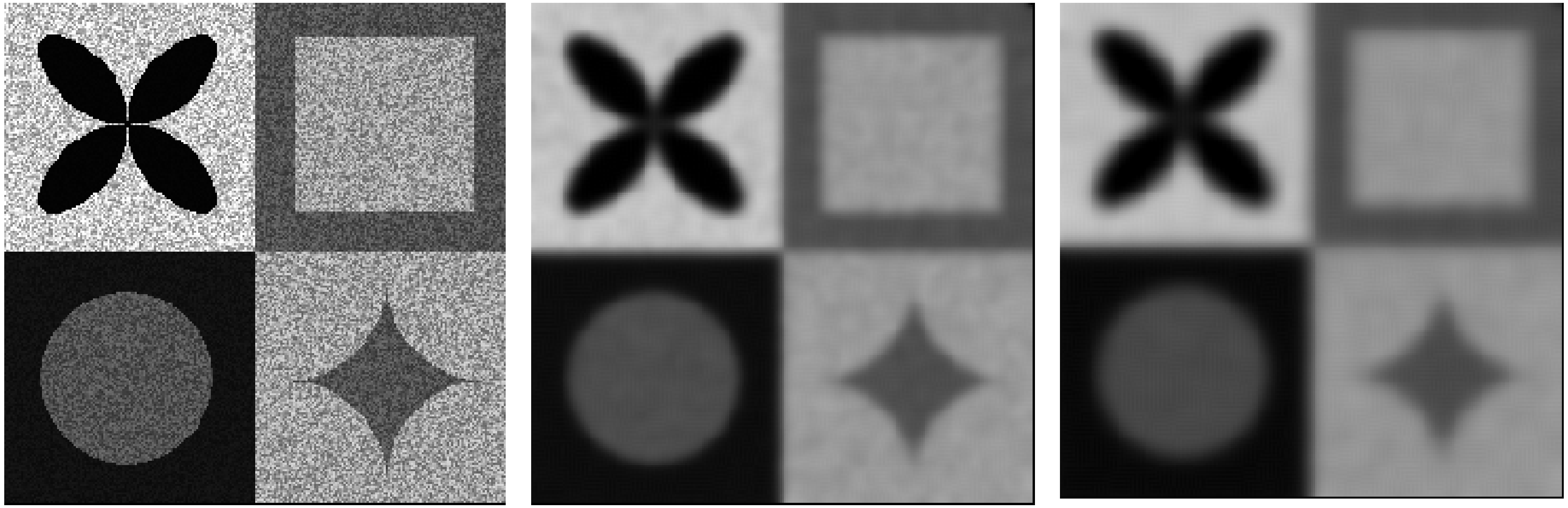
University of Dortmund
Apr 2005

Overview

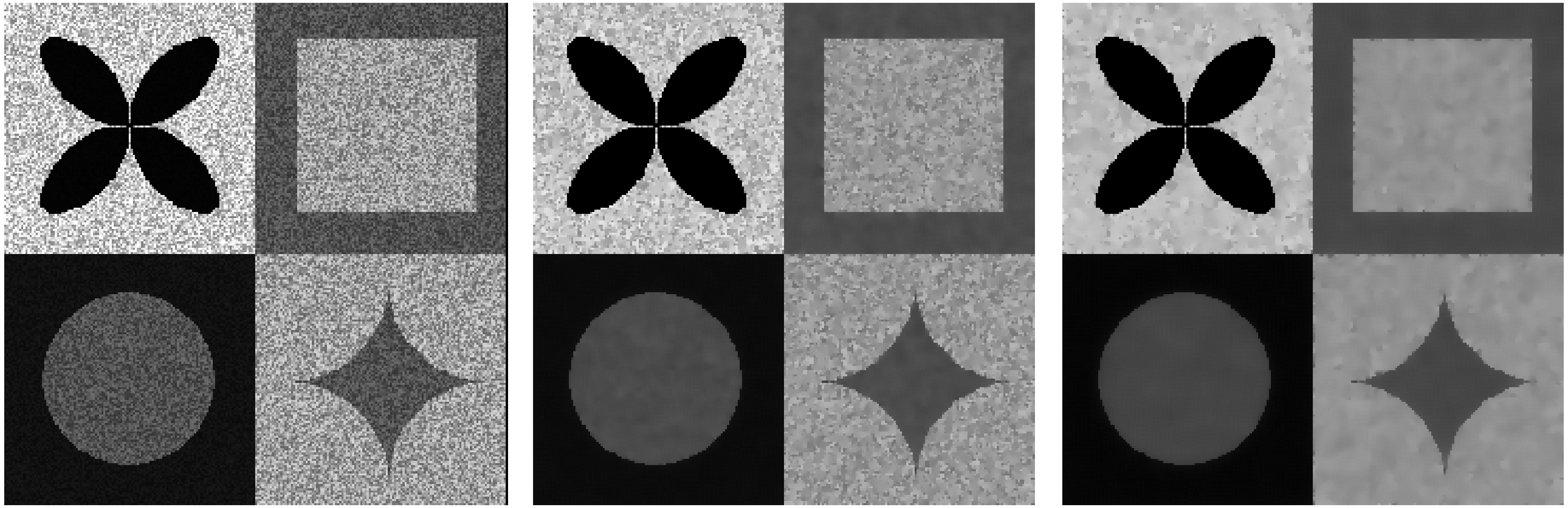
- **Image Processing**
  - Denoising
  - Segmentation
  - Registration

- **Computer Vision**
  - Object recognition
  - Object classification
  - Motion estimation

Denoising by a linear and a non-linear diffusion process



linear diffusion



non-linear diffusion

Diffusion PDE

Initial image $u_0 : \Omega \to [0,1]$

$$\partial_t u - \operatorname{div}\left(G(\nabla u_\sigma)\nabla u\right) = 0 \qquad \text{in } \Re^+ \times \Omega$$

$$u(0) = u_0 \qquad \text{in } \Omega$$

$$\partial_\nu u = 0 \qquad \text{on } \Re^+ \times \partial\Omega$$

- linear

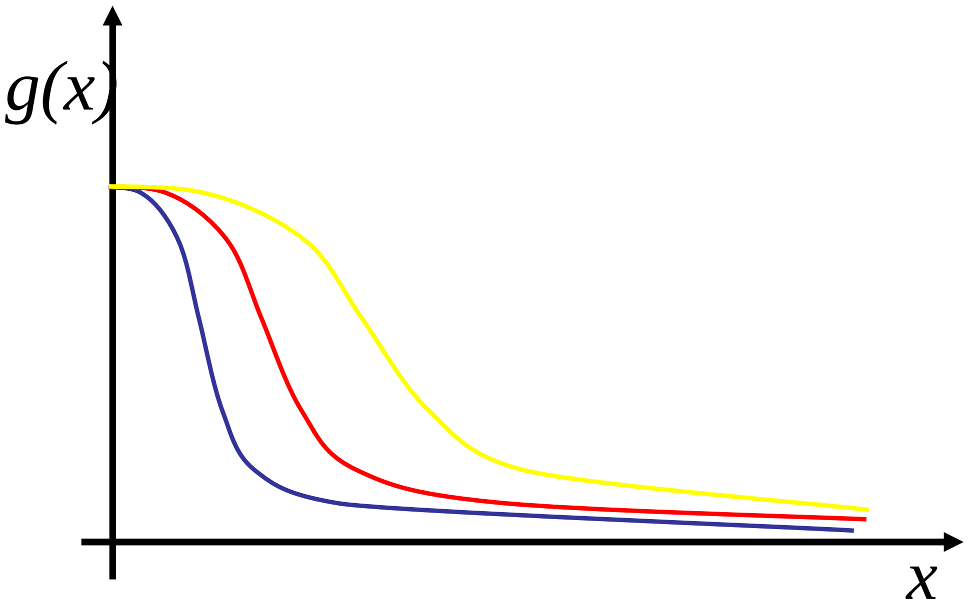
$G(v) := 1$

- isotropic non-linear

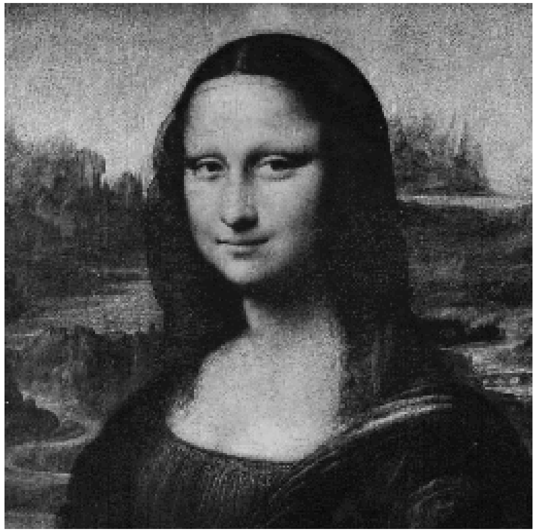
$G(v) := g(\|v\|)$ scalar

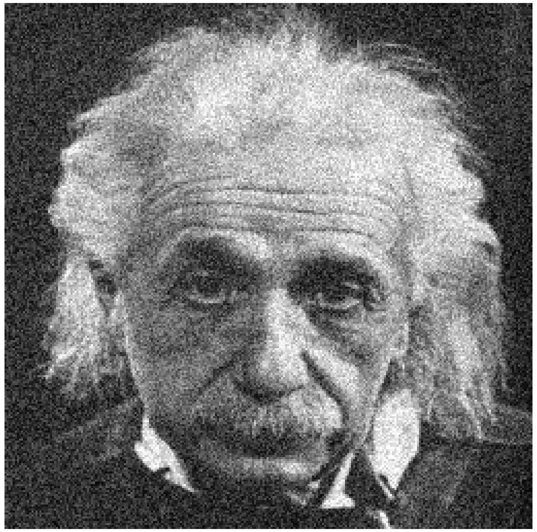
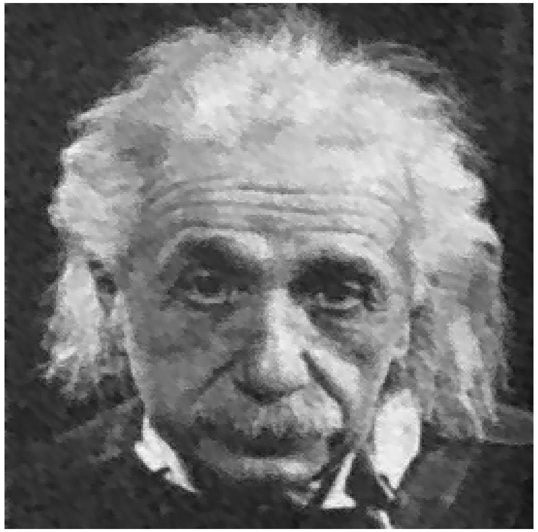
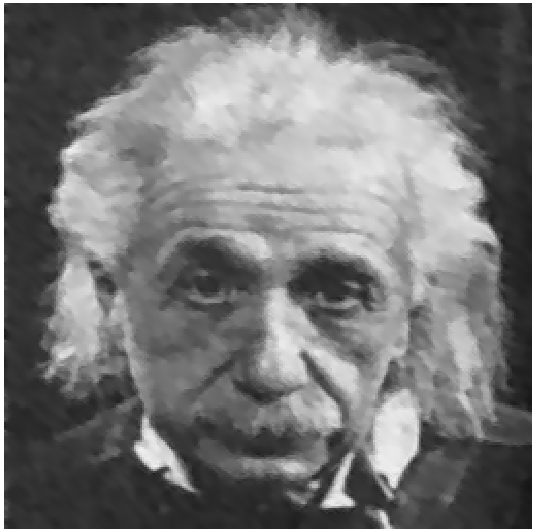
- anisotropic

$$G(v) := B^T(v)\begin{pmatrix} g_1(\|v\|) & 0 \\ 0 & g_2(\|v\|) \end{pmatrix} B(v)$$

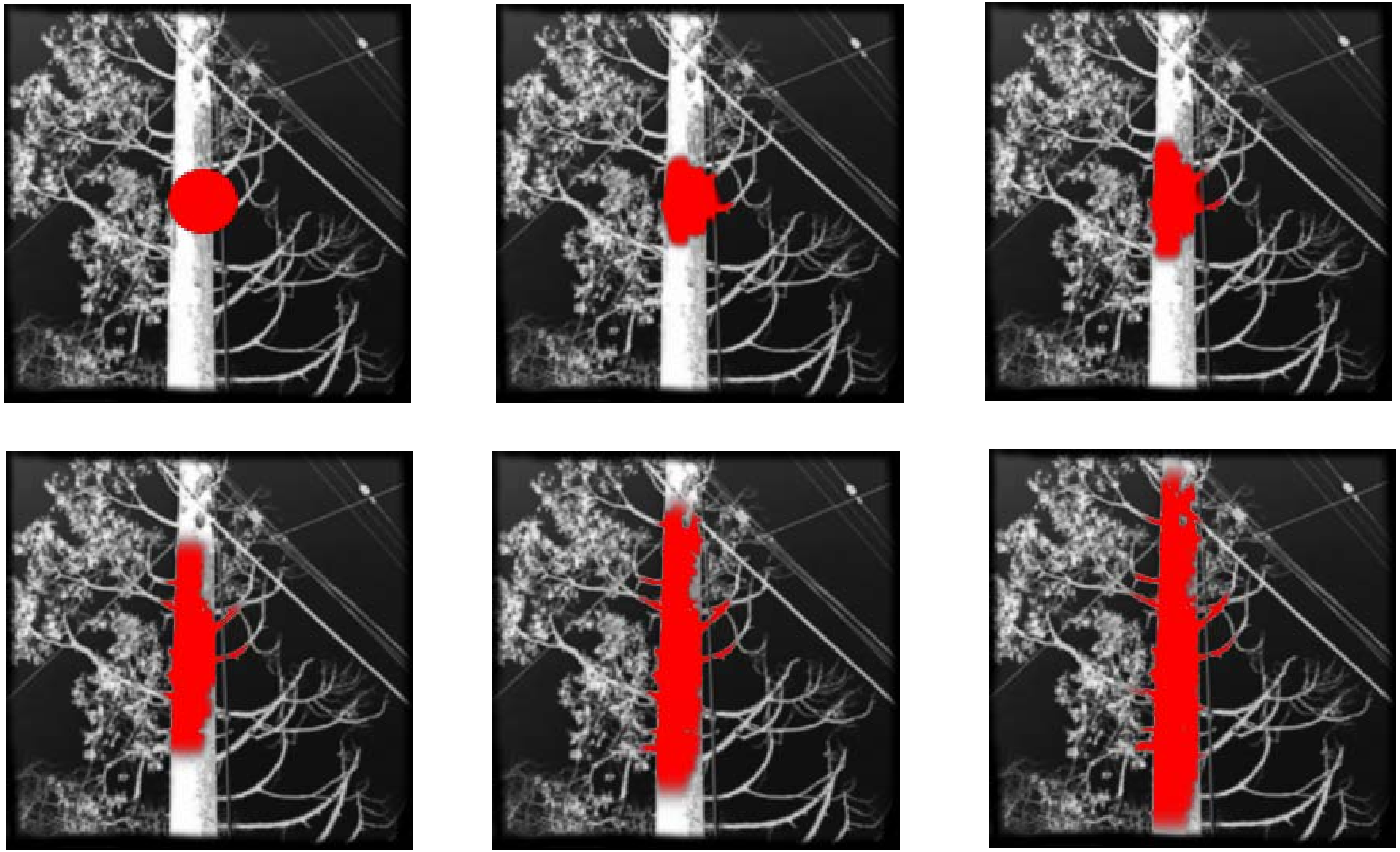


Denoising by anisotropic diffusion



Segmentation by the level-set method



Level-set equation

Initial image and level-set function $p, \varphi_0 : \Omega \to [0,1]$

$$\partial_t \varphi + f^{\sigma}[p,\varphi] \cdot \nabla \varphi = 0 \qquad \text{in } \Re^+ \times \Omega$$

$$\varphi(0) = \varphi_0 \qquad \text{in } \Omega$$

The level-set is driven by different forces $f^{\sigma}[p,\varphi] :=$

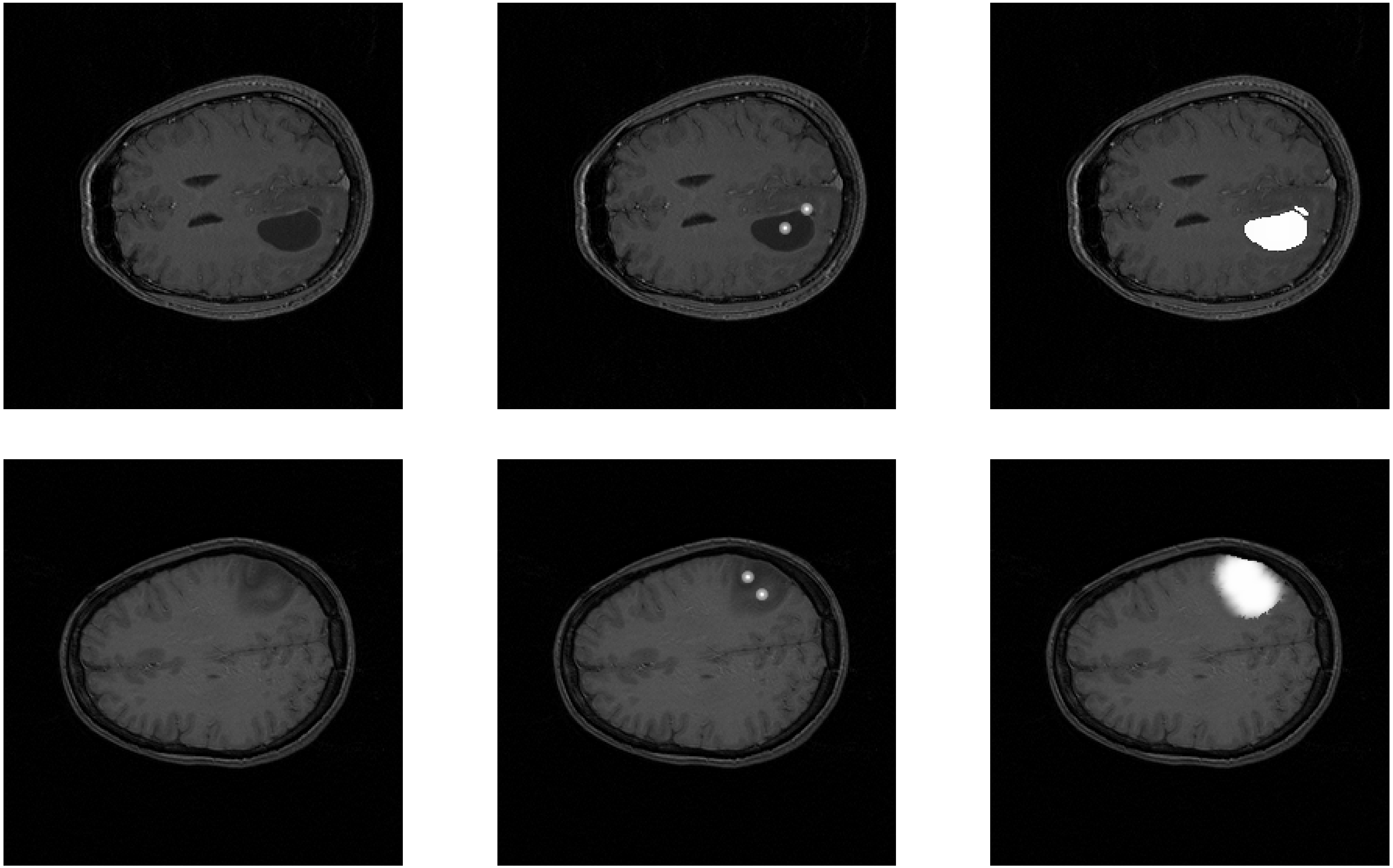
$$f_g^{\sigma}[p] \frac{\nabla \varphi}{\|\nabla \varphi\|} \quad + \quad f_{\kappa}[\varphi] \frac{\nabla \varphi}{\|\nabla \varphi\|} \quad + \quad f_V$$

image based forces dependent on *p*,Š*p*

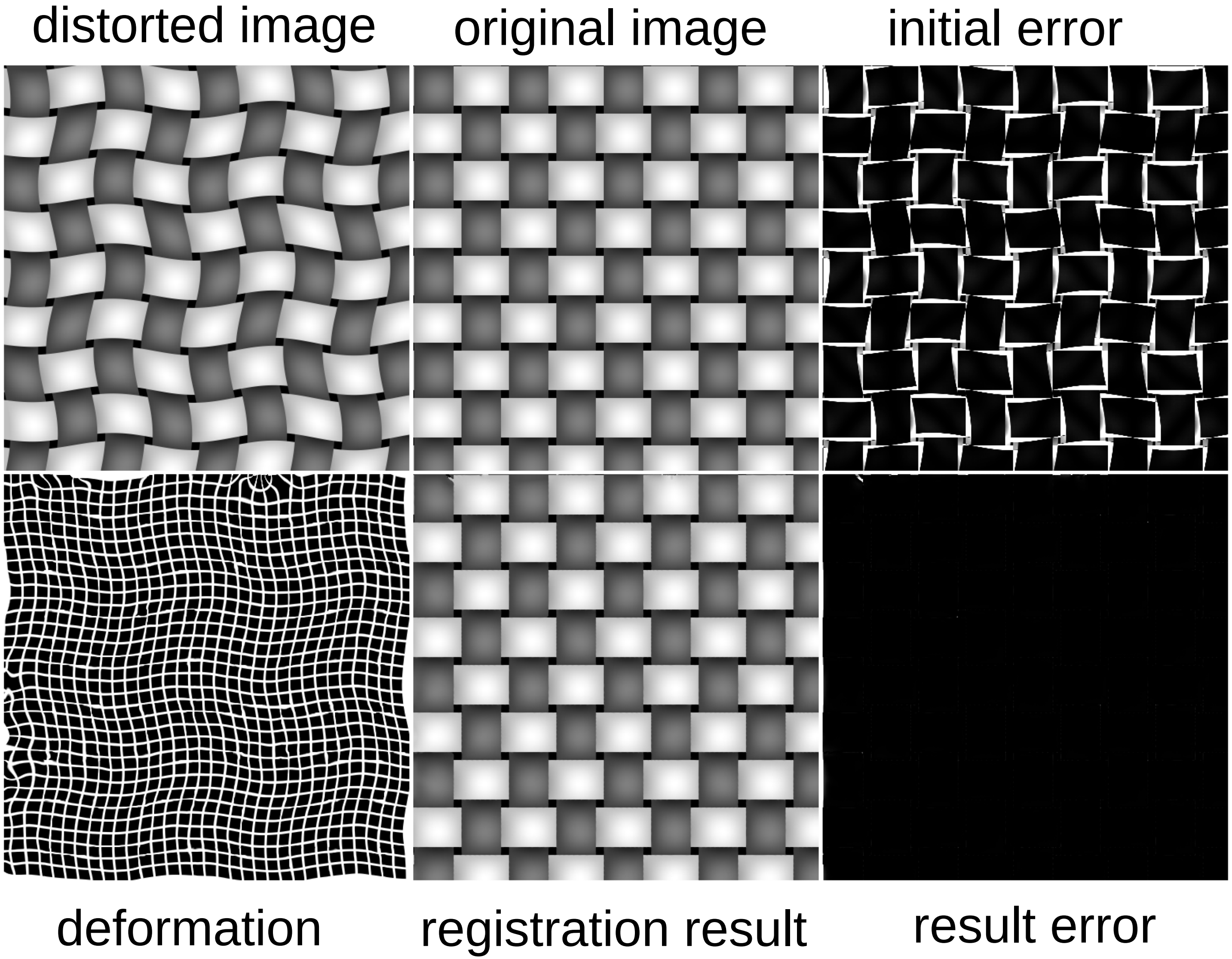
internal forces dependent on the form of level-sets, e.g. curvature n[m]

external forces, e.g. an advection field from a simulation

Segmentation by the level-set method



Registration by a reguralized gradient flow



Cascaded gradient flow on a multi-scale hierarchy

Input images $T, R : \Omega \to [0,1]$

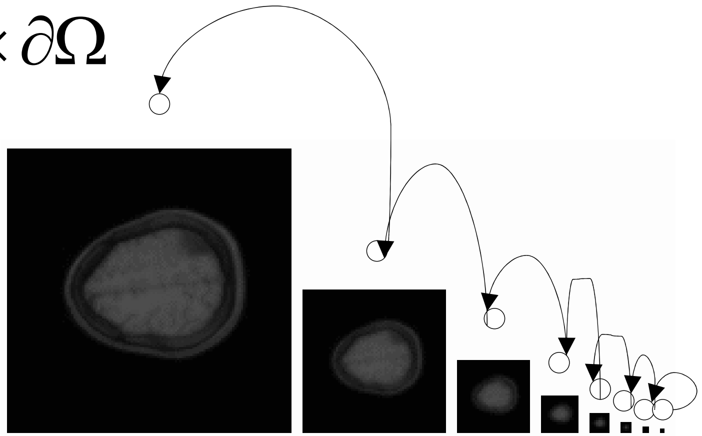
Energy measure $\quad E[u] = \frac{1}{2}\int_\Omega \left| T \circ (\mathbf{1}+u) - R \right|^2$

Gradient regularization $\quad A(\sigma) = \mathbf{1} - \frac{\sigma^2}{2}\Delta$

$$\partial_t u + A(\sigma)^{-1} \operatorname{grad}_{L^2} E[u] = 0 \quad \text{in } \Re^+ \times \Omega$$

$$u(0) = 0 \quad \text{in } \Omega$$

$$\partial_\nu u = 0 \quad \text{on } \Re^+ \times \partial\Omega$$

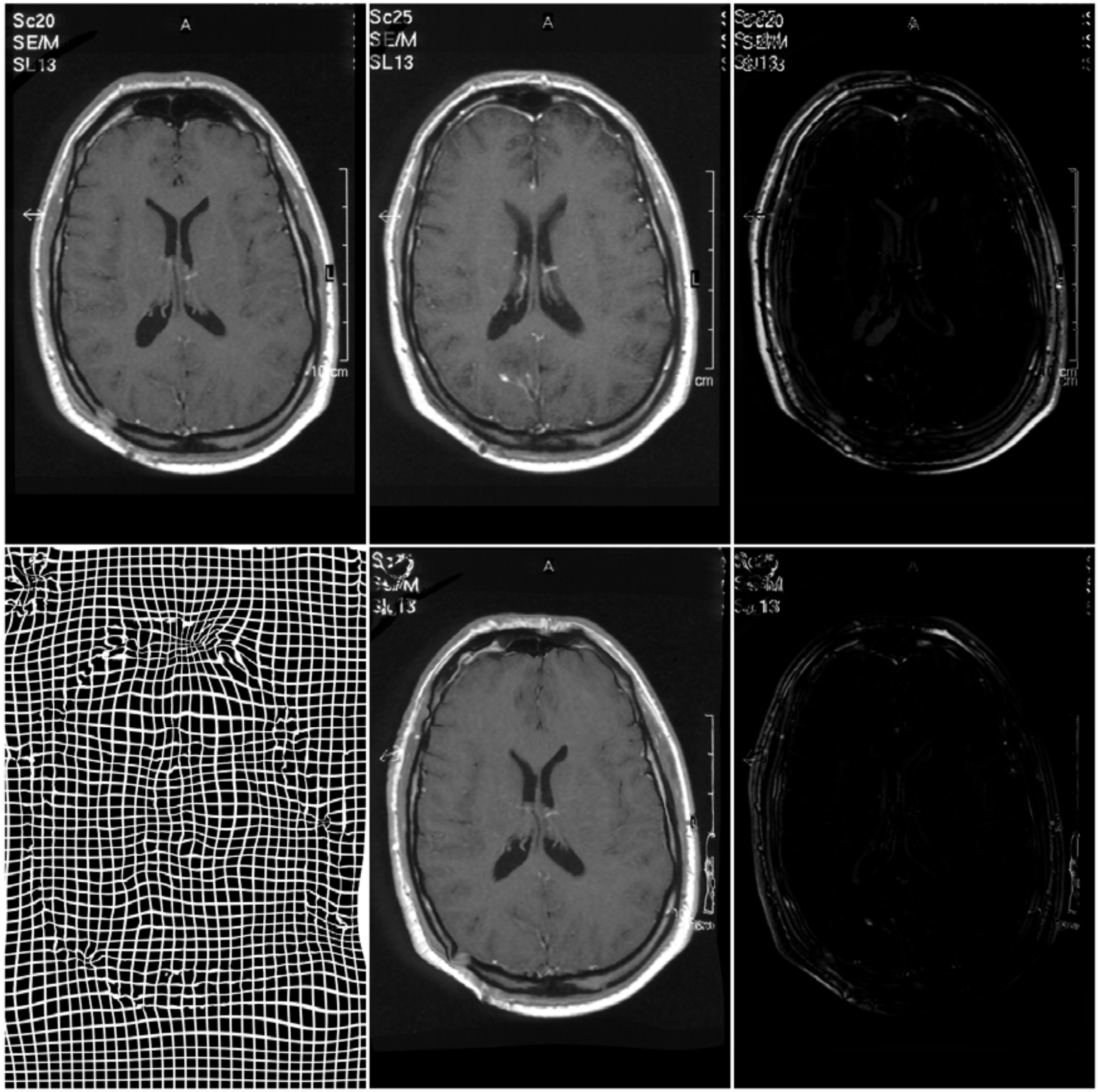


Multi-scale on multi-grid regularization

$$T_{\varepsilon^i} = S(\varepsilon^i)T, \quad R_{\varepsilon^i} = S(\varepsilon^i)R$$

$$E_{\varepsilon^i}[u] = \frac{1}{2}\int_\Omega \left| T_{\varepsilon^i} \circ (\mathbf{1}+u) - R_{\varepsilon^i} \right|^2$$

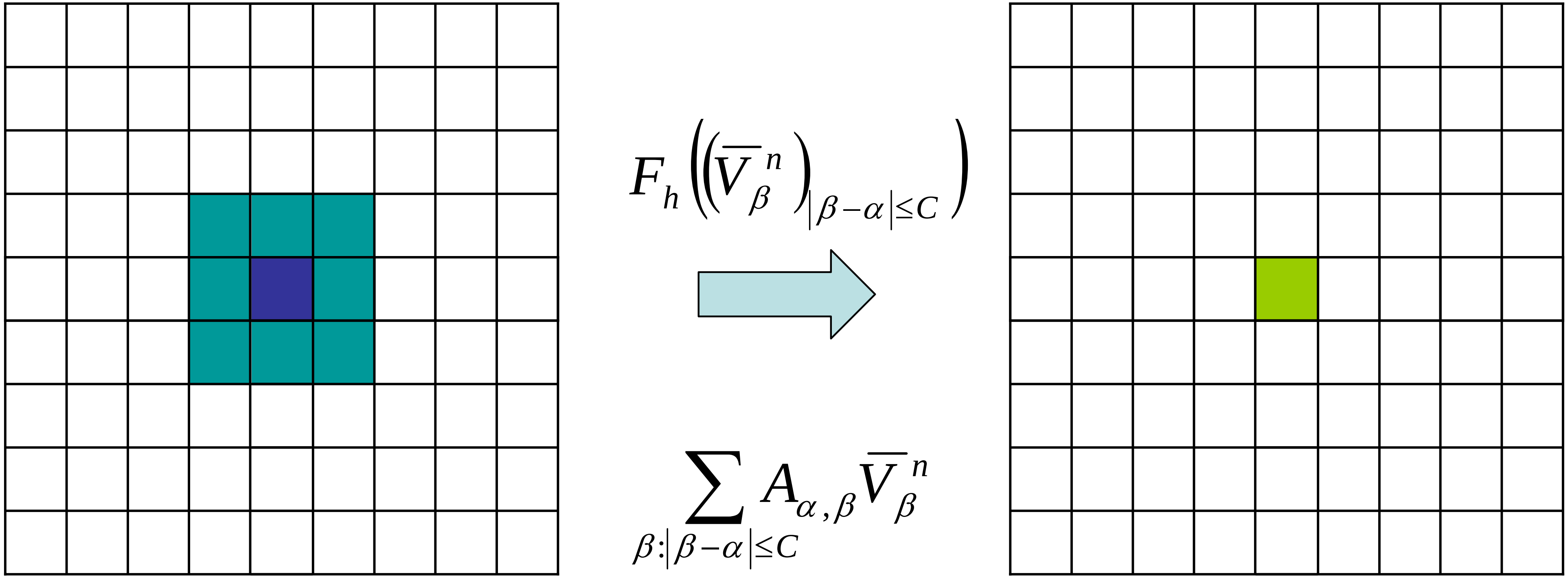
Registration by a reguralized gradient flow



Local Gather Operation

Step n

Step n+1



$$F_h\left(\left(\overline{V}_\beta^{\,n}\right)_{|\beta-\alpha|\leq C}\right)$$

$$\sum_{\beta:|\beta-\alpha|\leq C} A_{\alpha,\beta}\overline{V}_\beta^{\,n}$$

$$\left(\overline{V}_\beta^{\,n}\right)_{|\beta-\alpha|\leq C}$$

$$\overline{V}_\alpha^{\,n+1}$$

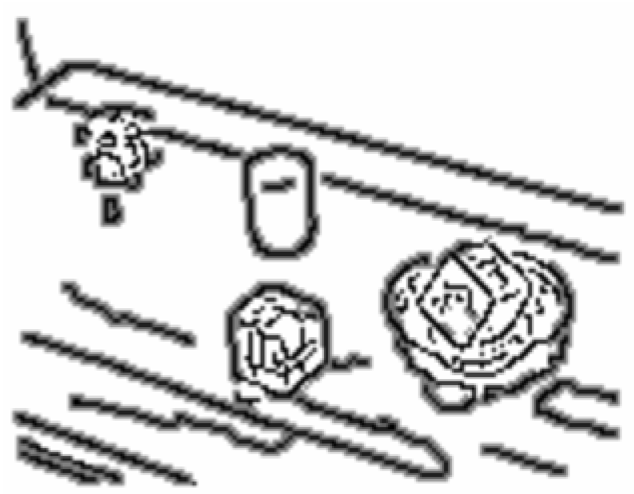
Overview

- **Image Processing**
  - Denoising
  - Segmentation
  - Registration

- **Computer Vision**
  - Object recognition
  - Object classification
  - Motion estimation

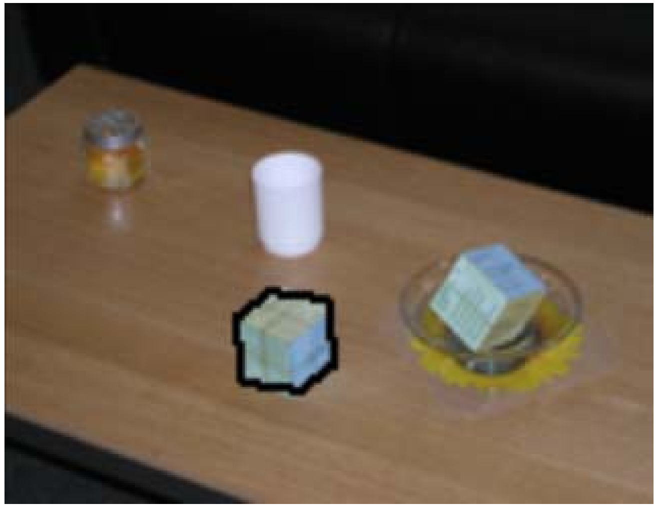
Object recognition by the Generalized Hough Transform

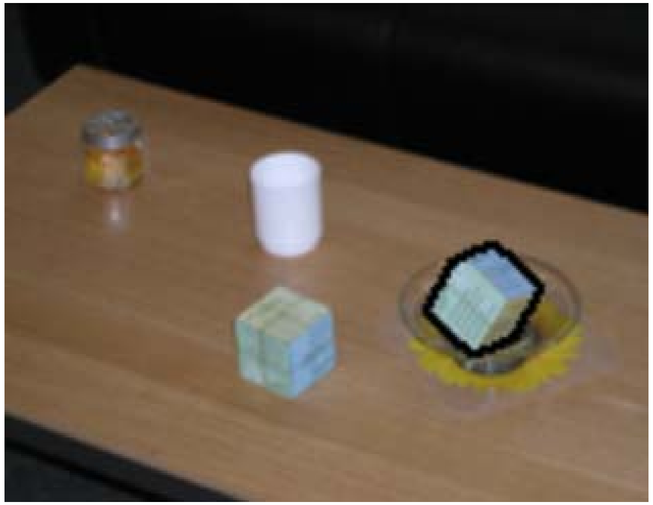
original image



edge image

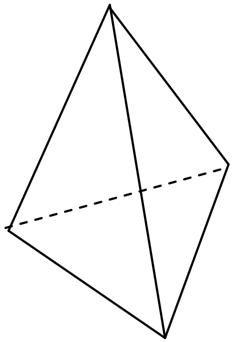
first detected cube





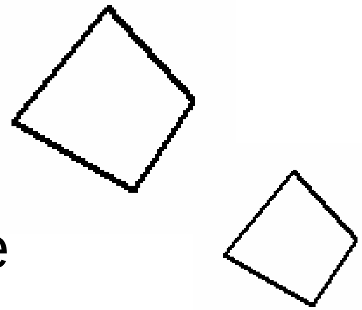
second detected cube

Generalized Hough Transform

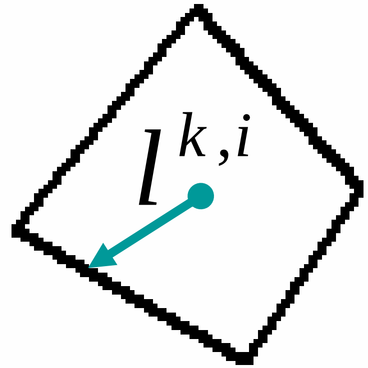


3D geometry

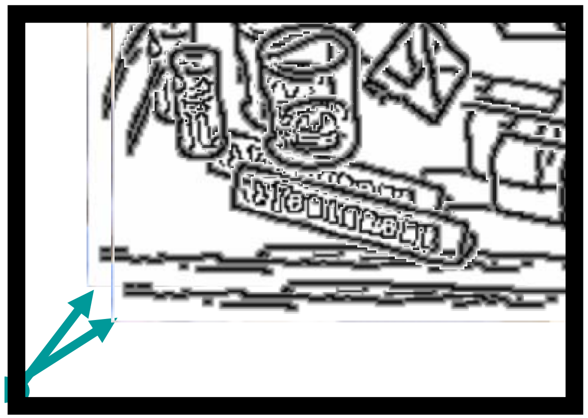
Generate poses of different perspective and scale



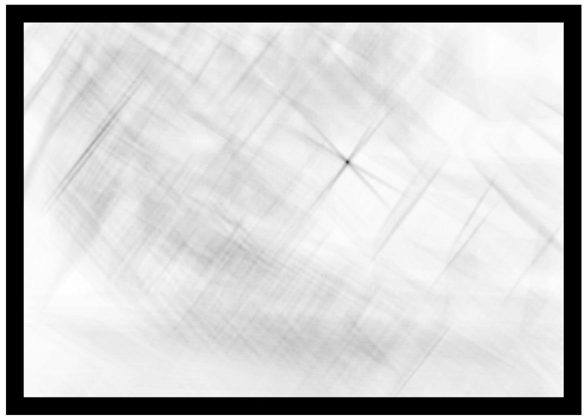
Store pose k as a list of offset vectors $l^{k,i}$



Draw image with offsets



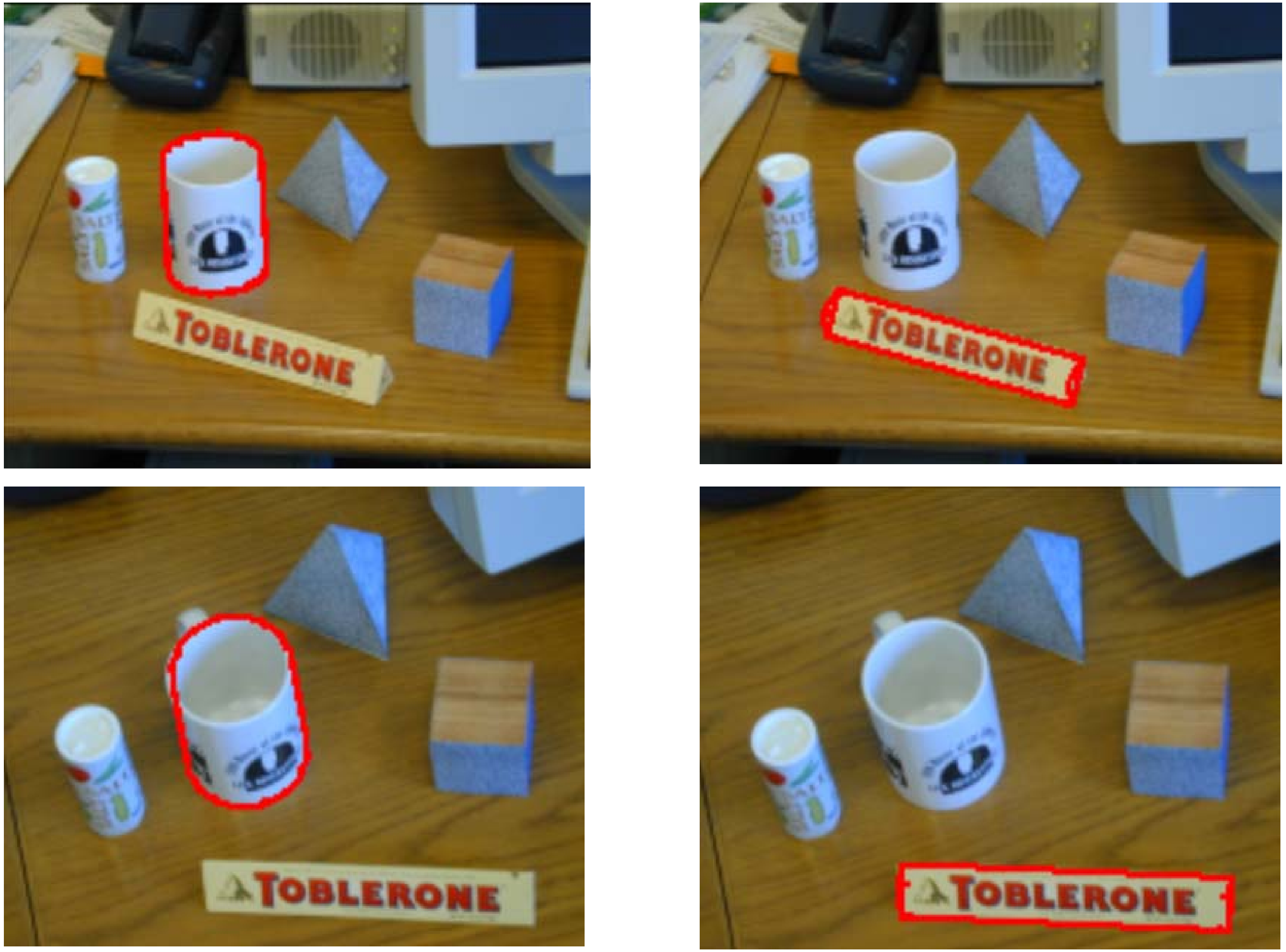
normalize the result



$$\overline{C}^k(x,y) = \sum_{i=1}^{|l^k|} I^{\text{edge}}(x + l_x^{k,i}, y + l_y^{k,i})$$

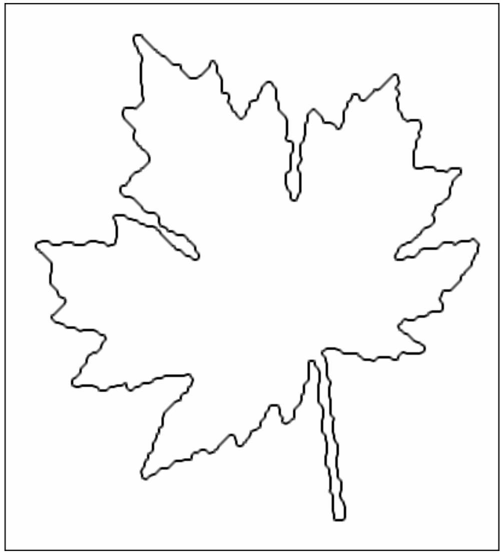
$$C^k(x,y) = \overline{C}^k(x,y) / |l^k|$$

Object recognition by the Generalized Hough Transform



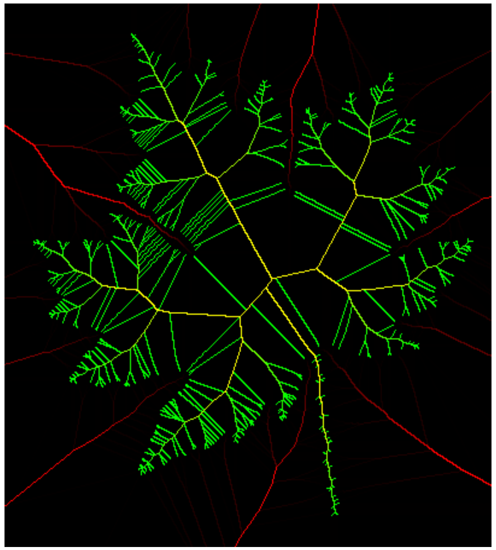
Object classification by skeletons

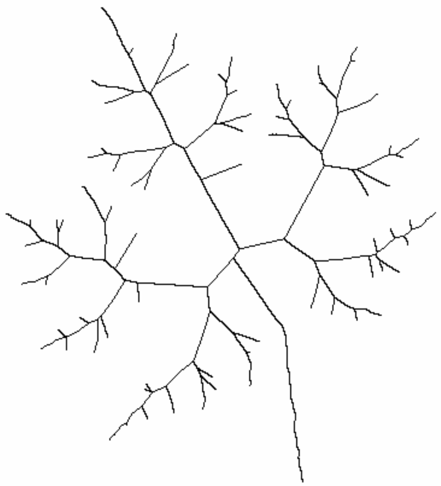
original boundary



distance transform

fine skeleton

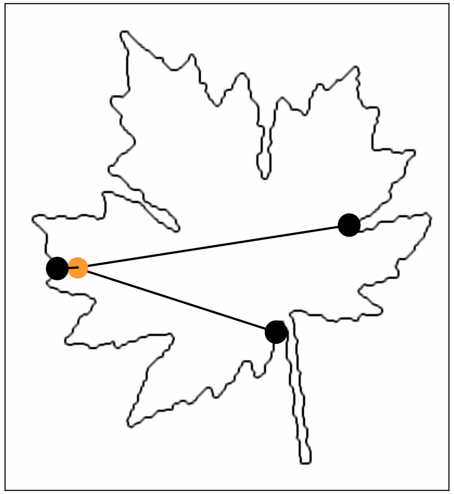




trimmed skeleton

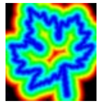
Adaptive Generalized Distance Transform

boundary



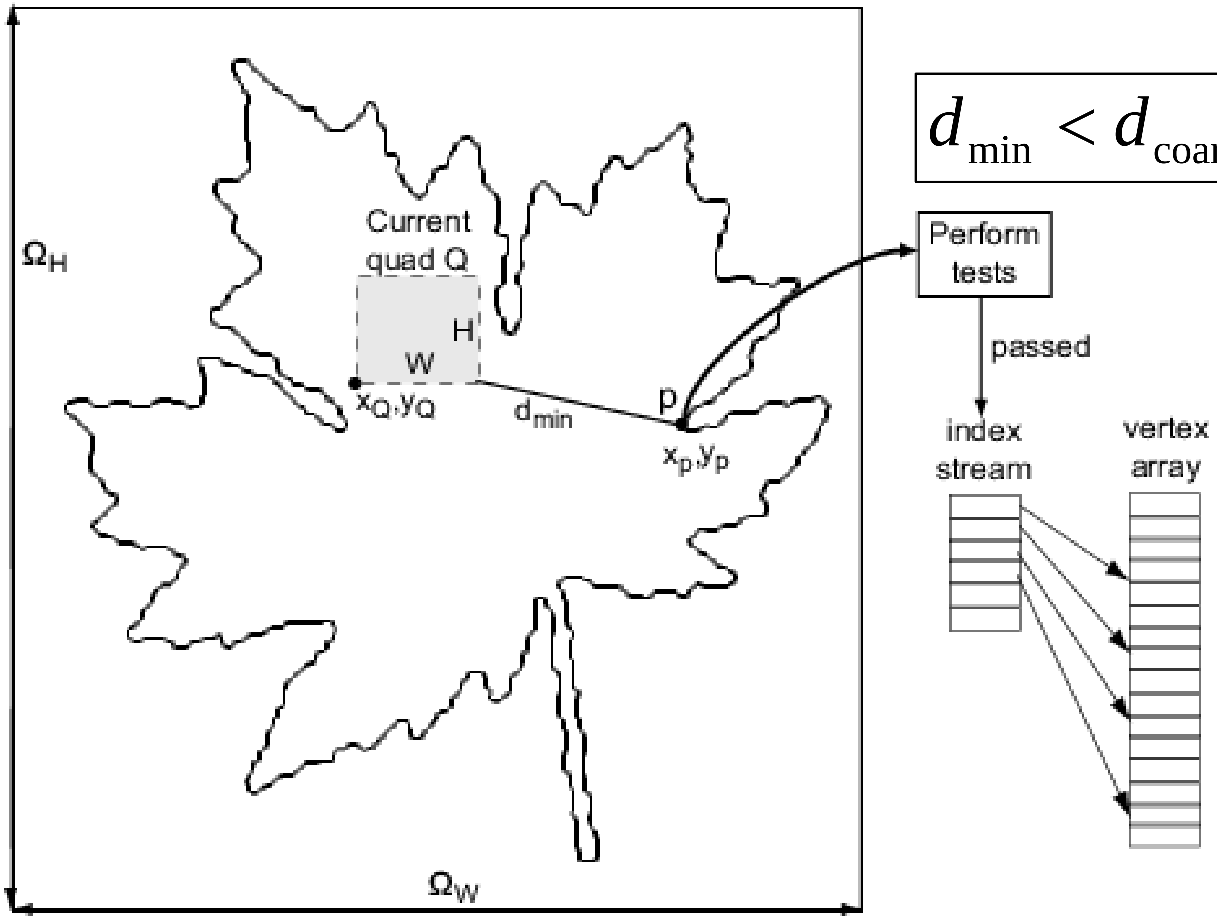
distance transform

coarse image



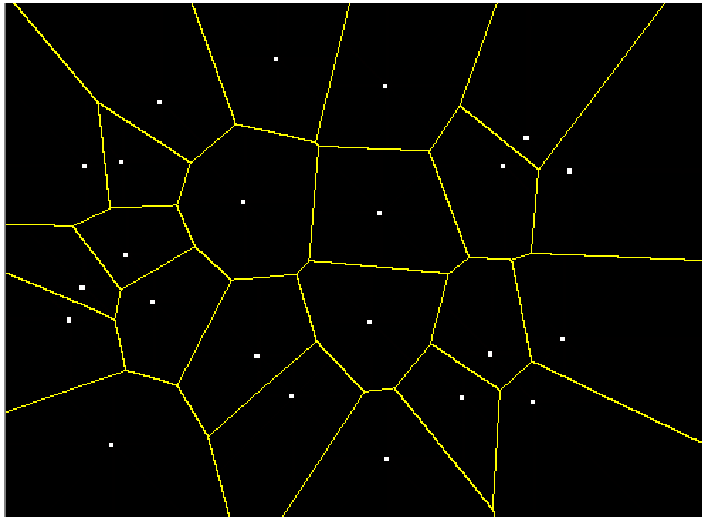
d_{coarse} coarse solution

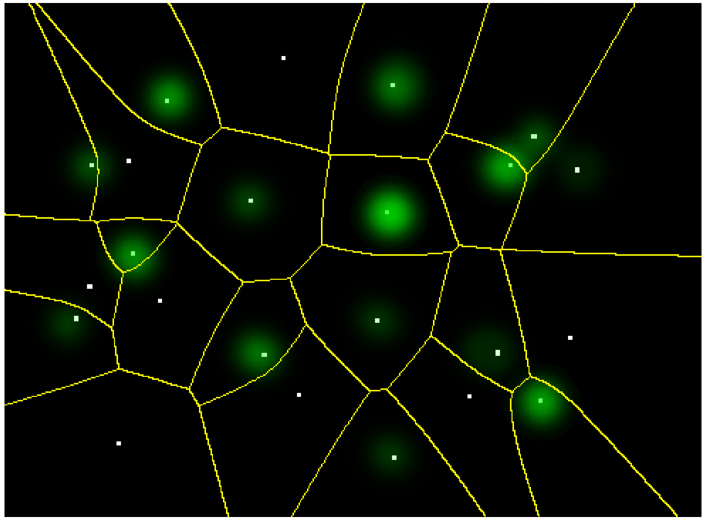
full solution

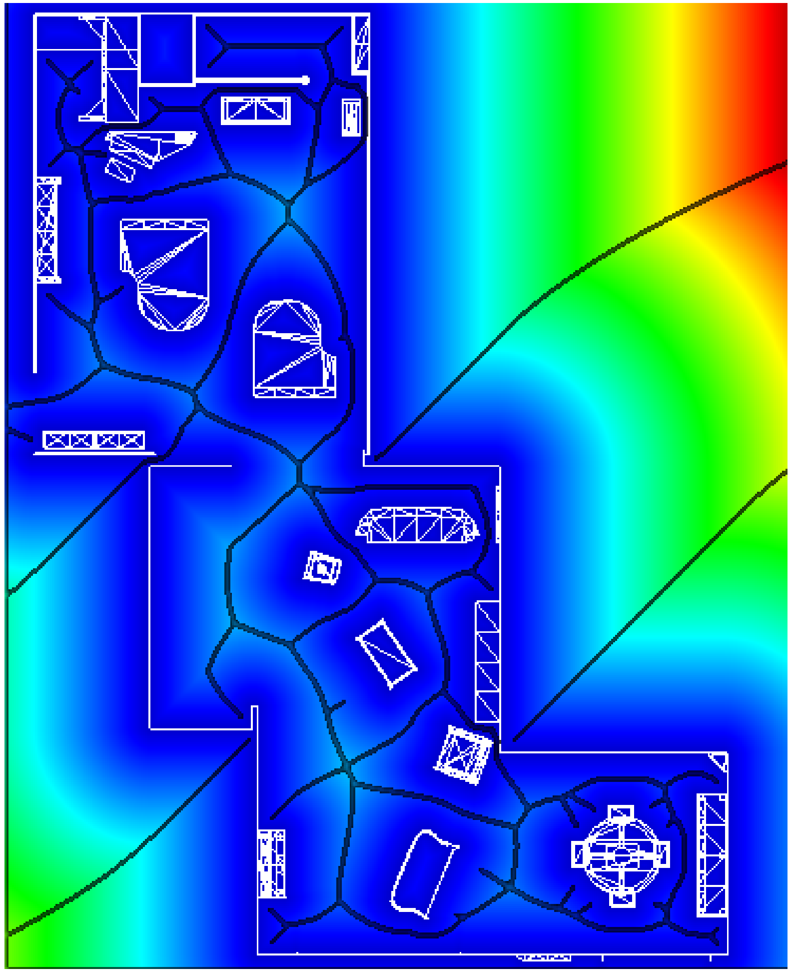


$$d_{\min} < d_{\text{coarse}}$$

Distance Transforms and Voronoi Diagrams

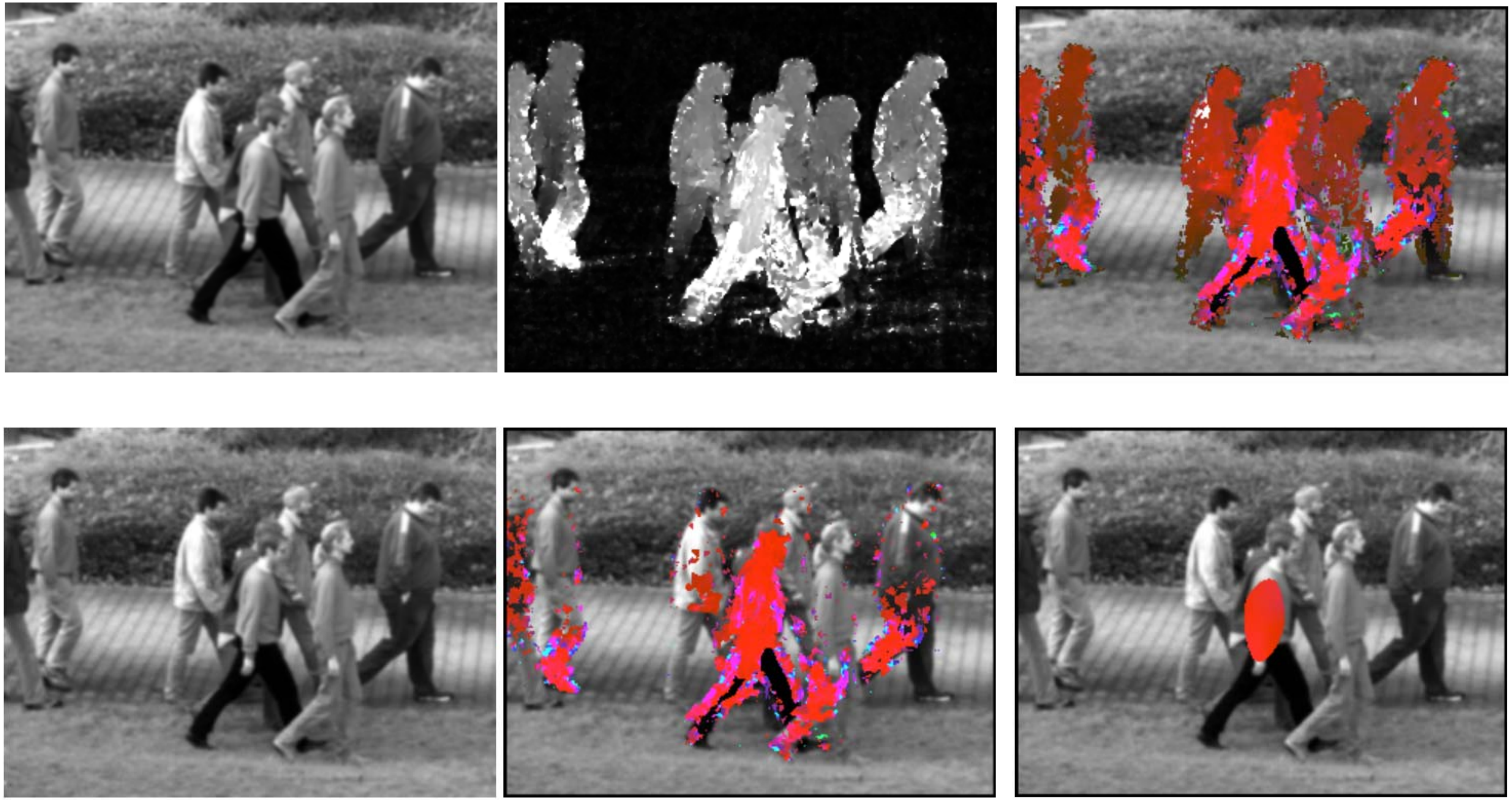






⬅ Generalized weighted Voronoi diagram

Motion estimation by an eigenvector analysis of the spatio-temporal tensor



Motion estimates as weighted least square minimizers

Input image sequence $u(\xi): \Xi \rightarrow [0,1], \quad \Xi := (\Omega, \Re^{+}), \;\; \xi := (x,t)$

Brightness change constraint equation, one equation two unknowns

$$0 = \frac{du}{dt} = \left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial t}\right) \cdot \left(\frac{\partial x}{\partial t}, \frac{\partial y}{\partial t}, 1\right)^{T} =: d^{T} p$$

Local continuity assumption of the flow p gives the minimization problem

$$\int_{\Xi} w(\xi - \xi')\left(d^{T}(\xi')\, p(\xi')\right)^{T}\left(d^{T}(\xi')\, p(\xi')\right) d\xi' \approx p^{T}(\xi) J(\xi)\, p(\xi) \rightarrow \min$$

$$J(\xi) := \int_{\Xi} w(\xi - \xi') d(\xi') d^{T}(\xi')\, d\xi' \qquad w: \Xi \rightarrow [0,1] \;\text{ weight function}$$

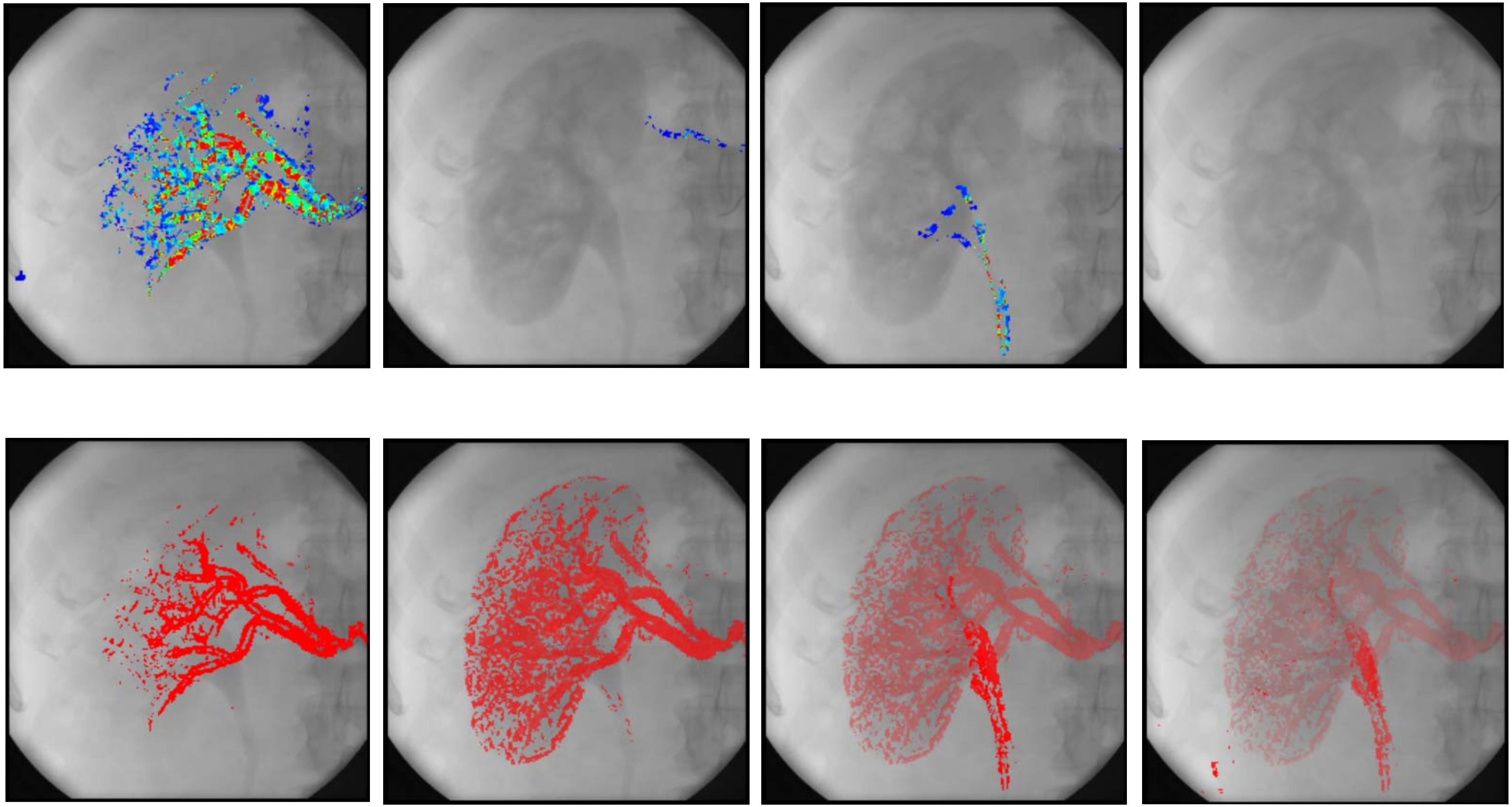
The diagonalization of the symmetric spatio-temporal 3x3 tensor

$$J = V \Lambda V^{T}, \quad V \text{ eigenvector basis, } \Lambda \text{ diagonal eigenvalue matrix}$$

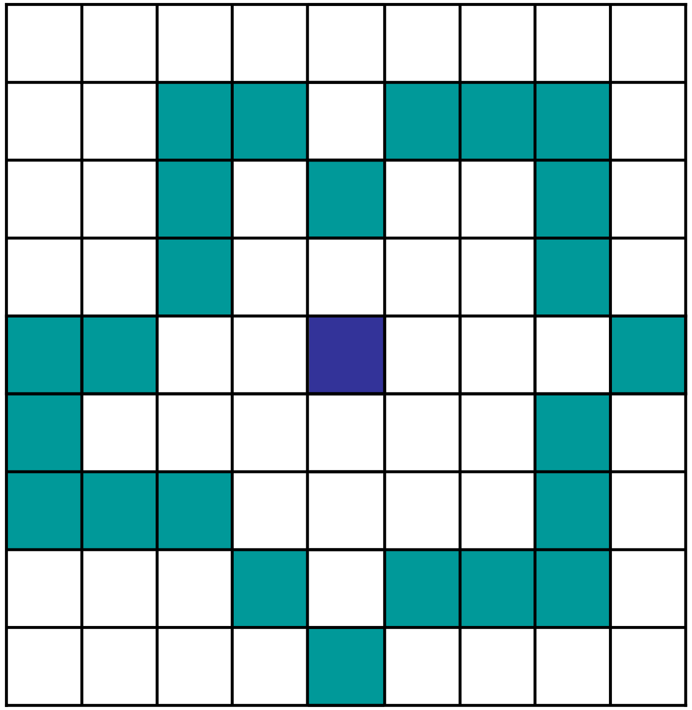
gives a motion estimation as the solution to the minimization problem

$$p = \left(\frac{\partial x}{\partial t}, \frac{\partial y}{\partial t}, 1\right)^{T} = \frac{(v_x, v_y, v_t)^{T}}{v_t}, \quad v \text{ eigenvector to smallest eigenvalue}$$

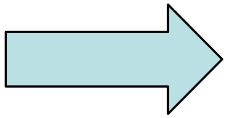
Motion estimation by an eigenvector analysis of the spatio-temporal tensor



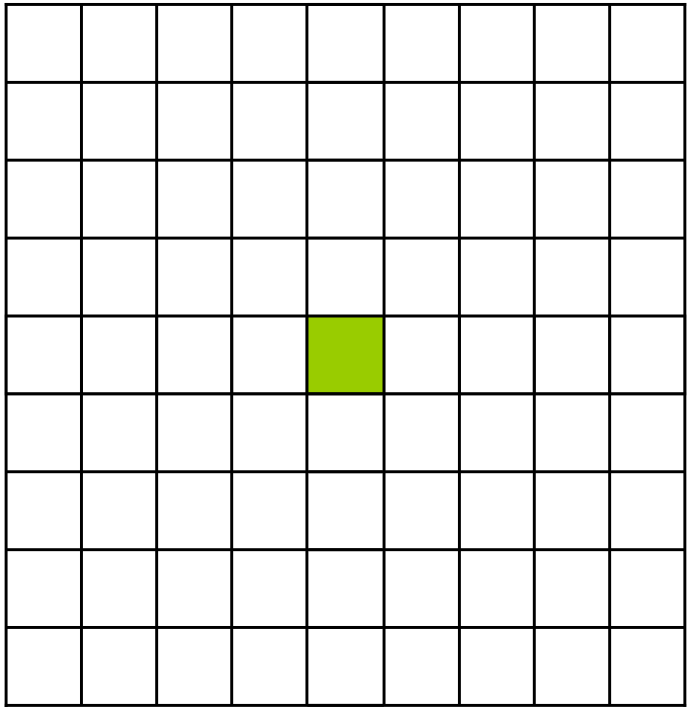
Global Gather Operation



$$F_h\left(\left(\overline{V}_\beta^{\,k}\right)_{\beta\in\gamma^k}\right)$$



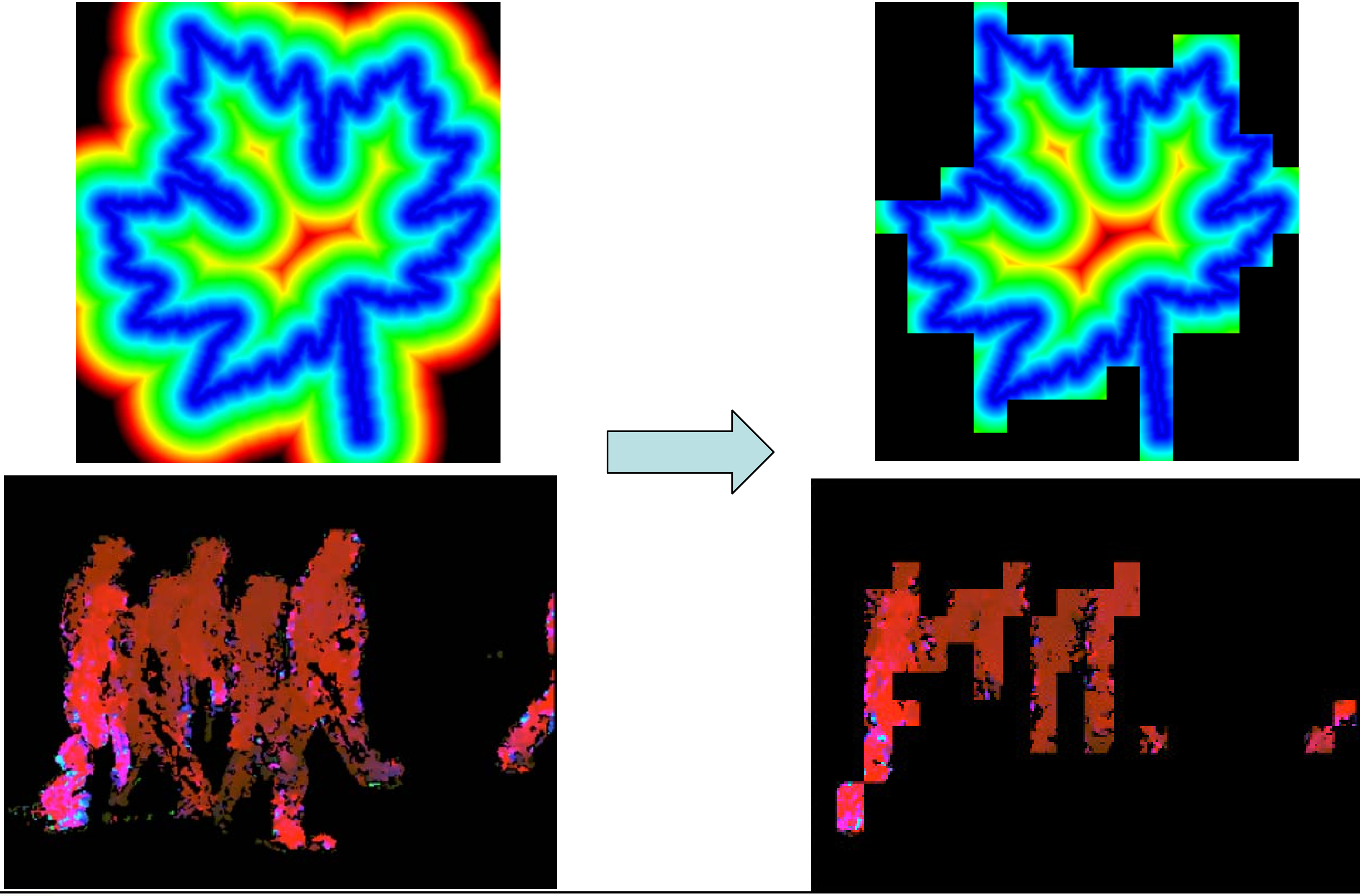
$$\sum_{\beta\in\gamma^k} A_{\alpha,\beta}\overline{V}_\beta^{\,k}$$



$$\left(\overline{V}_\beta^{\,k}\right)_{\beta\in\gamma^k}$$

$$\overline{V}_\alpha^{\,k}$$

Coarse Adaptivity – Tiling



Scientific Computing on GPUs Examples in Image Processing

Robert Strzodka

caesar research center
Bonn, Germany

University of Dortmund

Apr 2005