Mixed-Precision GPU-Multigrid Solvers with Strong Smoothers

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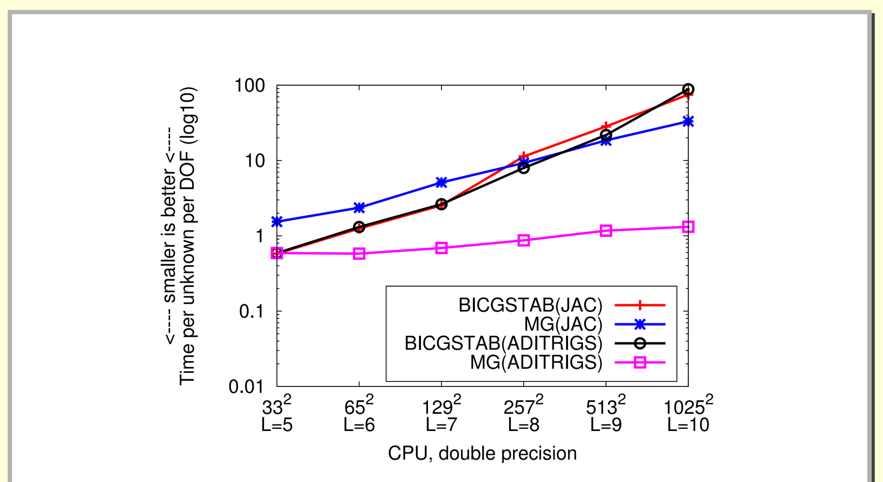
Motivation

- Sparse iterative linear solvers are **the** most important building block in (implicit) schemes for PDE problems
- In FD, FV and FE discretisations
- Lots of research on GPUs so far for Krylov subspace methods, ADI approaches and multigrid
- But: Limited to simple preconditioners and smoothing operators
- Numerically strong smoothers exhibit inherently sequential data dependencies (impossible to parallelise?)
- Strong smoothers required in practice: Anisotropies (mesh, operator), localised nonlinearities from the PDEs etc. increase ill-conditioning of the systems drastically
- Multigrid is asymptotically optimal, all other iterative schemes suffer from h-dependencies
- In our context: Multigrid = geometric multigrid

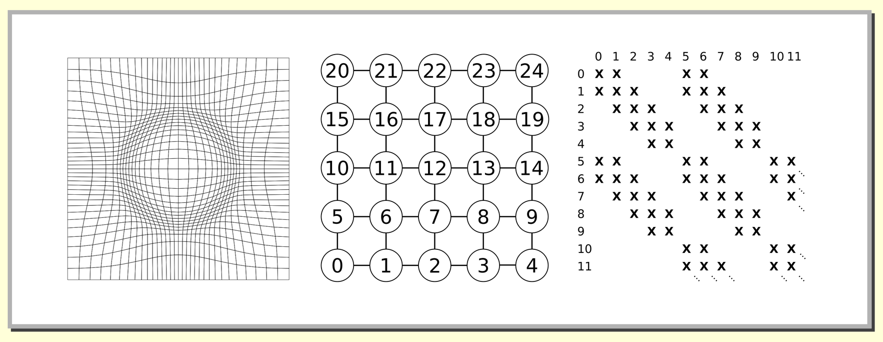


Example: Time (in ms) per unknown per digit, double precision, generalised Poisson problem with anisotropic diffusion, serial CPU test. **Only** MG with a strong smoother performs optimally.

⇒ Goal: Robust multigrid solvers for ill-conditioned sparse linear systems

Generalised Tensor Product Grids

Tensor product meshes $\neq$ cartesian meshes

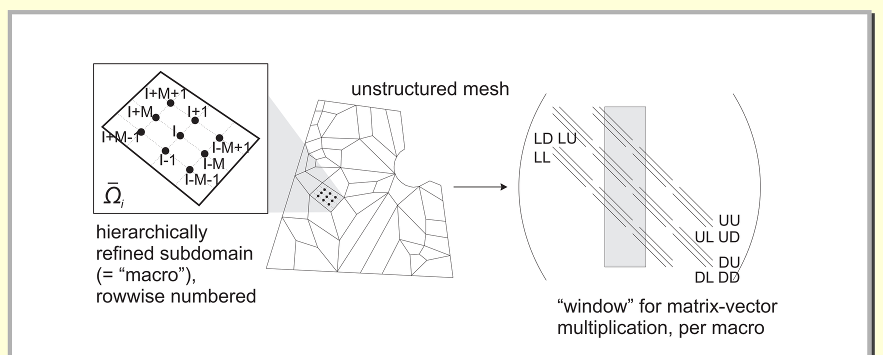


Row-wise numbering of the unknowns

⇓

Banded matrix, fixed structure but varying coefficients

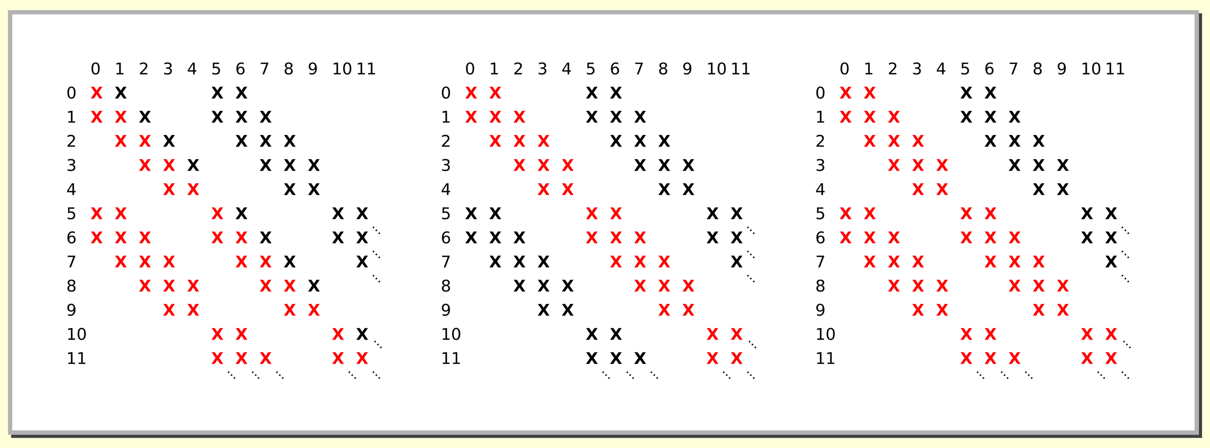
The Bigger Picture



Massively parallel MPI solver, unstructured collection of structured subdomains. This poster addresses the local component, see references for how this is included in the large-scale solver.

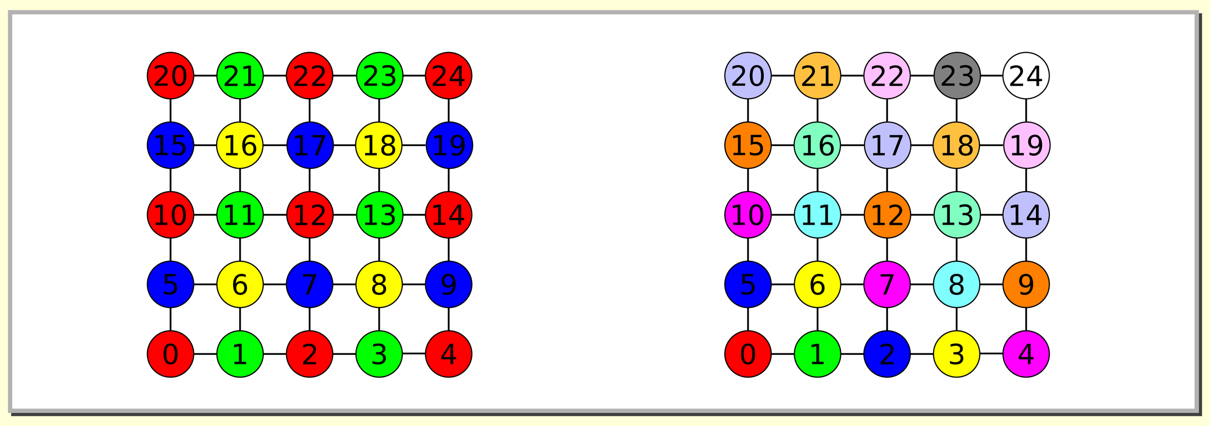
Preconditioner Construction

Use 'easily invertible' subsets of the system matrix as preconditioner, from left to right: Matrix bands used for Gauß-Seidel (GS), tridiagonal (TRIDI) and combined (TRIGS) preconditioner.



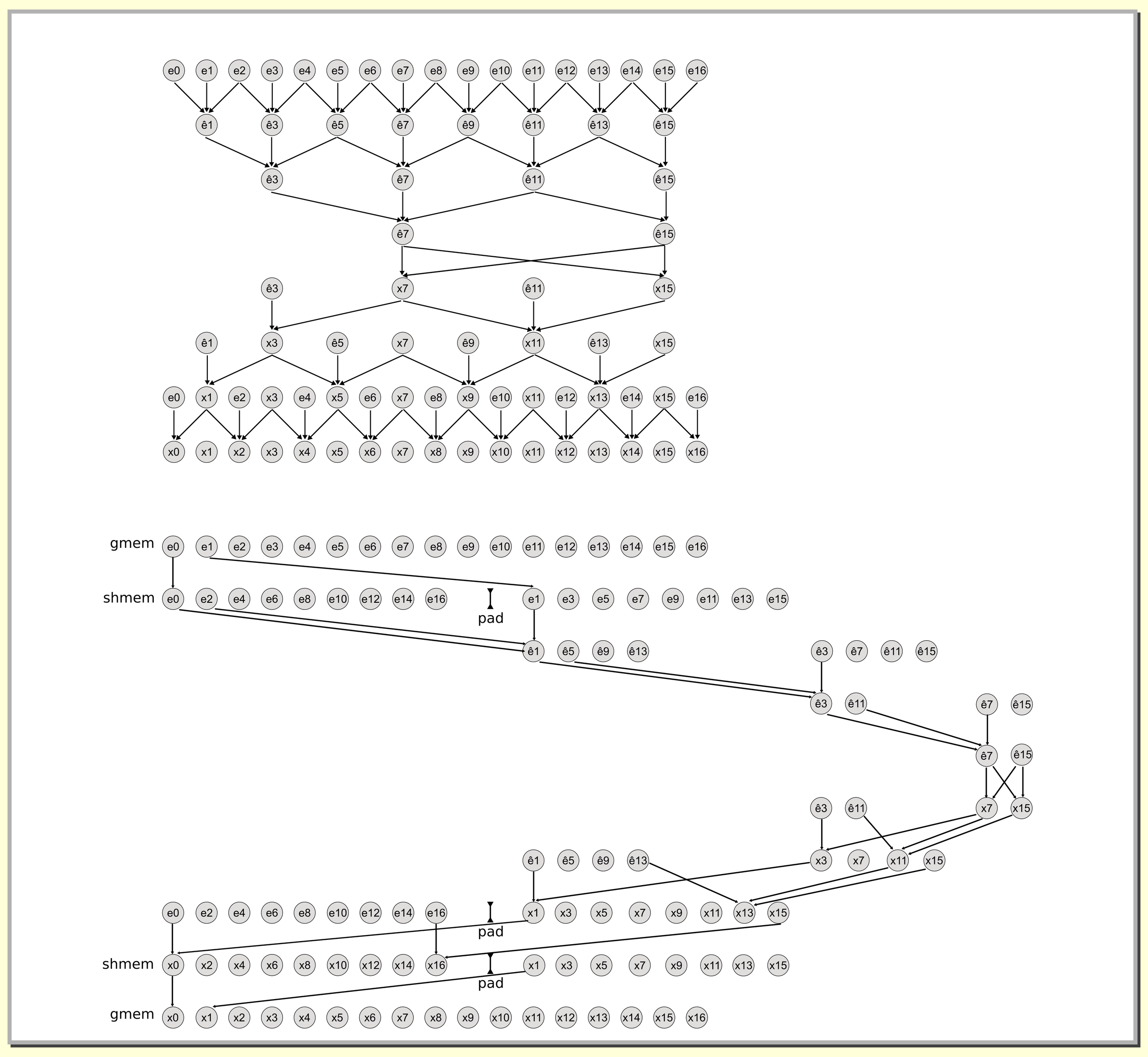
Multicolored Gauß-Seidel

Inexact parallelisation (decouple sequential dependencies into independent sweeps corresponding to a renumbering) yields more independent work than exact wavefront approach: Parallelism in multicolouring (left), lack of parallelism in natural ordering GS (right). Fuse two colours into one kernel to maximise coalescing.



Tridiagonal Solves

Parallelisation via cyclic reduction (exact, stable, cost-efficient). Challenge: Classical approach (top) parallelises operations and not memory accesses. Novel data placement (bottom) 2x-4x faster.



Combination: TRI-GS

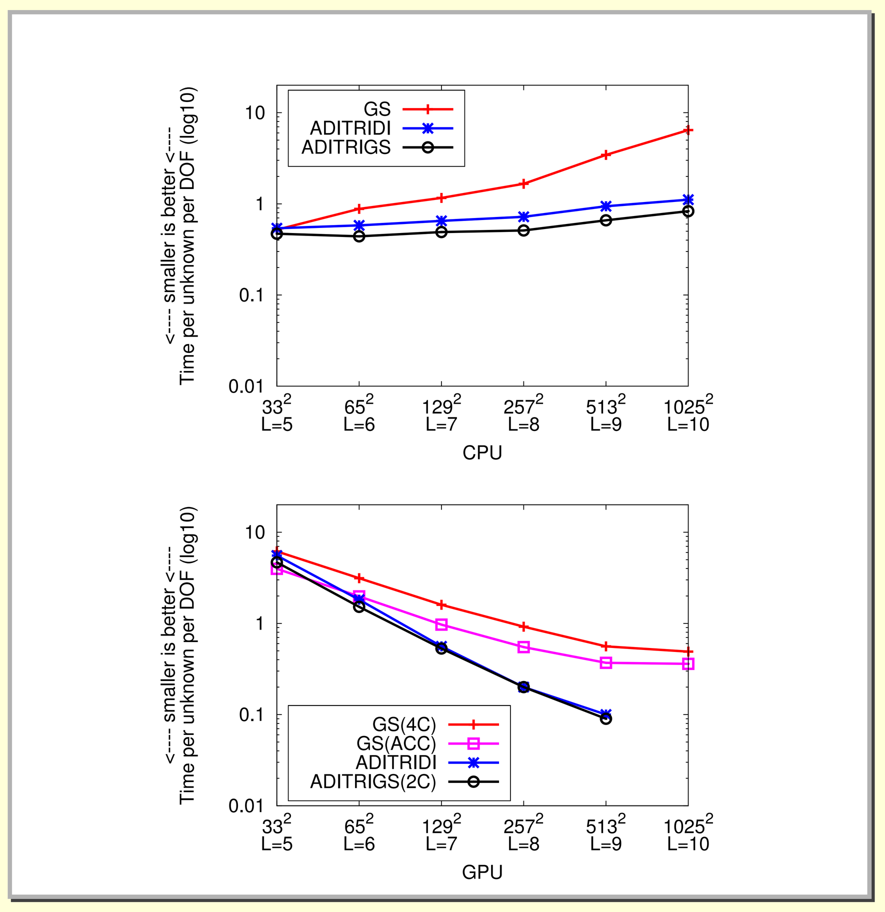
Combination of multicoloured GS and novel, fast TRIDI. Sequential: GS-shift of solution from previous row to RHS, tridiagonal solve. Parallel: New multicoloured row scheme balances numerical performance and parallelism.

Alternating Direction Implicit Method

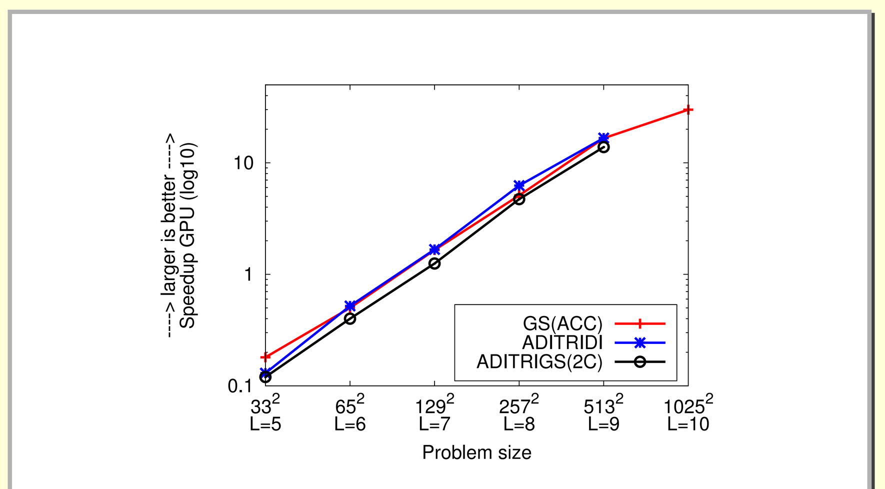
Preconditioner construction depends on the numbering of the unknowns. To resolve anisotropies in different directions, multiple numbering variants are applied in succession and we speak of an Alternating Direction Implicit Method.

Numerical Results

Generalised Poisson problem with anisotropic diffusion, mixed precision multigrid solvers: Time (in ms) per DOF per gained digit on CPU (top) and GPU (bottom).



GPU Speedup



Speedup calculated based on data in the box above

Current Work

Extension to unstructured patches: Treat prolongation and restriction in multigrid as SpMV (independent of FE space and dimension). Speedup of 10-15x vs. Nehalem Quadcore. Challenge: Strong smoothers!

References

[1] Dominik Göddeke. *Fast and Accurate Finite-Element Multigrid Solvers for PDE Simulations on GPU Clusters*. PhD thesis, Technische Universität Dortmund, Fakultät für Mathematik, May 2010. http://hdl.handle.net/2003/27243.

[2] Dominik Göddeke and Robert Strzodka. Cyclic reduction tridiagonal solvers on GPUs applied to mixed precision multigrid. *IEEE Transactions on Parallel and Distributed Systems, Special Issue: High Performance Computing with Accelerators*, 22(1):22–32, January 2011.

[3] Dominik Göddeke and Robert Strzodka. Mixed precision GPU-multigrid solvers with strong smoothers. In Jack J. Dongarra, David A. Bader, and Jakub Kurzak, editors, *Scientific Computing with Multicore and Accelerators*, chapter 7. CRC Press, January 2011. to appear.