technische universität dortmund

fakultät für mathematik

Mathematisches Kolloquium

am Mittwoch, **15. April 2026**

um **16.15 Uhr:**

Dr. Christoph Lohmann

Fakultät für Mathematik

Discretely divergence-free Q_2-P_1 finite elements for incompressible flow problems

A geometric multigrid approach for solving the three-dimensional Navier-Stokes equations

Der Vortrag findet in deutscher Sprache statt.

Der Vortrag findet im **Hörsaal E29 im Mathematikgebäude** statt.

Interessierte sind herzlich eingeladen.

Das Dekanat der Fakultät für Mathematik lädt herzlich zu diesem Vortrag ein.

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Discretely divergence-free Q_2-P_1 finite elements for incompressible flow problems

A geometric multigrid approach for solving the three-dimensional Navier-Stokes equations

Abstract:

This talk presents a geometric multigrid algorithm for the incompressible Navier-Stokes equations based on a discretely divergence-free variant of the well-known Q_2-P_1 finite element pair. The key idea of this discretization technique is the construction of vector-valued basis functions for the velocity field that automatically satisfy the discrete incompressibility condition. As a result, the saddle-point structure of system matrices commonly encountered in mixed finite element formulations is eliminated. Moreover, the overall problem size is reduced and the computation of the pressure variable is postponed to an inexpensive post-processing step. This allows for the design of efficient solution strategies that do not have to deal with saddle-point structures.

The talk outlines the main ideas behind discretely divergence-free finite elements and discusses possible challenges associated with this approach. In particular, we consider complex geometries in which the distribution of fluxes between different branches of the domain is not known a priori. In such cases, we argue that so-called 'global' finite element functions must be introduced to predict the correct flow behavior. Finally, a geometric multigrid solver is presented that efficiently incorporates the influence of these 'global' finite element functions. Numerical examples illustrate the performance of this solution strategy for three-dimensional flow simulations.

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