Bézout domains and Pythagoras numbers of fields

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Abstract

For a field F, the *Pythagoras number* of F, denoted by $p(F)$, is the minimal number m of squares such that any sum of squares in F is a sum of m squares in F. For several fields, an upper bound for the Pythagoras number of the form $2^n + 1$ has been found. For example, let K be a field such that $p(E) \leq 2^n$ for every algebraic function field E over K. Then $p(F) \leq 2^n + 1$ for every algebraic function field F over $K(\!(t)\!)$. This was shown by Becher, Grimm and Van Geel in 2014. In the same paper, they also proved that $p(F) \leq 3$ for an algebraic function field F over $\mathbb{R}(\!(t_1)\!)(\!(t_2)\!)\dots(\!(t_n)\!)$. Another example of such a bound was given by Yong Hu, who showed in 2015 that $p(F) \leq 3$ for any finite extension F of $\mathbb{R}(\!(t_1, t_2)\!)$.

The original proofs of these results were based on a local-global principle available for $2^n + 2$-dimensional quadratic forms. It is possible however to rely only on a local-global principle for Pfister forms, which is easier to obtain, by exploiting a common feature of these fields: each of them has a subring that is a semilocal Bézout domain, and which contains all the information about the sums of 2^n squares. In the talk it will be shown that if a field F has such a subring, then $p(F) \leq 2^n + 1$. Finally, a sufficient condition for a field to have such a subring will be presented.