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Multimaterial Moment-of-Fluid Interface Reconstruction [‡]

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1 Background and Rationale

In modeling multimaterial flows, materials in each computational cell c are represented by set of non-overlapping pure-material polygons ω_m^c (m is material index) such that $\cup_m \omega_m^c = c$. The standard requirement is that the volume of the reconstructed material polygon is equal to the corresponding target material volume, V_m^c - $|\omega_m^c| = V_m^c$. Target material volumes are obtained as a result of volume tracking. In this abstract we will assume that target material volumes are given and $\sum_m V_m^c = V_c$, where V_c is the volume of the entire multimaterial (MM) cell. Pure-material polygons, ω_m^c, are obtained by sequential cuts from cell c. That is, there is an explicit assumption that there is a prescribed order of materials $m_1, m_2, , \ldots m_n$, where n is the total number of the materials in cell c. The material polygon $\omega_{m_1}^c$ is obtained by cutting off cell c, which is equivalent to the intersection of cell c with some half-plane - P_{m_1} such that $|c \cap P_{m,1}| = V_m^c$. Next, to obtain material polygon $\omega_{m_2}^c$ one must intersect what is left from cell c after cutting off $\omega_{m_1}^c$, that is, $c \setminus \omega_{m_1}^c$ with another plane and so on. The points $\mathbf{r} = (x, y)$ in half-plane P_m^c, which defines material polygon ω_m^c satisfy the following inequality $\mathbf{n}_m^c \cdot \mathbf{r} + d_m^c \geq 0$, where $\mathbf{n}_m^c$ is the unit outward normal and d_m^c is the signed distance from origin to the corresponding straight line $\mathbf{n}_m^c \cdot \mathbf{r} + d_m^c = 0$. The normal $\mathbf{n}_m^c$ is defined by one parameter φ_m^c which is the angle between the normal and one of the coordinate axes. For a given normal, $\mathbf{n}_m^c$, the parameter d_m^c is uniquely defined by the requirement that the volume of the corresponding cutoff is equal to target volume.

In the interface reconstruction (IR) methods used in the standard volume-of-fluid (VOF) methods, one first estimates the direction of the normal $\mathbf{n}_m^c$ using information about target volumes (volume fractions) of material m in the cell c itself and its neighbors. That is, communication with neighboring cells is required on this stage. For more than two materials, one needs to specify the order of materials and in VOF methods it has to be a global order (that is, the same order for all multimaterial cells). But even in this case there is no clear and meaningful procedure for how to estimate a normal for each material. Moreover, for VOF methods there is no qualitative measure to compare results of IR with different material ordering.

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2 Moment-of-Fluid Interface Reconstruction

The Moment-of-Fluid (MoF) method, [1, 2] can can be characterized as a moment tracking method in which one tracks the zero moment - $M_{0,m}^c = V_m^c = \int_{\omega_m^c} 1\, dV$ (as in VOF method) and also the first moment - $\mathbf{M}_{1,m}^c = \int_{\omega_m^c} \mathbf{x}\, dV$. We assume that for each cell c we are given a set of target zeroth moments $M_{0,m}^c$, such that $\sum_m M_{0,m}^c = V_c$, and also a set of first moments $\mathbf{M}_{1,m}^c$. We will call $\mathbf{M}_{1,m}^c$ - by the *reference* first moment for material m. In the MOF method the IR is performed separately for each MM cell - there is no information from neighboring cells required. The construction of each cutoff is formulated as a constrained optimization problem: minimize first moment discrepancy between the reference first moment and the first moment of the actual cutoff polygon ω_m^c subject to an exact preservation of material volume. In 2D, this is optimization with respect to one variable φ_m^c, which defines normal. The optimal material ordering in cell c produces the smallest overall discrepancy for first moments for all material polygons.

3 Applications

Our primary application for MOF is MM Arbitrary Lagrangian-Eulerian (ALE) methods for compressible flows. In Fig. 1 we demonstrate superiority of MOF IR method with respect to traditional VOF IR for the ALE modeling of three-material vortex formation problem.
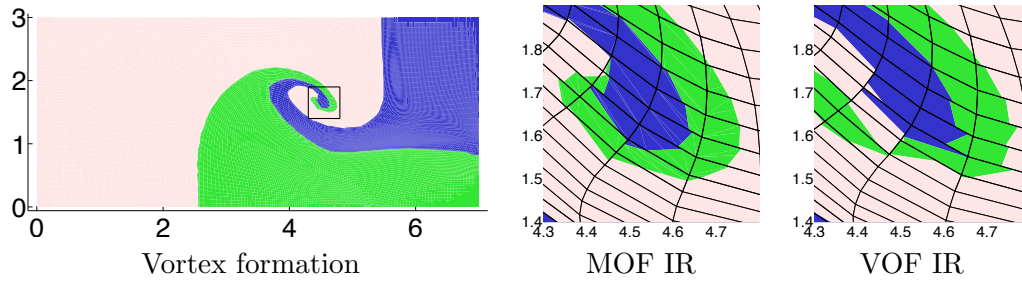

Figure 1: Vortex formation - comparison of MOF IR and VOF IR.

In Fig. 2 we show result of ALE modeling of Kelvin-Helmholtz instability.
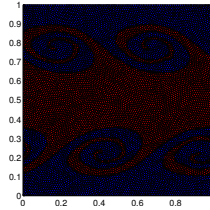

Figure 2: MOF IR for Kelvin-Helmholtz instability problem

In [3] one can find application of MOF for modeling incompressible flows.

The details of MOF method and examples of other applications will be presented in the lecture.

References

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