Minimising the Canham-Helfrich energy

Sascha Eichmann

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Abstract

Helfrich (1973) and Canham (1970) introduced the following geometric curvature energy to modell the shape of human red blood cells. The idea is that the two dimensional boundary layer $\Sigma \subset \mathbb{R}^3$ of such a cell minimises

$$\int_\Sigma |H - H_0|^2 \, dA$$

under suitable constraints on e.g. the enclosed volume and surface area. Here H is the scalar mean curvature of Σ and $H_0 \in \mathbb{R}$ is a parameter called the spontaneous curvature, which represents an asymmetry in the boundary layer of the cell. This induces a prefered curvature of the cell.
The goal is now to implement the direct method of variational calculus to show existence. Compactness under varifold convergence can be easily obtained, but unfortunately lower-semicontinuity of the Helfrich energy under this varifold convergence is in general not correct by a counterexample of Große-Brauckmann (1993). Nevertheless we can actually show a lower-semicontinuity estimate for the minimising sequence itself. We will demonstrate the main ideas of the proof.
In the last part of the talk, we will discuss directions for future research in this area, i.e. some open problems and some modifications of the Canham-Helfrich energy.