Diffusion-driven unbounded growth and dynamical spike patterns in reaction-diffusion-ode models

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In this talk we explore a mechanism of pattern formation arising in processes described by a system of a single reaction-diffusion equation coupled to ordinary differential equations. Such systems of equations arise, for example, in modeling of interactions between cellular processes and diffusing growth factors. Our theory applies to a wide class of pattern formation models with an autocatalytic non-diffusing component.

We show that the lack of diffusion in some model components may lead to singularities which result in instability of all regular stationary patterns. Interestingly, the degeneration of the system yields a continuous spectrum of the linearization operator, which contains positive values. We show that, under some conditions, also all discontinuous stationary solutions are unstable. However, in numerical simulations, solutions having the form of periodic or irregular spikes are observed. We explain this phenomenon using a shadow-type reduction of the reaction-diffusion-ode model. For the resulting system of integro-differential equations, we prove convergence of the model solutions to singular unbounded spike patterns, which location depends on the initial condition.

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