The Effective Putinar's Positivstellensatz on the Hypercube

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The Positivstellensätze of Schmüdgen and Putinar show that any polynomial f positive on a compact semialgebraic set can be represented using sums of squares. Recently, there has been great interest in proving effective versions of these results, namely to show bounds on the required degree of the sums of squares in such representations. The problem is particularly interesting because such representations are non-stable, i.e. it is possible to find families of polynomials with constant degrees, that require arbitrarily large degrees for their Putinar-type and Schmüdgen-type representations.

In this talk, we start recalling the existing results about the lower and upper degree bounds for these Positivstellensätze, and we then focus on Putinar-type representations for the hypercube $[-1,1]^n$.

We present an upper degree bound of the order $O(f_{\max}/f_{\min})$, where $f_{\max}$, $f_{\min}$ are the maximum and minimum of f on $[-1,1]^n$, respectively. Previously, specialized results of this kind were available only for Schmüdgen-type representations. We also show a lower degree bound in $\Omega(\sqrt[8]{f_{\max}/f_{\min}})$. This is the first lower bound for Putinar-type representations on a semialgebraic set with nonempty interior described by a standard set of inequalities.

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