An Introduction to the Stability of Positivity-Preserving Schemes

Abstract

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Positivity-preserving schemes are of high importance and gained more and more attention in recent years. However, they often result in nonlinear iteration schemes even when applied to a linear test problem $\mathbf{y}' = \mathbf{A}\mathbf{y}$. Hence, a stability analysis becomes more complex. Moreover, the scheme often has to preserve additionally other properties of the analytic solution such as steady states and linear invariants. As a result, steady states of the test problem become non-hyperbolic fixed points of the method. In general, the stability analysis thereby turns into a case by case study.

In this talk, we present sufficient conditions for stability as well as local convergence, and hence, overcome this case by case study for a large class of steady state preserving schemes to which reasonable positivity-preserving schemes belong.

To illustrate the theoretical results, we consider so-called modified Patankar-type schemes that are positive and conservative numerical methods when applied to a positive and conservative production-destruction system. Applying the main theorem we are able to define stability functions which are the basis for the stability analysis. Finally, numerical experiments are presented to confirm the theoretical results.

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References

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