What is a limit of structure preserving schemes for compressible flows ?

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Many problems in computational fluid dynamics are described via the compressible Euler or Navier-Stokes (NS) equations. Recently, dissipative weak (DW) solutions have been introduced as a generalization to classical solution concepts. In a series of works, DW have been established as a meaningful concept from the analytical and numerical point of views. They do not have to fulfill the equations weakly but up to some defect and oscillations measures. They are a natural extension of classical solutions since DW solutions coincide with them if either the classical solution exists.
In this talk, we give an introduction to the concept of DW solutions and summarize recent results inside this framework. We consider, in particular, high-order FE schemes and prove convergence to DW solutions for the multi-dimensional Euler equations. To this end, it is crucial that structure preserving properties, such as positivity preservation and entropy inequality hold, and the schemes are consistent with the underlying PDE. We show how to ensure these properties and in numerical simulations, we verify our theoretical findings. Importantly, the applicability of DW solutions extends beyond the Euler and Navier-Stokes equations. As we conclude, we offer a glimpse into the future, discussing potential developments in this evolving field.

References

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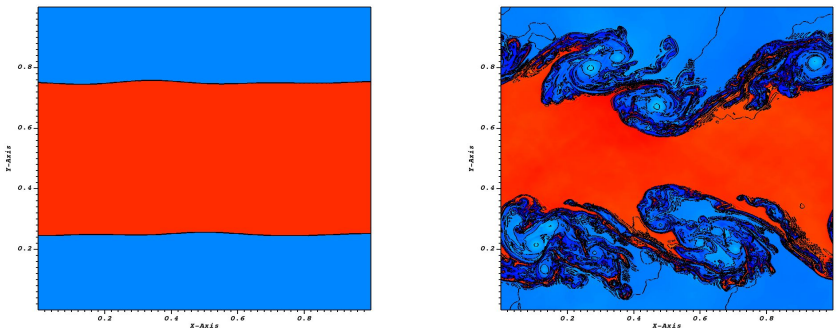

Figure 1: Kelvin-Helmholz instability.

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