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“Division of Fractions” in Japanese elementary school

Today, many researchers and teachers point out that education of “Division of Fractions” is the issue in Japanese elementary school. The issue point is that children can’t understand the meaning of calculation of it though they can memorize calculation method of it easily. That is, the present contents of it are not enough for children to understand it. Therefore, we tried to make original teaching contents of Division of Fractions. And we tried to teach the contents for 6th grade elementary school children (Japanese elementary school system : from 1st grade (6 years old) to 6th grade(12 years old)). So, we would like to show the results in this paper.

Teaching Contents of Division of Fractions in Japan

Today, contents of Division of Fractions are taught at 6th grade in elementary school in Japan. The teaching contents in Japanese 6th grade textbooks are treated it as follows. (All Japanese text books are based on the course of study. Therefore, all Japanese children study very similar contents though school mathematics textbooks are published by six publishing companies.)

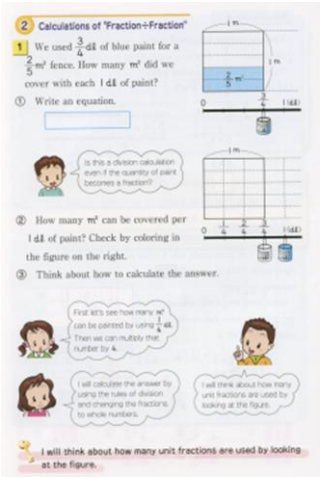


Fig.1 an introduction part & a part of hints for solving the question

The explanation of Division of Fractions in school textbook (Masakazu Aoyagi et al (ed.), (2005)) is as follows (the upper section of Fig.1).

(1)An introduction part: First, there is a question (Children try to found a good idea for solving it in this part).

The Question is "We used 3/4 dL of blue paint for a 2/5 m^2 fence. How many m^2 did we cover with each 1dL of paint?”

(As you know the question is very connected real life. So, many contents of mathematics are connected real life in Japanese school.)

(2)A part of hints for solving it: Next, there are three children’s ideas (the lower section of Fig.1) (Children try to understand each idea in this part).

(3)A part of solving the question: There are three methods are based on children's ideas for solving the question.

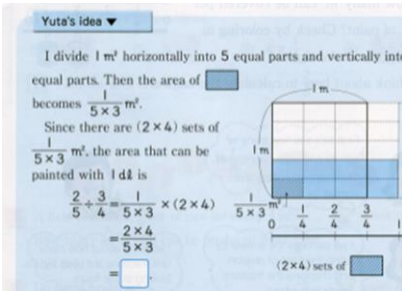
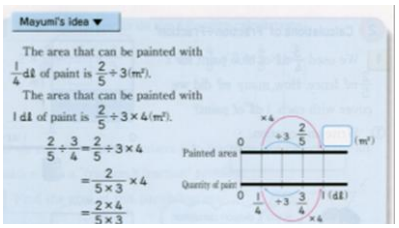


Fig.2 method 1

Method1 is as follows (Fig.2).

"I divide 1 m^2 horizontally into 5 equal parts and vertically into 3 equal parts. Then the area of ■ becomes 1/ (5×3) m^2. Since there are (2×4) sets of 1/(5×3)m^2, the area that can be painted with 1dL is 2/5÷3/4=1/(5×3)×(2×4)=(2×4)/(5×3)".



Method2 is as follows (Fig.3).

"The area that can be painted with 1/4dL of paint is 2/5÷3 (m^2). The area that can be painted with 1dL of paint is 2/5÷3×4(m^2).

2/5÷3/4=2/5÷3×4=2/(5×3)×4 = (2×4)/(5×3)".

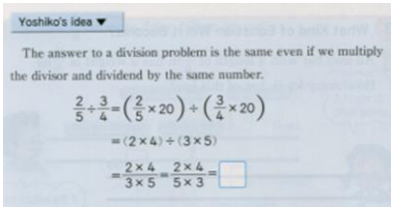


Fig.4 method 3

Method 3 is as follows (Fig.4).

"The answer to a division problem is the same even if we multiply the divisor and dividend by the same number.

2/5÷3/4=(2/5×20)÷(3/4×20)=(2×4)÷(3×5) =(2×4)/(3×5)=(2×4)/(5×3)".

These explanations are proof of calculation method of Division of Fractions. Method1 and 2 are very inductive and method3 is similar deductive (but it is not completing deductive). Thus there are detail explanations in a textbook. But many children can't understand these contents. That is, only few students can explain the method of calculation of Division of Fractions.

Original Teaching Contents of Division of Fractions

And the above issue point has been noted for a long time. Nevertheless, why do many teachers teach children these contents for a long time in Japan? The reason is as follows. The present school textbook has almost contents of inductive thinking. So, 5,6th graders (11, 12 years old children) seldom study the contents of deductive thinking in Japan.

But we think that 5,6th graders must study the contents of deductive thinking, because it is just time for them to acquire abstract thinking. In other

words, they can probably understand contents of deductive thinking more easily than contents of inductive thinking.

For that reason, we made new contents of Division of Fractions to solve the issue. And it was based on deductive thinking. Then, we taught 6th graders it. The practice of the teaching is as follows.

[Practice of the teaching]

1) Teaching object: Public elementary school 6th graders (29 children) in Osaka, Japan

2) Practical side of the teaching: Date: May, 2003, Time: 5 school hours (1 school hour = 45 min.)

3) Teaching Contents

The teaching contents have 2 units. One is "Multiplication of fractions". The other is "Division of fractions".

Fig.5 algebraic property of multiplication

<Multiplication of fractions>

"unit fractions","a fraction×an integral number","a fraction×a unit fraction","a fraction×a fraction","a fraction×a fraction×a fraction","a reciprocal number","property of a reciprocal number","algebraic property of multiplication(Fig.5)"

Fig.6 relation of multiplication and division

<Division of fractions>

"relation of multiplication and division (Fig.6)", "division of fractions "

Explanation of method of Division of Fractions is as follows.

• $3 / 5 \div 1 / 3 = \square \rightarrow \square \times 1 / 3 = 3 / 5 \rightarrow \square \times 1 / 3 \times (3 / 1) = 3 / 5 \times (3 / 1) \rightarrow \square = 3 / 5 \times 3 / 1 \therefore 3 / 5 \div 1 / 3 = 3 / 5 \times 3 / 1$

4) Results

We tested the contents of Division of Fractions to the children after our last teaching on the same day. The results of the test are that over about 90% children could understand it. That is, many children became to be able to prove a calculate method of Division of Fractions. And many children be-

came to be able to calculate numerical calculations of Division of Fractions too.

Discussion

In this paper, we tried to solve the issue of Division of Fractions in Japanese elementary school. So we made new contents that were based on deductive thinking. And we verified whether our made teaching contents is effective or not. And, we tried a practice for 6th graders by using our made contents.

The results clearly show the next points.

6th graders could understand our made contents that was based on deductive thinking. That is, 6th grader can understand more easily deductive proof than inductive proof. Therefore, the results should suggest that the issue point of Division of Fractions in Japanese elementary school is that the contents are not based on deductive thinking but based on inductive thinking.

But we couldn't test them after that. So, we don't know that they keep the understanding. Therefore, we must investigate it. And we would like to create new various another teaching contents for deductive thinking.

Acknowledgment

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Literature

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