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An Educational Significance of the Sundial and Examples of Teaching in Mathematical Modelling

1. Introduction

The sundial is used the teaching materials only to teach time in elementary school in Japan. But, the sundial has much educational significance. In this paper I would like to study on the educational significance of the Sundial in mathematics education from several points of view; the improvement of geometry education, the development of mathematical modelling, the cultural history of mathematics and the interrelationship of mathematics and science.

The Sundial has three types as follows;

- The horizontal sundial which the board of time lines keeps horizontality and we often find in Japan.
- The vertical sundial which the board of time lines keeps vertical on wall and we often find in Europe.
- The equator sundial which the board of time lines keeps a plane parallel to the equator's plane and we sometimes find in China and Korea.

Fig.1 Horizontal sundial(Japan)

Fig.2 Vertical sundial(Germany)

Fig.3 Equator sundial(China)

2. Educational significance

1) Teaching time

We use sundial when we teach time. Children put a milk bottle on the paper and record its shadow par hour on the paper. They learn time depended on the moving of the Sun.

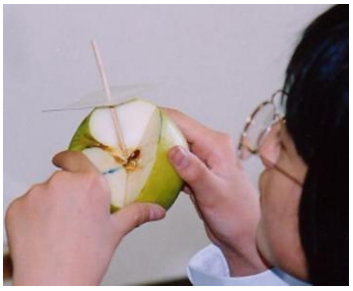

Fig.4 Studying latitude and longitude

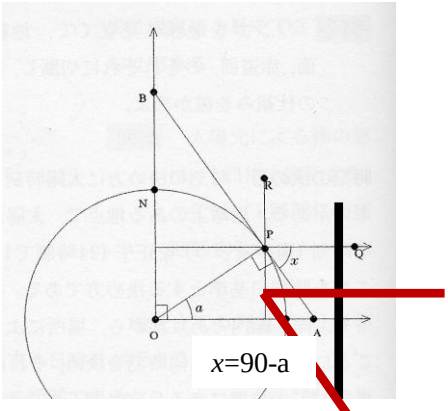

Fig.5 A principle of the equator sundial

2) The improvement of geometry education

Principle of sundial includes plane, solid and space geometry. For this research we taught the equator sundial to pupils of 1st grade in Junior high

school. At first the pupils studied definition of latitude and longitude by cutting an apple. Next, they studied principle of an equator sundial using learned proof of geometry. At last, pupils made original sundial. It is simple and easy for them to draw time lines on the board of equator sundial, because the sun revolves round the gnomon for one day, then they only drew the time line of the radiating of 15 degrees out from the center.

3) The development of mathematical modelling

The sundial exists around pupils on daily life. But they don't understand the sundial's structure and the principle, and the interrelationship between mathematics and sundial. We would like to teach the sundial making mathematical model. Teaching Sundial is effective to teach developing from Elementary Model to Mathematical Development Model in mathematical modelling (Fig.6).

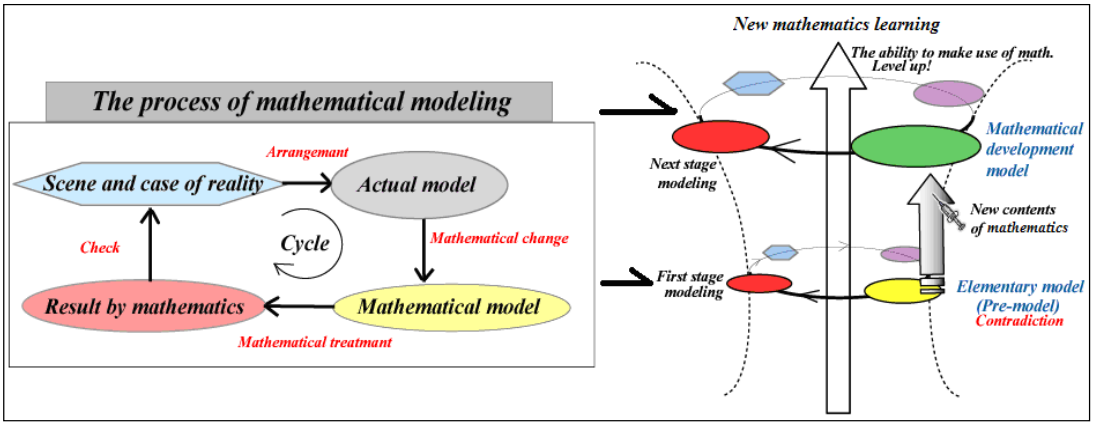

Fig.6 T.Kawasaki & S.Moriya (2011)

Sundial as teaching material is worth developing mathematics model. The understanding on principle and making Equator Sundial is the base for modelling. Time boards of Horizontal and Vertical Sundial are developed from the time board of Equator Sundial. At this time, it is different that mathematics which is used according to ages of pupils. For example, pupils in a primary school and junior high school use construction of a right triangle. And pupils in a high school use calculation using trigonometric function.

i) The developing to Horizontal sundial in class in junior high school

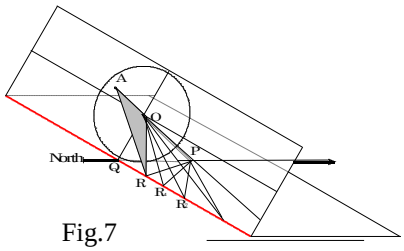

Fig.7

After pupils made the equator sundial, we gave them a new problem; "How are time lines drawn on the time board of the horizontal Sundial?" One solution by pupils is the following; the shadow of AP is RP, because shadow of AP in horizontal plane is line of intersection with △ARP and horizontal plane. They draw time lines at intervals of 15

degrees on the board which is parallel to equator plane. Next they extend these lines to line of intersection of two planes. And they draw the points of intersection, R, R1, R2, etc. At last, they connect these points and P. Because length of OQ, ∠Q=90-latitude, ∠O=90 => Draw △OQP, they could determine the length of PQ. When OQ is 5cm, then they take PQ is from 8.0cm to 8.3cm. And they made horizontal sundial (Fig.9). During the time, we can evaluate whether pupils understand on tests for congruent triangles.

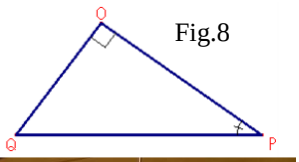

Fig.8


Fig.9 The work of pupil

ii) Estimating place from time lines of Horizontal sundial as real classroom in junior high school.

The mathematical model develops further when pupils research the latitude based on a time board of Horizontal Sundial. In Elementary Model, pupils used teaching materials prepared by the teacher. But in Mathematical Development Model, they used construction or trigonometric function and solved the problem by themselves. The rubbing in Fig. 10 is time board of sundial made by Mr. Hayashi who is very famous researcher on geography at 1792 in Edo period to make a contribution to Shiogama Shinto shrine in Shiogama city. We gave the pupils a new problem; "Can we use this time board in Shiogama of 38.3 degrees of the north latitude?" In Elementary Model, pupils traced the lines of the rubbing of this sundial to a transparent sheet B. Sheet A1, A2, A3... are the lines of the sundial in latitude 38, 36, 34, 32… degrees. They compared sheet A to sheet B, and they examined whether sheet B accord with sheet A. B accorded with A of latitude 34 degrees. In Mathematical Development Model, they thought of the right angled triangle which had been used to make the equator sundial. They could estimate the latitude of the rubbing by the method of geometric construction. Their solution is the following; at first, they determined O by this method. Next, Given Length of PQ, Length of OQ, Angle O is 90 degrees, then triangle is determined. They could know the angle of latitude. The latitude that they estimated was 31.0 to 34.1degrees. And the average was 33.3 degrees (Fig. 10).

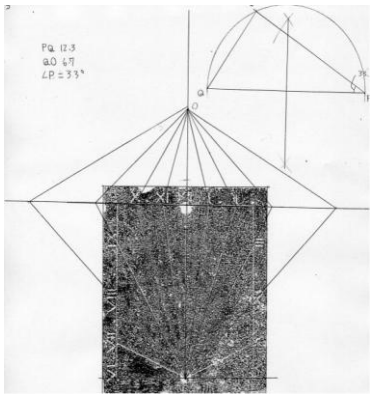
Fig. 10 The work of pupil

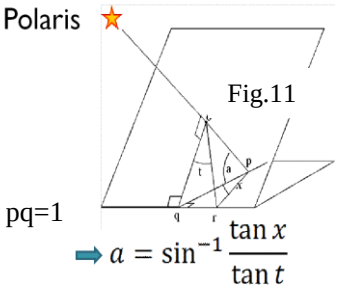

Fig.11

If pupils use trigonometric function like in Fig.11, then it is more mathematical development model than model of construction.

4) The cultural history of mathematics

The degrees of latitude which the horizontal sundial of Shiogama was used was estimated 33.3 degrees by pupils. But latitude of Shiogama is 38.3 degrees! Pupils inferred that this horizontal sundial was not produced to set at Shiogama, and was produced to set Nagasaki of 34 degrees of the north latitude. They thought that Mr. Hayashi maybe couldn't make original sundial to set at Shiogama, and he didn't know the principal of sundial. Pupils came to realize that mathematical researching of sundial was used to study history.

It is good problems for pupils to solve why shape of equator sundial is different, why there are sundials on the walls of East, West and North in London, and why Korea's sundials are similar to Chinese sundials

5) The interrelationship of mathematics and science.

Education in Japan draws a distinction between mathematics education and science education. But the sundial as an educational material has the interrelationship of teaching time's concept and moving the Sun, the interrelationship of teaching proofs of geometry and making the sundial, and explication on the interrelationship of the Earth and the Sun. If we relate mathematics education and science education, levels of mathematical contents will increase.

3. Conclusion

Why do we use the sundial? There are sundials all over the world. The sundial has common principles and differences in appearance. We can teach the sundial to pupils of all grades. The sundial includes Euclid's geometry and analytic geometry as geometry contents in three-dimensional shape. Pupils can experience the developed mathematical modelling through learning the sundial. The sundial connects mathematics, science and culture. Pupils can experience practical use of mathematics by using and making sundials. The sundial is a very good teaching material in Mathematics Classroom

References

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