technische universität
dortmund

16th GAMM-Seminar on Microstructures

TU Dortmund
January, 20-21, 2017

The **16th GAMM-Seminar on Microstructures** is hosted by the TU Dortmund from January 20 to January 21, 2017.

Location	All lectures take place in room 1.001 at the first floor of the Seminarraumgebäude 1, Friedrich-Wöhler-Weg 6, 44227 Dortmund.
Keynote Speakers	Grégoire Allaire (Ecole Polytechnique)
	Gilles Francfort (Université Paris-Nord)
	Marc Geers (TU Eindhoven)
	Bernd Schmidt (Augsburg University)
	Augusto Visintin (University of Trento)

Local Organizers

Postal Address:	16th GAMM-Seminar on Microstructures
	Prof. Dr. Ben Schweizer
	Maik Urban
	TU Dortmund
	Fakultät für Mathematik, Lehrstuhl 1
	Vogelpothsweg 87
	44227 Dortmund, Germany
Email:	gamm-micro@math.tu-dortmund.de
Website:	www.mathematik.uni-dortmund.de/lsi/Gamm-2017

Conference Office	Saskia Stockhaus
	In front of room 1.001 in the Seminarraumgebäude 1 or office M 635 in the mathematics building
Phone:	(+49 231) 755-3053

Important information

Start of the conference	Friday, 9:00
End of the conference	Saturday, 15:40
GAMM activity group meeting	Friday, 17:30 - 18:30, in room 1.004 of the Seminarraumgebäude 1
Informal dinner	Thursday, 19:30, at the restaurant *Hövels Hausbrauerei*, Hoher Wall 5-7, 44137 Dortmund
Conference dinner	Friday, 19:00, at the restaurant *View*, Leonie-Reygers-Terrasse, 44137 Dortmund
Lunch (Friday)	Canteen of the TU Dortmund, Campus Nord
Lunch (Saturday)	kostBar, Emil-Figge-Straße 40

We recommend the TU Dortmund-App for your smart phone (iOS and Android 4.x) for navigation on the campus.
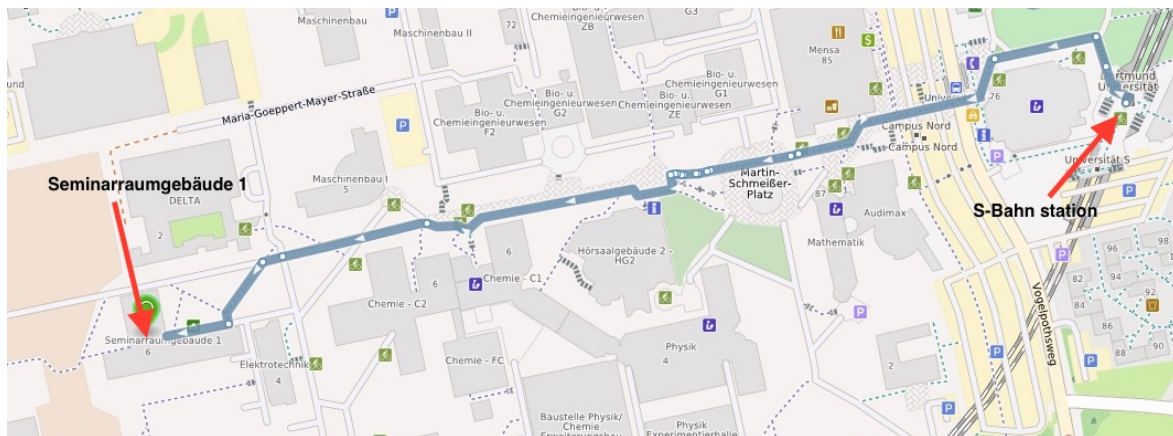
Directions

By public transport
From Dortmund main station:

1. Take the S-Bahn (train) S1 (usually from platform 7)

2. Exit the train at "Dortmund Universität" (3rd stop)

3. Follow the route on the map below

Tickets:
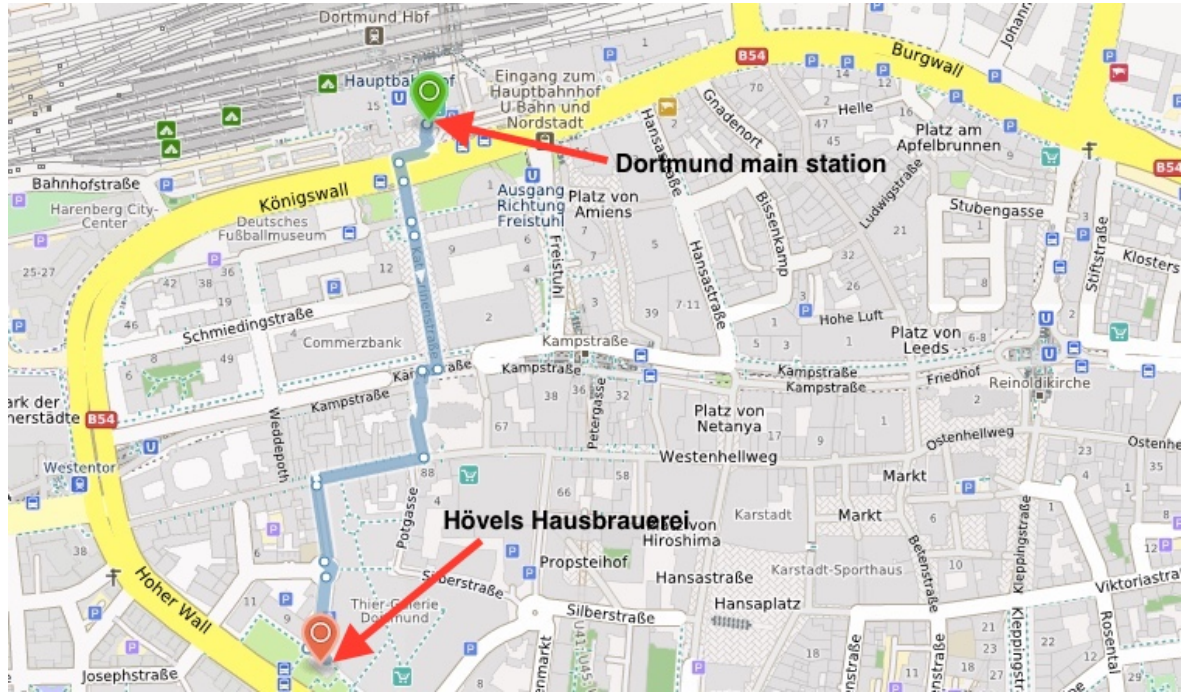You need a ticket "Erwachsene Preisstufe A"

For your way back to the city center, you can take the train S1 to "Dortmund Hbf" (Dortmund main station).

Informal dinner

On Thursday evening, January 19, at 19:30 there is an informal meeting at the restaurant *Hövels Hausbrauerei*, Hoher Wall 5-7, 44137 Dortmund (see the map below).

Conference dinner

The conference dinner takes place at the restaurant *View* in Dortmund's famous landmark "the U", Leonie-Reygers-Terrasse, 44137 Dortmund. From Dortmund main station, it is a 5 minutes walk to the restaurant (see the map below).
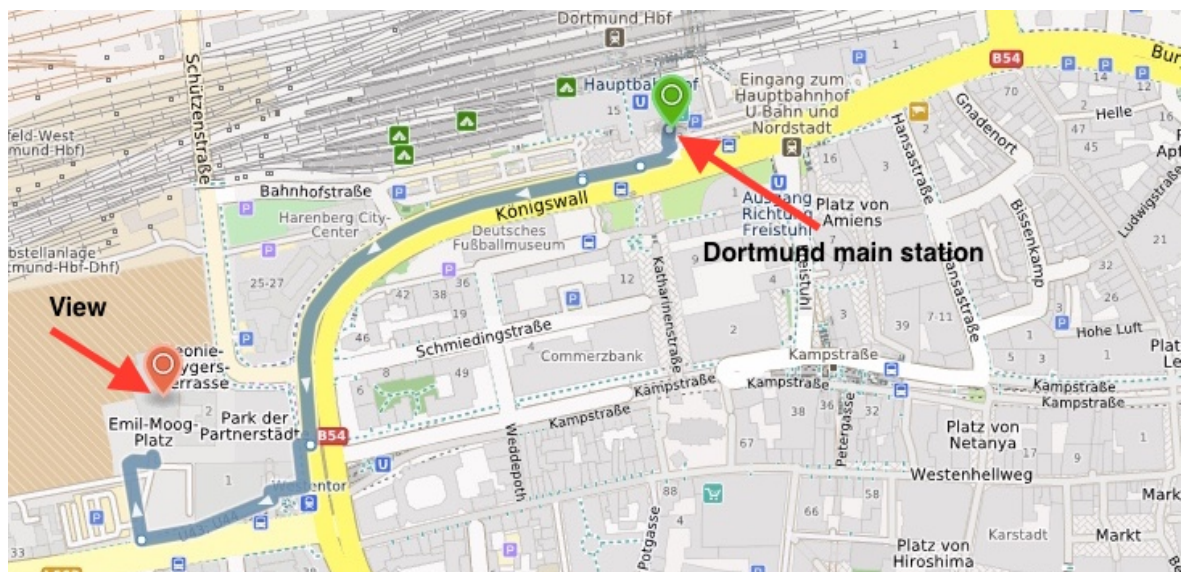
16th GAMM-Seminar on Microstructures

Program

Friday, January 20, 2017, 9:00-18:30

8:30- 9:00		**registration**
9:00- 9:15		**opening**
9:15- 9:55	Gilles Francfort	A criticism of finite elasto-plasticity
9:55-10:15	Andre Keip	A Phase-Field Approach to Micro-Magneto-Mechanics at Finite Deformations
10:15-10:35	Stephan Teichtmeister	Variational framework of phase field modeling of ductile fracture in isotropic and anisotropic porous solids at finite strains
10:35-11:05		**coffee break**
11:05-11:45	Bernd Schmidt	On a quantitative piecewise rigidity result and Griffith-Euler-Bernoulli functionals for thin brittle beams
11:45-12:05	Carolin Kreisbeck	Asymptotic rigidity and homogenization of layered materials with stiff components
12:05-12:25	Thilo Simon	Rigidity of Shape Memory Alloys undergoing cubic-to-tetragonal transformations
12:25-14:00		**lunch break** Canteen of the TU Dortmund

14:00-14:40	Marc Geers	Upscaling microfluctuation fields towards emergent phenomena in the mechanics of microstructures
14:40-15:00	Marco Morandotti	Formulas for relaxed disarrangement densities
15:00-15:20	Mathias Schäffner	Quantitative homogenization in nonlinear elasticity
15:20-15:40	Matti Schneider	An FFT-based fast gradient method for elastic and inelastic unit cell homogenization problems
15:40-16:10		**coffee break**
16:10-16:30	Swantje Bargmann	Computational modeling of hierarchical composites
16:30-16:50	Konrad Schneider	Fully periodic RVEs of matrix inclusion composites in practical multiscaling: not worth the effort
16:50-17:10	Matthias Labusch	Application of a magnetostrictive 3D Preisach model for the simulation of magneto-electric composites
17:10-18:10		**GAMM activity group meeting**
19:00		**Conference dinner** *View*, Leonie-Reygers-Terrasse , 44137 Dortmund

Saturday, January 21, 2017, 9:00 - 15:40

9:00- 9:40	Grégoire Allaire	Optimization of dispersive coefficients in the homogenization of the wave equation in periodic structures
9:40-10:00	Takayuki Yamada	High Order Homogenization for Elastic Wave Problems Using the Asymptotic Expansion Method
10:00-10:20	Wolf-Patrick Düll	Analysis of the embedded cell method for the numerical homogenization of metal-ceramic composite materials
10:20-10:40	Hans Knüpfer	Magnetic Domains in Thin Ferromagnetic Films with Strong Perpendicular Anisotropy
10:40-11:00		**coffee break**
11:00-11:20	Jan Nagel	A relaxation approach to the modeling of the stochastic behavior of elastic materials
11:20-11:40	Claudia Raithel	A Large-Scale Regularity Theory for Random Elliptic Operators on the Half- Space
11:40-12:00	Sergiy Nesenenko	Stochastic homogenization of rate-dependent systems of monotone type
12:00-12:20	Marco Veneroni	Stochastic homogenization of a maximal monotone relation
12:20-13:40		**lunch break** kostBar, Emil-Figge Straße 40
13:40-14:20	Augusto Visintin	Analogical Models and Homogenization Disagree
14:20-14:40	Stephan Wulfinghoff	Conditions for phenomenological anisotropic damage models derived from micro- mechanics
14:40-15:00	Elvira Zappale	Optimal design of fractured media with prescribed macroscopic strain
15:00-15:20	Philipp Junker	An evolution equation based approach to topology optimization
15:20-15:40	Anja Schlömerkemper	About an evolutionary model for magnetoelasticity in Eulerian description

16th GAMM-Seminar on Microstructures

Participants

Prof. Dr. **Grégoire Allaire**	Ecole Polytechnique
Dr. **Keith Anguige**	University of Freiburg
Tobias Asmanoglo	TU Dortmund
Prof. Dr. **Swantje Bargmann**	University of Wuppertal
Dr. **Thorsten Bartel**	TU Dortmund
Julia Bergmann	University of Duisburg-Essen
Christian Bleiler	University of Stuttgart
Dr.**Dominik Brands**	University of Duisburg-Essen
Karsten Buckmann	TU Dortmund
Prof. Dr. **Carsten Carstensen**	HU Berlin
Prof. Dr. **Sergio Conti**	University of Bonn
Nils Dabrock	TU Dortmund
JProf. Dr. **Tomáš Dohnal**	TU Dortmund
Prof. Dr. **Patrick Dondl**	University of Freiburg
PD Dr. **Wolf-Patrick Düll**	University of Stuttgart
Prof. Dr. **Julian Fischer**	IST Austria
Prof. Dr. **Gilles Francfort**	University of Paris North
Dr. **Manuel Friedrich**	University of Vienna
Prof. Dr. **Marc Geers**	TU Eindhoven
Prof. Dr. **Klaus Hackl**	Ruhr-University Bochum
Dr. **Martin Heida**	WIAS Berlin
Lisa Helfmeier	TU Dortmund
Dr. **Philipp Junker**	Ruhr-University Bochum
JProf. Dr. **Marc-Andre Keip**	University of Stuttgart
JProf. Dr. **Sandra Klinge**	TU Dortmund
Prof. Dr. **Hans Knüpfer**	Heidelberg University
Dr. **Carolin Kreisbeck**	Utrecht University
Matthias Labusch	University of Duisburg-Essen
Dr. **Agnes Lamacz**	TU Dortmund
Veronika Lemke	University of Duisburg-Essen

Prof. Dr. **Paolo Mariano**	University of Florence
Prof. Dr. **Andreas Menzel**	TU Dortmund
Prof. Dr. **Christian Meyer**	TU Dortmund
Dr. **Marco Morandotti**	TU Munich
Prof. Dr. **Jörn Mosler**	TU Dortmund
Dr. **Jan Nagel**	TU Munich
Aref Nateghi	University of Stuttgart
Dr. **Sergiy Nesenenko**	TU Berlin
Prof. Dr. **Stefan Neukamm**	TU Dresden
Dr. **Julia Orlik**	Fraunhofer ITWM
Dr. **Richard Ostwald**	TU Dortmund
Klaas Hendrik Poelstra	TU Dortmund
Prof. Dr. **Pedro Ponte Castañeda**	University of Pennsylvania
Claudia Raithel	MPI for Mathematics in the Sciences
PD Dr. **Andreas Rätz**	TU Dortmund
Prof. Dr. **Matthias Röger**	TU Dortmund
Dr. **Mathias Schäffner**	TU Dresden
Prof. Dr. **Anja Schlömerkemper**	University of Würzburg
Prof. Dr. **Bernd Schmidt**	Augsburg University
Dr. **Matti Schneider**	Fraunhofer ITWM
Konrad Schneider	Hamburg University of Technology
Prof. Dr. **Jörg Schröder**	University of Duisburg-Essen
Robin Schulte	TU Dortmund
Prof. Dr. **Ben Schweizer**	TU Dortmund
Thilo Simon	Max-Planck-Institute for Mathematics in the Sciences
Leon Sprave	TU Dortmund
Stephan Teichtmeister	University of Stuttgart
Maik Urban	TU Dortmund
Prof. Dr. **Marco Veneroni**	University of Pavia
Prof. Dr. **Augusto Visintin**	University of Trento
Prof. Dr. **Georg Weiss**	University of Duisburg-Essen
Dr. **Stephan Wulfinghoff**	RWTH Aachen University
Dr. **Takayuki Yamada**	Kyoto University
Dr. **Elvira Zappale**	University of Salerno
Carsten Zwilling	TU Dortmund

16th GAMM-Seminar on Microstructures

Abstracts

Keynote speakers

Optimization of dispersive coefficients in the homogenization of the wave equation in periodic structures

Grégoire Allaire
Ecole Polytechnique, Palaiseau, France

We study dispersive effects of wave propagation in periodic media, which can be modelled by adding a fourth-order term in the homogenized equation. The corresponding fourth-order dispersive tensor is called Burnett tensor and we numerically optimize its values in order to minimize or maximize dispersion. More precisely, we consider the case of a two-phase composite medium with an 8-fold symmetry assumption of the periodicity cell in two space dimensions. We obtain upper and lower bound for the dispersive properties, along with optimal microgeometries. This is a joint work with T. Yamada (Kyoto).

A criticism of finite elasto-plasticity

Gilles Francfort
Université Paris-Nord, Villetaneuse, France

In this talk, I will propose a slightly different approach to finite elasto-plasticity which may (or may not) fare better than the traditional $F = F_e F_p$ decomposition. I will then proceed to show that, in any case, all of this makes little sense in the absence of regularization (hardening, gradient plasticity), except possibly when restricting the slip directions (that is for crystal plasticity).

Upscaling microfluctuation fields towards emergent phenomena in the mechanics of microstructures

Marc Geers, Ashwin Sridhar, Bart Vossen, Varvara Kouznetsova, Olaf van der Sluis
Eindhoven University of Technology, Eindhoven, The Netherlands

The bottom-up design of advanced materials with unprecedented mechanical properties is a grand challenge, requiring reliable multiscale methods. The mechanical performance of engineering materials at the large scale often reveals particular emergent properties that originate from the complexity at the underlying fine scale level. Relating and predicting the macroscopic emergent behaviour is a non-trivial task, for which classical homogenization methods may fall short. This presentation addresses this multi-scale challenge, whereby the microfluctuation fields at the fine scale are key in understanding and solving the problem. Two problems are addressed to illustrate the problem and potential solutions.

The first problem concerns the multiscale modelling of acoustic metamaterials, which are tailored to attenuate sound wave propagation for certain frequencies. The unique features of these metamaterials originate from the complex interaction of transient phenomena at the microscopic and macroscopic scales, with local resonance occurring within (one of the) micro-constituents resulting in effective band gaps at the macro-scale. To capture the complex multi-scale interaction, an enriched transient computational homogenization scheme is required, which homogenizes the evolution in space and in time of materials with a non-steady state microstructure [1]. The conventional separation of scales hypothesis is relaxed, using an enriched description of the micro-macro kinematics that allows for large spatial microfluctuations, originating from local resonance phenomena. The proposed multi-scale solution method enables innovative designs of finite-size microstructures for locally resonant acoustic metamaterials. Material nonlinearities can be incorporated in a natural manner, which is a new development in the field of mechanical metamaterials. In the particular case of a metamaterial with linear elastic phases, the homogenization method allows to eliminate the fine scale, yielding an extended (micromorphic) continuum [2].

The second problem concerns the multi-scale behaviour of delaminating cohesive interfaces of polymer-metal laminates, as used for stretchable electronics applications. Fibrillation mechanisms at the fine scale give rise to pronounced discretized microfluctuations that have a major contribution to the macroscopic work-of-separation. Standard homogenization frameworks fall short [2] and it is shown that the role of the discrete fluctuating fields originating from the fine scale are again essential.

References

[1] K. Pham, V.G. Kouznetsova, M.G.D. Geers, JMPS 61 (2013) 2125.
[2] A. Sridhar, V.G. Kouznetsova, M.G.D. Geers, Computational Mechanics 57 (2016), 423, DOI: 10.1007/s00466-015-1254-y
[3] B.G. Vossen, P.J.G. Schreurs, O. van der Sluis, M.G.D. Geers, JMPS 66 (2014) 117.

On a quantitative piecewise rigidity result and Griffith-Euler-Bernoulli functionals for thin brittle beams

Bernd Schmidt
University of Augsburg, Augsburg, Germany

We study a planar thin brittle beam subject to elastic deformations and cracks described in terms of a nonlinear Griffith energy functional acting on SBV deformations of the beam. In particular we consider the case in which elastic bulk contributions due to finite bending of the beam are comparable to the surface energy which is necessary to completely break the beam into several large pieces. In the limit of vanishing aspect ratio we rigorously derive an effective Griffith-Euler-Bernoulli functional which acts on piecewise $W^{2,2}$ regular curves representing the midline of the beam. The elastic part of this functional is the classical Euler-Bernoulli functional for thin beams in the bending dominated regime in terms of the curve's curvature. In addition there also emerges a fracture term proportional to the number of discontinuities of the curve and its first derivative.

Analogical Models and Homogenization Disagree

Augusto Visintin
University of Trento, Trento, Italy

We shall show that the outcome of the theory of homogenization is at variance with a classical engineering technique for constructing rheological models of mechanical materials.
Analogical models. The mechanical constitutive behavior of several materials is often represented via discrete models, that are built up by arranging few elements. These are representative of the basic behavior of the material, viz., elasticity, viscosity and plasticity in continuum mechanics. In the case of univariate models, one may also use the pictorial image of parallel and serial arrangements. Similar constructions are also used elsewhere, e.g., in electro-magnetism.
As apparently these models are not derived from any representation of the mesoscopic structure, here we address the following question:

may these models be retrieved by homogenizing an underlying lower-scale structure?

We shall answer in the negative: the main reason stays in an (unjustified) *mean-field hypothesis,* that is at the basis of these constructions.
Homogenization via Scale Transformations. An alternative approach is illustrated, that rests on the following steps:
(i) formulation of a (space-distributed) model for an inhomogeneous material, with constitutive data that depend periodically on a x/η;
(ii) derivation of a *two-scale model* as $\eta \to 0$;
(iii) *upscaling,* i.e. derivation of a coarse-scale model by averaging the mesoscopic fields over a reference set $\mathcal{Y}$;
(iv) *downscaling,* i.e., retrieval of a two-scale solution from a coarse-scale solution.
This shows that the single-scale and two-scale models are equivalent. This entitles us to regard the coarse-scale model as the genuine *homogenized model.*
Variational Approach. This program is here developed for an initial- and boundary-value problem for a class of elasto-visco-plastic materials, see [4].

This process is given a variational formulation, which allows one to restate the homogenization result in terms of De Giorgi's Γ-*convergence.*

This is just an example of a more general set-up: analogous results have been derived for scale transformation for elasto-visco-plastic material of Prandt-Reuss-type, for electro-magnetic materials, for phase-transitions, see references.

References

[1] A.V.: *Homogenization of the nonlinear Kelvin-Voigt model of visco-elasticity and of the Prager model of plasticity.* Continuum Mech. Thermodyn. **18** (2006) 223-252

[2] A.V.: *Homogenization of a doubly-nonlinear Stefan-type problem.* S.I.A.M. J. Math. Anal. 39 (2007), 987–1017

[3] A.V.: *Homogenization of nonlinear visco-elastic composites.* J. Math. Pures Appl. 89 (2008), 477–504

[4] A.V.: *Homogenization of the nonlinear Maxwell model of visco-elasticity and of the Prandtl-Reuss model of elasto-plasticity.* Royal Soc. Edinburgh Proc. **A 138** (2008) 1363-1401

[5] A.V.: *Scale-transformations in the homogenization of nonlinear magnetic processes.* Archive Rat. Mech. Anal. **198** (2010) 569-611

[6] A.V.: *Scale-transformations and homogenization of maximal monotone relations, with applications.* Asymptotic Analysis 82 (2013), 233–270

Contributed talks

Computational modeling of hierarchical composites

Swantje Bargmann[1], Edgar Husser[2], Celal Soyarslan[1]
[1] Chair of Solid Mechanics, University of Wuppertal, Germany
[2] Institute of Continuum Mechanics and Material Mechanics, Hamburg University of Technology, Germany

The impact of small scale geometric confinement on deformation mechanisms is subject of intense research in materials sciences nowadays. Nanoporous metals have a micro-structure with an extremely high volume-specific surface content. Due to a very high local strength and a relatively regular interconnection of the nanoconstituents as well as a low mass density, nanoporous metals are very good candidates for strong and light-weight structural materials.

The gold-polymer composite considered in this contribution is hierarchical. To date, no established understanding of how to design a microstructured composite exists. As a consequence, there is a strong need for experimental as well as numerical studies at all scales.

References

[1] B. Elsner, S. Müller, S. Bargmann, J. Weißmüller Surface excess elasticity of gold: ab initio coefficients and impact on the effective elastic response of nanowires accepted

[2] C. Soyarslan, E. Husser, S. Bargmann Effect of surface elasticity on the elastic response of

nanoporous gold submitted

[3] J. Wilmers, A. McBride, S. Bargmann Interface elasticity effects in polymer-filled nanoporous metals submitted

[4] L. Lührs, C. Soyarslan, J. Markmann, S. Bargmann, J. Weißmüller Elastic and plastic Poissons ratio of nanoporous gold Scripta Materialia 110, 65-69, 2016

[5] S. Bargmann, C. Soyarslan, E. Husser, N. Konchakova Materials based design of structures: computational modeling of the mechanical behavior of gold-polymer nanocomposites Mechanics of Materials 94, 53-65, 2016

Analysis of the embedded cell method for the numerical homogenization of metal-ceramic composite materials

Wolf-Patrick Düll[1], Bastian Hilder[1], Guido Schneider[1]
[1] Universität Stuttgart, Stuttgart, Germany

In this talk, we analyze the embedding cell method, an algorithm which has been developed for the numerical homogenization of metal-ceramic composite materials. We show the convergence of the iteration scheme of this algorithm and the coincidence of the material properties predicted by the limit with the effective material properties provided by the analytical homogenization theory in three situations, namely for a one dimensional linear elasticity model, a simple one dimensional plasticity model and a two dimensional model of linear hyperelastic isotropic materials with constant shear modulus and slightly varying first Lamé parameter.

References
[1] W.-P. Düll, B. Hilder, G. Schneider. Analysis of the embedded cell method for the numerical homogenization of metal-ceramic composite materials. Submitted for publication, 2016, arXiv:1609.07644

An evolution equation based approach to topology optimization

Philipp Junker[1], Dustin R. Jantos[2], Klaus Hackl[2]
[1] Continuum Mechanics, Ruhr-University Bochum, Germany
[2] Mechanics of Materials, Ruhr-University Bochum, Germany

The objective of topology optimization is to find a mechanical structure with maximized stiffness by minimizing the work of external forces for given boundary conditions[1]. In contrast to the common approaches, we solve this problem by introducing a variational problem for the field of the mass density defined by an internal variable [2]. Employing Hamilton's principle for dissipative processes, we are able to find an evolution equation for the density field in a comparable manner to, e.g., plasticity. To this end, we introduce a viscous-like dissipation potential for the internal variable and add a Lagrange multiplier to control the total mass within the model space. A regularization with a discontinuous Galerkin approach for the internal variable enables us to suppress the well-known checkerboarding phenomena while evaluating the evolution equation within each finite element separately [3]. Thus, the numerical solution can be provided in a single finite element environment – no additional optimization tool is necessary. Since the density field is no additional field unknown but a Gaußpoint quantity, the calculation effort is strongly reduced. We end our presentation by presenting

solutions of optimized structures for different boundary problems to show the potential of our model.

References

[1] M.P. Bendsøe and O. Sigmund. "Topology optimization: theory, methods and applications", Springer (2003),

[2] P. Junker and K. Hackl. "A variational growth approach to topology optimization", Structural and Multidisciplinary Optimization (2015),

[3] P. Junker and K. Hackl. "A discontinuous phase field approach to variational topology optimization", Structural and Multidisciplinary Optimization (2016).

A Phase-Field Approach to Micro-Magneto-Mechanics at Finite Deformations

M.-A. Keip, A. Sridhar, C. Miehe
Institute of Applied Mechanics (CE), Chair of Material Theory, University of Stuttgart, Pfaffenwaldring 7, 70569 Stuttgart, Germany, E-mail: marc-andre.keip@mechbau.uni-stuttgart.de.

The talk addresses a variational formulation for finite-deformation micro-magneto-mechanics [1]. In line with our previous work [2], we develop a variationally consistent phase-field approach to micro-magneto-elasticity that incorporates the magnetization as order parameter. Key motivation is the generalization of micro-magnetic phase-field models to the general case of *finite deformations*. This will allow to solve micro-magnetically informed large-strain problems, which are found in a number of technological applications. A central example is given by magnetorheological elastomers (MRE), which are characterized by a microstructure composed of a soft elastomeric matrix and quasi-rigid, micron-sized ferromagnetic inclusions [3,4]. The effective behavior of MREs is driven by complex phenomena that span over multiple length scales: At micro-scale we observe magnetic domain wall motion within the embedded ferromagnetic particles. Particle-to-particle interactions as well as particle rotations then give rise to deformations within the soft matrix [5]. At macro-scale we eventually observe effective magneto-mechanical deformations that result from both micro- and macro-structural interactions [6,7].

In a series of numerical examples we will demonstrate the versatility of the model formulation. A particular focus will be on the modeling of ferromagnetic microstructure evolution in magnetorheological elastomers. Here we investigate the macroscopically driven micro-magnetic domain-wall motion as well as the resulting interactions between individual spherical and ellipsoidal particles which lead to an overall compression or expansion of the MRE.

References

[1] A. Sridhar, M.-A. Keip & C. Miehe, "A large-deformation phase-field approach to micro-magneto-elasticity", *in preparation.*

[2] A. Sridhar, M.-A. Keip & C. Miehe [2016], "Homogenization in micro-magneto-mechanics", Computational Mechanics **58**, 151–169.

[3] P. Ponte Castañeda & E. Galipeau [2011], "Homogenization-based constitutive models for magnetorheological elastomers at finite strain", J. Mech. Phys. Solids **59**, 194–215.

[4] K. Danas, S. V. Kankanala & N. Triantafyllidis [2012], "Experiments and modeling of iron-particle-filled magnetorheological elastomers", J. Mech. Phys. Solids **60**, 120–138.

[5] E. Galipeau & P. Ponte Castañeda [2013], "Giant field-induced strains in magnetoactive elastomer

composites", Proc. R. Soc. A **469**, 20130385.

[6] M.-A. Keip & M. Rambausek [2016], "A multiscale approach to the computational characterization of magnetorheological elastomers", Int. J. Numer. Meth. Engng. **107**, 338–360.

[7] M.-A. Keip & M. Rambausek [2016], "Computational and analytical investigations of shape effects in the experimental characterization of magnetorheological elastomers", *submitted for publication.*

Magnetic Domains in Thin Ferromagnetic Films with Strong Perpendicular Anisotropy

H. Knüpfer
Institute of Applied Mathematics, Heidelberg, Germany

We consider a regime of large and ultra-thin ferromagnetic films with strong anisotropy and easy axis pointing out of the film plane. Starting from the full three-dimensional micromagnetic model, we identify (to leading order) the critical scaling where the phase transition from single domain states to multi-domain states such as bubble or maze patterns occurs. Furthermore, we derive the asymptotic behavior of the energy in the single domain regime within the framework of Gamma-convergence. This is joint work with C. Muratov und F. Nolte.

Asymptotic rigidity and homogenization of layered materials with stiff components

Fabian Christowiak[1], **Carolin Kreisbeck**[2]
[1] Fakultät für Mathematik, Universität Regensburg, Regensburg, Germany
[2] Mathematisch Instituut, Universiteit Utrecht, Utrecht, The Netherlands

In the context of finite-strain elastoplasticity, we investigate the effective behavior of variational models for bilayered composite materials featuring large elastic constants in one component. Our particular interest lies in understanding whether the presence of the stiff layers forces a rigid macroscopic material response. The answer to this question is expected to depend on the scaling relation between stiffness and layer thickness.

In this talk, we characterize the limit deformations of sequences of uniformly bounded energy as the thickness of the layers tends to zero, and identify two different scaling regimes. If the elastic constants diverge sufficiently fast, the observed macroscopic deformations coincide with those in the special case of completely rigid layers. As it is shown in [1], the latter correspond exactly to global rotations of shear deformations in layer direction, provided they are locally volume preserving. One major step in the proof is to quantify the layers' stiffness with the help of the geometric rigidity estimate established in [2]. To show optimality of this regime, an explicit construction of a family of deformations based on bending of the individual stiffer layers is given, which yields macroscopically softer material behavior.

These findings serve as a basis for proving a rigorous homogenization result via Γ-convergence.

References

[1] F. Christowiak and C. Kreisbeck [2016], "Homogenization of layered materials with rigid components in single-slip finite crystal plasticity", Preprint arXiv:1604.03483.

[2]　G. Friesecke, R. D. James and S. Müller. [2002], "A theorem on geometric rigidity and the derivation of nonlinear plate theory from three-dimensional elasticity", Comm. Pure Appl. Math. **55**, 1461–1506.

Application of a magnetostrictive 3D Preisach model for the simulation of magneto-electric composites

M. Labusch, J. Schröder
Institute of Mechanics, Department of Civil Engineering, Faculty of Engineering, University of Duisburg-Essen, Universitätsstraße 15, 45141 Essen

In this contribution we focus on the magneto-electric (ME) coupling of multiferroic materials. They combine two ferroic characteristics and exhibit an interaction between magnetic and electric fields. This magneto-electric coupling can find applications in sensor technology or in electric field-controlled magnetic data storage devices [1]. Since most ME single-phase materials show an interaction between electric polarization and magnetization far below room temperature and therefore outside of a technical relevant temperature range, the manufacturing of two-phase composites, consisting of a ferroelectric matrix with magnetostrictive inclusions, becomes important. They generate the ME coupling at room temperature as a result of the interaction of their constituents. Hence, the ME coupling of composite materials significantly depends on the material behavior of both phases. In order to determine the effective properties with respect to both aspects, a multiscale finite element (FE^2) homogenization approach is performed, which combines via a scale bridging the macro- and microscopic level. Therefore, the microscopic morphology is considered by using a representative volume element [2,3]. Furthermore, the nonlinear properties of both phases are approximated with appropriate material models. On the one hand, the switching behavior of spontaneous polarizations of barium titanate unit cells are taken into account [4], which reproduces after a homogenization step the typical dielectric and butterfly hysteresis loops of the ferroelectric matrix material. On the other hand, the nonlinear remanent magnetizations of the magnetostrictive inclusions are described with a Preisach operator [5]. Both models affect the strain-induced coupling, such that we obtain a nonlinear behavior of the ME coefficient.

References
[1]　N.A. Spaldin, M. Fiebig [2005], "The renaissance of magnetoelectric multiferroics", Materials Science **309**, 391–392,
[2]　J. Schröder, M. Labusch, M.-A. Keip [2016], "Algorithmic two-scale transition for magneto-electro-mechanically coupled problems. FE^2-scheme: Localization and Homogenization", Computer Methods in Applied Mechanics and Engineering **302**, 253–280,
[3]　M. Labusch, M. Etier, D.C. Lupascu, J. Schröder, M.-A. Keip [2014], "Product properties of a two-phase magneto-electric composite: Synthesis and numerical modeling", Computational Mechanics **54**, 71–83,
[4]　S.C. Hwang, C.S. Lynch, R.M. McMeeking [1995], "Ferroelectric/ferroelastic interactions and a polarization switching model", Acta Metallurgica **43**, 2073–2084,
[5]　M. Kaltenbacher, B. Kaltenbacher, T. Hegewald, R. Lerch [2010], "Finite Element Formulation for Ferroelectric Hysteresis of Piezoelectric Materials", Journal of Intelligent Material Systems and Structures **21**, 773–785.

Formulas for relaxed disarrangement densities

M. Morandotti
Technische Universität München, München, Germany

Structured deformations provide a multiscale geometry that captures the contributions at the macrolevel of both smooth geometrical changes and non-smooth geometrical changes (disarrangements) at submacroscopic levels. Recently, Owen and Paroni evaluated explicitly some relaxed energy densities arising in Choksi and Fonseca's energetics of structured deformations. In this talk we will show that a different approach to the energetics of structured deformations, that due to Baía, Matias, and Santos, confirms the roles of the relaxed densities established by Owen and Paroni. In doing so, we give an alternative, shorter proof of Owen and Paroni's results, and we establish additional explicit formulas for other measures of disarrangements. This is joint work with A. C. Barroso, J. Matias, and D. R. Owen.

A relaxation approach to the modeling of the stochastic behavior of elastic materials

Philipp Junker[1], **Jan Nagel**[2]
[1] Ruhr-Universität Bochum, Bochum, Germany
[2] Technische Universität München, München, Germany

The material behavior even in the simple linear elastic range is not deterministic but rather fluctuates randomly around some expectation values. The knowledge about this characteristic is obviously trivial from an experimentalist's point of view. However, it is not considered in the vast majority of material models in which "only" the deterministic behavior is taken into account.
One very promising approach to the inclusion of stochastic effects to the modeling of materials is provided by the so-called Chaos Polynomial Expansion. It had been used, e.g., for the derivation of the so-called stochastic finite element method. This method yields results which are exactly of the desired kind but unfortunately at drastically increased numerical costs.
This contribution aims at proposing a new ansatz which is also based on a stochastic series expansion at the Gauß point level. Appropriate energy relaxation allows for a synthesized (deterministic) stress measure while simultaneously offering stochastic properties as e.g. the standard derivation. The total procedure only needs negligibly more computation effort as compared to a simple elastic calculation.

Stochastic homogenization of rate-dependent systems of monotone type

Martin Heida[1], **Sergiy Nesenenko**[2]
[1]Weierstrass Institute for Applied Analysis and Stochastics, Berlin, Germany
[2] Fakultät II, Institut für Mathematik, Technische Universität Berlin, Berlin, Germany

In this work we are concerned with the stochastic homogenization of the initial boundary value problems of monotone type. The models of monotone type under consideration describe the deformation behaviour of inelastic materials with a microstructure which can be characterised

by random measures. Based on the Fitzpatrick function concept we reduce the study of the asymptotic behaviour of monotone operators associated with our models to the problem of the stochastic homogenization of convex functionals within an ergodic and stationary setting. The concept of Fitzpatrick's function helps us to introduce and show the existence of the weak solutions for rate-dependent systems. The derivations of the homogenization results presented in this work are based on the stochastic two-scale convergence in Sobolev spaces. We present also the convergence results, which are related to the classical Γ-convergence results.

A Large-Scale Regularity Theory for Random Elliptic Operators on the Half-Space

Claudia Raithel, based on joint work with Julian Fischer
MPI for Mathematics in the Sciences, Leipzig, Germany

We are interested in large-scale regularity properties of linear elliptic operators with random coefficients. It is well-known that for general heterogeneous coefficient fields a, functions that are a-harmonic in the whole-space may fail to be Lipschitz or (in the systems case) even locally bounded. As was already noted in the 80's by Avellaneda and Lin in the context of periodic coefficients, the situation improves when homogenization occurs. The philosophy here is to transfer the regularity properties of the constant coefficient homogenized operator to the heterogeneous coefficient operator. More recently, a large body of literature devoted to extending the strategy of Avellaneda and Lin to the setting of random coefficient fields has developed.

The main emphasis of this talk will be on boundary regularity: We consider operators on the half-space with coefficients that are the restriction of a random coefficient field on the whole-space. In the case of homogeneous Dirichlet (or Neumann) boundary data, under certain assumptions on the random coefficient field a, we obtain a first-order Liouville principle, which states that the space of subquadratic a-harmonic functions vanishing on the boundary has the same dimension as in the Euclidean case. This Liouville principle arises as a corollary of a large-scale $C^{1,\alpha}$-decay of an appropriate version of tilt-excess, the proof of which relies critically on the availability of a sublinear half-space-adapted homogenization corrector. The main contribution of our work is the construction of such a sublinear half-space-adapted corrector given a whole-space corrector satisfying a slightly quantified sublinearity condition (the existence of which may be ensured with a quantitative ergodicity assumption). To the best of our knowledge, while within the context of periodic homogenization boundary correctors have already been introduced by Avellaneda and Lin in the Dirichlet case and Kenig, Lin, and Shen in the Neumann case, this is the first use of boundary correctors in the stochastic setting.

References
[1] A. Gloria, S. Neukamm, and F. Otto [2014], "A regularity theory for random elliptic operators", Preprint, arXiv:1409.2678v3
[2] J. Fischer and C. Raithel [2016], "Liouville principles and a large-scale regularity theory for random elliptic operators on the half-space", Preprint, arXiv:1604.02717v2
[3] M. Avellaneda and F.-H. Lin [1987], "Compactness Methods in the Theory of Homogenization", Comm. Pure Appl, Math. **40.6**, 803–847
[4] C. Kenig, F.-H. Lin, Z. Shen [2013], "Homogenization of Elliptic Systems with Neumann Boundary

Conditions", J. Amer. Math. Soc. **26.4**, 901–937

Quantitative homogenization in nonlinear elasiticity

Stefan Neukamm, **Mathias Schäffner**
Department of Mathematics, Technische Universität Dresden, Germany

We consider a nonlinear elastic composite with a periodic micro-structure described by the non-convex integral functional

$$I_\varepsilon(u) = \int_\Omega W\left(\frac{x}{\varepsilon}, \nabla u(x)\right) - f(x) \cdot u(x)\, dx,$$

where $u : \Omega \to \mathbb{R}^d$ is the deformation, $f : \Omega \to \mathbb{R}^d$ is an external force, $\varepsilon > 0$ denotes the size of the micro-structure, and $W(y, F)$ is a stored energy function which is periodic in y. As it is well-known, under suitable growth conditions, I_ε Γ-converges to a functional with a homogenized energy density $W_{\mathrm{hom}}(F)$, which is given by an *infinite-cell formula*. Under appropriate assumptions on W (namely, $p \geq d$-growth from below, frame indifference, minimality at identity, non-degeneracy and smoothness in a neighborhood close to the set of rotations) and on the microstructure, we show that in a neighbourhood of rotations the homogenized stored energy function W_{hom} is of class C^2 and characterized by a *single-cell homogenization formula*. Moreover, for small data, we establish an estimate on the homogenization error, which measures the distance between (almost) minimizers u_ε of I_ε and the minimizer of the homogenized problem. More precisely, we prove that the L^2-error as well as the H^1-error of the associated two-scale expansion decays with the rate $\sqrt{\varepsilon}$.

About an evolutionary model for magnetoelasticity in Eulerian description

B. Benešová[1], J. Forster [1], C. García Cervera[2], C. Liu[3], **A. Schlömerkemper**[1], J. Žabenský[1]
[1] University of Würzburg, Würzburg, Germany
[2] UC, Santa Barbara, USA
[3] Penn State University, University Park, USA

A fundamental issue in the modeling of magnetoelastic materials is that elasticity is phrased in Lagrangian coordinates whereas magnetism is phrased in Eulerian coordinates. We discuss an evolutionary model that is completely phrased in Eulerian coordinates and takes microstructures of the magnetization into account. The model presented is a system of partial differential equations that contains (1) a Navier-Stokes equation with magnetic and elastic terms in the stress tensor obtained by a variational approach, (2) a regularized transport equation for the deformation gradient and (3) the Landau-Lifshitz-Gilbert equation for the dynamics of the magnetization. We prove existence of weak solutions in two dimensions and study their uniqueness.

An FFT-based fast gradient method for elastic and inelastic unit cell homogenization problems

Matti Schneider
Fraunhofer ITWM, Kaiserslautern, Germany

Building upon the previously established [1] equivalence of the basic scheme of Moulinec-Suquet's FFT-based computational homogenization technique [2] with a gradient descent method, this work concerns the impact of the fast gradient method of Nesterov [3] in the context of computational homogenization. Nesterov's method with speed restarting [4] leads to a significant speed up compared to the basic scheme for linear problems with moderate contrast, and compares favorably to the (Newton-)conjugate gradient (CG) method for problems in digital rock physics and (small strain) elastoplasticity. We present an efficient implementation requiring twice the storage of the basic scheme, but only half of the storage of the CG method.

References

[1] M. Kabel, T. Böhlke, M. Schneider [2014], "Efficient fixed point and Newton-Krylov solvers for FFT-based homogenization of elasticity at large deformations", Computational Mechanics **54** (6), 1497–1514.

[2] H. Moulinec, P. Suquet [1994], "A fast numerical method for computing the linear and nonlinear mechanical properties of composites", C. R. Acad. Sci. Ser. II **318** (11), 1417–1423.

[3] Y. Nesterov [1983], "A method for solving the convex programming problem with convergence rate $O(1/k^2)$", Dokl. Aka. Nauk SSSR **269** (3), 543–547.

[4] W. Su, S. Boyd, E. Candes [2014], "A Differential Equation for Modeling Nesterov's Accelerated Gradient Method: Theory and Insights", in: Z. Ghahramani, M. Welling, C. Cortes, N. Lawrence, K. Weinberger (Eds.), Advances in Neural Information Processing Systems **27**, Curran Associates, Inc., 2510–2518.

[5] B. Beta [2002], "A Survey of the Beta–Method", International Journal of Templates **2**, 2–22.

Fully periodic RVEs of matrix inclusion composites in practical multiscaling: not worth the effort

Konrad Schneider[1], Benjamin Klusemann[2,3], Swantje Bargmann[4]
[1] Institute of Continuum Mechanics and Material Mechanics, Hamburg University of Technology, Hamburg, Germany
[2] Institute of Product and Process Innovation, Leuphana University of Lüneburg, Lüneburg, Germany
[3] Institute of Materials Research, Helmholtz-Zentrum Geesthacht, Geesthacht, Germany
[4] Chair of Solid Mechanics, University of Wuppertal, Wuppertal, Germany

The identfication of effective material responses or macroscopic constitutive laws is on of the major goals in computational micromechancis. State of the art are FE2-methods or homogenization approaches that utilize representative volume elements (RVEs) which incorporate the microstructural information. In this regard, periodic model setups featuring a fully periodic RVE topology and discretization are known to perform the best. However, generating such RVEs and setting up a proper model might become a cumbersome task. Therefore, it is highly questionable if the tremendous effort of generating such complex RVEs is justified over other simplification current in an engineering analysis. In the present work we investigate the effect of utilizing simpler RVE model setups to determine effective material parameters and responses. Especially the influences of different RVE topologies, discretizations

and boundary conditions are studied. The case of a fully periodic RVE with exact periodic boundary conditions will act as a reference solution for the evaluations. Special emphasis is paid to so called matrix-inclusion composites, widely found in industrial applications. General trends for linear and non-linear material behavior will be presented.

References

[1]   K. Schneider, B. Klusemann and S. Bargmann [2016], "Automatic three-dimensional geometry and mesh generation of periodic representative volume elements for matrix-inclusion composites", Advances in Engineering Software **99**, 177–188.

[2]   K. Schneider, B. Klusemann and S. Bargmann [2016], "Fully periodic RVEs for technological relevant composites: not worth the effort", *submitted*.

Rigidity of Shape Memory Alloys undergoing cubic-to-tetragonal transformations

Thilo Simon
Max-Planck-Institute for Mathematics in the Sciences, Leipzig, Germany

Microstructures in Shape Memory Alloys (SMAs) undergoing cubic-to-tetragonal transformations typically exhibit phase transitions parallel to six different hyperplanes relative to the crystal orientation. The simplest case is laminar structures involving two martensite variants called twins. The same phenomenon also occurs, however, when transitioning between a twinned structure and austenite (habit planes). The orientations of these hyperplanes can phenomenologically be explained by requiring the involved strains to be rank-one connected.

Building on work by Dolzmann and Müller [2], a more quantitative analysis of twins in the framework of linearized elasticity has been given by Capella and Otto [1]. Starting from a variational model including a term penalizing surface area between phases, they identified the energy regime where, asymptotically, only twins can occur. Their result is sharp in the sense that the regime reaches up to the energy of the well-known Kohn-Müller branching construction [3] of habit planes.

Starting from the result of Capella and Otto, we present our findings on the rigidity of SMAs in the regime where branching may occur. While planes may cross, we do recover the predicted orientations if we additionally assume that the strain either lies in BV or is independent of some arbitrary direction. We also give some insight into the regularity of possible microstructures by using tools from microlocal analysis to track the "internal" orientation of the twins. Roughly speaking, the results suggest that in three space dimensions habit planes may cluster on a set of Haussdorff dimension $3 - \frac{2}{3}$.

References

[1]   A. Capella, F. Otto [2012], "A quantitative rigidity result for the cubic-to-tetragonal phase transition in the geometrically linear theory with interfacial energy", Proc. Roy. Soc. Edinburgh Sect. A **142**, **No. 2**, 273–327,

[2]   G. Dolzmann, S. Müller [1995], "The influence of surface energy on stress-free microstructures in shape memory alloys", Meccanica, **30**, **No. 5**, 527–539,

[3]   R. Kohn, S. Müller [1994], "Surface energyv and microstructure in coherent phase transitions", Communications on Pure and Applied Mathematics, **47**, **No. 4**, 405–435.

Variational framework of phase field modeling of ductile fracture in isotropic and anisotropic porous solids at finite strains

S. Teichtmeister, D. Kienle, F. Aldakheel, C. Miehe
Institute of Applied Mechanics (CE), Stuttgart, Germany

This work outlines a rigorous variational-based framework for the phase field modeling of fracture in isotropic and anisotropic porous solids undergoing large elastic-plastic strains. It extends the recent work [1] to a formulation of porous plasticity with particulate microstructures characterized by spherical pores or by ellipsoidal voids which beside changing their size are also able to change their shape and orientation due to a macroscopic deformation [2].

The phase field approach regularizes sharp crack surfaces within a pure continuum setting by a specific gradient damage modeling with geometric features rooted in fracture mechanics. A gradient plasticity model for isotropic and anisotropic porous plasticity with a simple growth law for the evolution of the void fraction is developed, and linked to a failure criterion in terms of the local elastic-plastic work density that drives the fracture phase field [3]. It is shown that this approach is able to model basic phenomena of ductile failure such as cup-cone failure surfaces in terms of only two material parameters on the side of damage mechanics: a critical work density that triggers the onset of damage and a shape parameter that governs the postcritical damage up to fracture.

The coupling of gradient plasticity to gradient damage is realized by a constitutive work density function that includes the stored elastic energy and the dissipated work due to plasticity and fracture. With this viewpoint at hand, the thermodynamic formulation is outlined for gradient-extended dissipative solids with generalized internal variables.

The proposed canonical theory is shown to be governed by a rate-type minimization principle, which describes the coupled multifield evolution problem of plasticity-damage, where on the side of plasticity an additive split of the total Hencky strain is used. The variational nature of the formulated problem is exploited on the numerical side by inherent symmetry properties. Another aspect of this work is the regularization towards a micromorphic gradient plasticity-damage setting [4] by taking into account additional internal variable fields linked to the original ones by penalty terms. This enhances the robustness and facilitates the finite element implementation, in particular on the side of gradient plasticity.

References

[1] C. Miehe, S. Teichtmeister and F. Aldakheel [2015], "Phase-field modelling of ductile fracture: a variational gradient-extended plasticity-damage theory and its micromorphic regularization", Philos T Roy Soc A **374** (2066)

[2] M. Kailasam and P. Ponte Castañeda [1998], "A general constitutive theory for linear and nonlinear particulate media with microstructure evolution", J Mech Phys Solids **46**, 427–465.

[3] C. Miehe, D. Kienle, F. Aldakheel and S. Teichtmeister [2016], "Phase field modeling of fracture in porous plasticity: A variational gradient-extended Eulerian framework for the macroscopic analysis of ductile failure", Comput Method Appl M, in press

[4] S. Forest [1998], "Micromorphic Approach for Gradient Elasticity, Viscoplasticity, and Damage", J Eng Mech-ASCE **135**, 117–131.

Stochastic homogenization of a maximal monotone relation

M. Veneroni
Università di Pavia, Pavia, Italy

We study the stochastic homogenization of a quasilinear model of Ohmic electric conduction with Hall effect. We prove that, as the homogenization parameter vanishes, the solution of the random system converges to the solution of a deterministic system having the same structure. The proof relies on Birkhoff's ergodic theorem, on the maximal monotonicity of the nonlinear operator, and on the scale integration method introduced by A. Visintin. (Joint work with L. Lussardi and S. Marini)

Conditions for phenomenological anisotropic damage models derived from micromechanics

Stephan Wulfinghoff, Marek Fassin, Stefanie Reese
Institute of Applied Mechanics, RWTH Aachen, Aachen, Germany

The talk investigates the modeling of irreversible anisotropic brittle and ductile damage based on second and fourth order damage tensors and its connection to micromechanical material models. In particular, the conditions for strictly increasing damage are discussed. Restrictions on the form of the free energy function and the damage evolution law are proposed. These are motivated by energetical, scale-bridging considerations for growing cracks and pores. Moreover, conclusions are drawn related to the damage-induced evolution of the yield surface in plasticity.

High Order Homogenization for Elastic Wave Problems Using the Asymptotic Expansion Method

Takayuki Yamada[1], Grégoire Allaire[2]
[1] Kyoto University, Kyoto, Japan
[2] École Polytechnique, Palaiseau, France

Dispersion is a phenomenon appearing in heterogeneous media, by which waves of different wavelengths propagate at different velocities [1]. In the scalar wave equation for periodic media, the dispersive effect is modeled as an additional forth-order term in the homogenized wave equation [2,3,4]. This effect must be taken into account in industrial applications because the size of periodic microstructures is finite in manufactured products. In this study, the dispersive effect is formulated in the elastic wave problem for periodic media by using the asymptotic expansion method, in the same manner as for scalar wave propagation problems.

References

[1] F. Santosa, W. W. Symes [1991], "A Dispersive Effective Medium for Wave Propagation in Periodic Composites", SIAM Journal on Applied Mathematics, **51**, 984–1005.
[2] C. Conca, R. Orive, M. Vanninathan [2006], "On Burnett Coefficients in Periodic Media", Journal of Mathematics Physics, **47**, 032902.

[3]   A. Lamacz [2011], "Dispersive Effective Models for Waves in Heterogeneous Media", Math. Models Methods Appl. Sci., **21** (9), 1871-1899.

[4]   G. Allaire, T. Yamada [2016], "Optimization of Dispersive Coefficients in the Homogenization of the Wave Equation in Periodic Structures", **hal-01341082**.

Optimal design of fractured media with prescribed macroscopic strain

Elvira Zappale
Università di Salerno, Fisciano (SA), Italy

An optimal design problem for two-component fractured media, for which a macroscopic strain is prescribed, is studied. Within the framework of structured deformations, we derive an integral representation for the relaxed energy functional. We start from an energy functional accounting for bulk and surface contributions coming from both constituents of the material; the relaxed energy densities, obtained via a blow-up method, are determined by a delicate interplay between the optimization of sharp interfaces and the diffusion of microcracks. This model has the far-reaching perspective to incorporate elements of plasticity in optimal design of composite media.