

Numerical Lab 2007/08

# **AUTOMATIC CONTROL OF A HEAT EXCHANGER WITH CHANGING OPERATION CONDITIONS**

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## **1. Abstract**

In the term project of Numerical Lab 07/08, it is aimed to design and implement an automatic control system for a heat exchanger which is operating under varying and continuously changing conditions. The heat exchanger is modeled and the fluid flow is simulated with the help of FEATFLOW software that utilizes the principles of computational fluid dynamics and finite element analysis methods. FEATFLOW is an open source research code for an analysis of computational fluid dynamics. Compared to most other codes, it is based on finite element methods. Geometry of the heat exchanger is designed so as to provide the optimal and homogeneous heat exchange conditions. Prestudies are done to obtain the necessary control parameters after the decision of using PI controller. Three control approaches are used to observe and compare the outcomes by adjusting these parameters in such a way that the overshoot, settling time and the steady state error values of output are minimized. The control algorithm itself was implemented into FEATFLOW. User specific changes in the source code enables one to measure the outflow temperature for every time step and change operating conditions accordingly. The results are listed and compared, then the effects of control parameters are discussed to draw a conclusion.

## 2. Introduction

This project deals with how to control and obtain a pre-specified output temperature of a fluid flow system. After some discussions, it was decided to consider a tube system, which consists of two main parts.

The first part is main tube / pipe where water flows and heating operation is done. This pipe is long enough to let heated water mix and establish a homogeneous temperature at the output. The speed of the water is defined as constant.

The second part is the heating part. This part consists of 13 small pipes in order to heat water as quickly and as homogeneously as possible. The pipe temperature can change in a feasible pipe temperature interval, which is predefined by using physical facts. (i.e. no pipe can be 5000 °C )

The physical dimensions / geometry and other quantities are detailed in the following section.

The problem is implementing a control mechanism, which controls heating tubes temperatures in order to obtain a user defined water temperature at the output of the main pipe.

For that reason, different types of controllers are tested and the effects of different controllers are observed (i.e. Effects of Proportional or Integral parts of controllers). The aim is to obtain a suitable controller, which is a combination of fastest response (shortest settling time) and smallest overshoot (no overshoot if possible).

The FEATFLOW program is used to realize the problem and to implement the controller. The program allows to change heating tube temperatures and to measure output water temperature at the end of the main tube.

### FEATFLOW

The FEAST group has a long tradition in Numerical Analysis (finite element discretizations, multigrid solvers), High Performance Computing and Software Engineering, particularly in Computational Fluid Dynamics (CFD).

The current flow solver is FEATFLOW 1.2 - based on the FEAT2D and FEAT3D packages - which has proven to be an efficient simulation tool.

The FEATFLOW package has been used for years for education of students coming from Mathematics as well as from Computer Science, Engineering Sciences, Physics and more. As supporting multimedia tool, the Virtual Album may help to illustrate abstract mathematical techniques and numerical solution techniques for PDE's, particularly in Computational Fluid Dynamics, and can be employed to illustrate the facilities of modern Mathematics. [5]

The program package FEATFLOW is both a user oriented as well as a general purpose subroutine system for the numerical solution of the incompressible Navier Stokes equations in two and three space dimensions.

FEATFLOW is based on the FORTRAN77 finite element packages feat2d and feat3d which are not user oriented systems. They only provide subroutines for several main steps in a finite element program. [4]

## Techniques

Numerical solution techniques are considered for the non-stationary (or stationary, without the term  $u_t$  ( $u_t=0$ )) incompressible Navier Stokes equations,

$$\mathbf{u}_t - \nu \Delta \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p = \mathbf{f} \quad , \quad \nabla \cdot \mathbf{u} = 0 \quad , \quad \text{in } \Omega \times (0, T],$$

for a given force  $f$  and viscosity  $\nu$ , with any prescribed boundary values on the boundary of a Dirichlet or Neumann type and an initial condition at  $t=0$ . Solving this problem numerically still seems to be a considerable task in the case of long time calculations and higher Reynolds numbers, particularly in 3D.

The corresponding discretized nonlinear systems of equations may be treated by coupled approach in  $u$  and  $p$  which promises the best stability behavior, but also entails the largest numerical effort. Another variant, known as projection method decouples pressure and velocity, which reduces the problem to the solution of a sequence of "simple" (scalar) problems. However, at the same time, it leads to smaller time steps due to the inherently more explicit character and often suffers from spurious pressure boundary layers.

These different approaches lead to a large variety of schemes all of which are occurring in practice since years. Theoretical considerations could provide some ideas, concerning stability of these schemes, convergence rates for sub problems, necessary time step sizes, or qualitative behavior for large Reynolds numbers, but a complete analysis or quantitative prediction is not possible even today.

Therefore, the only way to make a judgment was to perform numerical tests, at least for some classes of problems which seem to be representative.

What is reached is a (finite element) discretization and a solution procedure such that:

1. The finite element spaces for  $u$  and  $p$  are stable; i.e., satisfy the LBB-condition.
2. A robust and efficient coupled solver is available
3. A robust and efficient solver of projection type is available
4. An efficient nonlinear solution strategy is available
5. An efficient time step control is available

The method which seems to satisfy all these requirements consists of discrete projection schemes with nonconforming linear or rotated multi linear finite elements for  $u$  and piecewise constant approximations for  $p$ . With this approach, very efficient solution schemes can developed of both coupled and projection type, with a special non-linear or linearized treatment of the advection. The resulting solutions are coincident (as soon as the time steps are small enough) and no spurious pressure oscillations occur. [4]

### 3. Introductory information about fluid dynamics and heat transfer

The object of this chapter is to present a fundamental knowledge and some basic notions of heat transfer and fluid dynamics which are in conformity with the ones our project implements.

These two topics are vast fields that include enormous amount of knowledge and information, so only a little basic portion of them are going to be explained here.

#### 3.1. Thermodynamic Systems and Processes

Thermodynamics involves the study of various systems. A *system* in thermodynamics is nothing more than the collection of matter that is being studied. A system could be the water within one side of a heat exchanger, the fluid inside a length of pipe, or the entire lubricating oil system for a diesel engine. Determining the boundary to solve a thermodynamic problem for a system will depend on what information is known about the system and what question is asked about the system.

Everything external to the system is called the *thermodynamic surroundings*, and the system is separated from the surroundings by the *system boundaries*. These boundaries may either be fixed or movable. In many cases, a thermodynamic analysis must be made of a device, such as a heat exchanger, that involves a flow of mass into and/or out of the device. The procedure that is allowed in such an analysis is to specify a control surface, such as the heat exchanger tube walls. Mass, as well as heat and work (and momentum), may flow across the control surface.[1]

##### 3.1.1. Types of thermodynamic systems

Systems in thermodynamics are classified as isolated, closed, or open based on the possible transfer of mass and energy across the system boundaries. An *isolated system* is one that is not influenced in any way by the surroundings. This means that no energy in the form of heat or work may cross the boundary of the system. In addition, no mass may cross the boundary of the system.

A thermodynamic system is defined as a quantity of matter of fixed mass and identity upon which attention is focused for study. A *closed system* has no transfer of mass with its surroundings, but may have a transfer of energy (either heat or work) with its surroundings.

An *open system* is one that may have a transfer of both mass and energy with its surroundings.[1]

### 3.1.2. Types of thermodynamic processes

Whenever one or more of the properties of a system change, a change in the state of the system occurs. The path of the succession of states through which the system passes is called the *thermodynamic process*. One example of a thermodynamic process is increasing the temperature of a fluid while maintaining a constant pressure. Another example is increasing the pressure of a confined gas while maintaining a constant temperature.

When a system in a given initial state goes through a number of different changes in state (going through various processes) and finally returns to its initial values, the system has undergone a *cyclic process or cycle*. Therefore, at the conclusion of a cycle, all the properties have the same value they had at the beginning. Steam (water) that circulates through a closed cooling loop undergoes a cycle.

A *reversible process* for a system is defined as a process that, once having taken place, can be reversed, and in so doing leaves no change in either the system or surroundings. In other words the system and surroundings are returned to their original condition before the process took place. In reality, there are no truly reversible processes; however, for analysis purposes, one uses reversible to make the analysis simpler, and to determine maximum theoretical efficiencies. Therefore, the reversible process is an appropriate starting point on which to base engineering study and calculation.

An *irreversible process* is a process that cannot return both the system and the surroundings to their original conditions. That is, the system and the surroundings would not return to their original conditions if the process was reversed.

An *adiabatic process* is one in which there is no heat transfer into or out of the system. The system can be considered to be perfectly insulated.[1]

### 3.2. Heat Transfer

Heat is energy transferred as the result of a temperature difference. When a temperature difference exists across a boundary, the Second Law of Thermodynamics indicates the natural flow of energy is from the hotter body to the colder body. The Second Law of Thermodynamics denies the possibility of ever completely converting into work all the heat supplied to a system operating in a cycle. The Second Law of Thermodynamics, described by Max Planck in 1903, states that:

*It is impossible to construct an engine that will work in a complete cycle and produce no other effect except the raising of a weight and the cooling of a reservoir.*

The second law says that if you draw heat from a reservoir to raise a weight, lowering the weight will not generate enough heat to return the reservoir to its original temperature, and eventually the cycle will stop. If two blocks of metal at different temperatures are thermally insulated from their surroundings and are brought into contact with each other the heat will flow from the hotter to the colder. Eventually the two blocks will reach the same temperature, and heat transfer will cease. Energy has

not been lost, but instead some energy has been transferred from one block to another. [2]

### **3.2.1. Modes of Heat Transfer**

Heat is always transferred when a temperature difference exists between two bodies or phases. There are three basic modes of heat transfer:

*Conduction* involves the transfer of heat by the interactions of atoms or molecules of a material through which the heat is being transferred.

*Convection* involves the transfer of heat by the mixing and motion of macroscopic portions of a fluid.

*Radiation*, or radiant heat transfer, involves the transfer of heat by electromagnetic radiation that arises due to the temperature of a body. [2]

Therefore, in our system, the heat transfer occurs between water and heating tubes by conduction. However, heat transfer by convection takes place between the water molecules to form homogenous heat distribution at the outlet.

### **3.2.2. Heat Exchangers**

The transfer of thermal energy between fluids is one of the most important and frequently used processes in engineering. The transfer of heat is usually accomplished by means of a device known as a heat exchanger. Common applications of heat exchangers in the nuclear field include boilers, fan coolers, cooling water heat exchangers, and condensers.

The basic design of a heat exchanger normally has two fluids of different temperatures separated by some conducting medium. The most common design has one fluid flowing through metal tubes and the other fluid flowing around the tubes. On either side of the tube, heat is transferred by convection. Heat is transferred through the tube wall by conduction.

Heat exchangers may be divided into several categories or classifications. In the most commonly used type of heat exchanger, two fluids of different temperature flow in spaces separated by a tube wall. They transfer heat by convection and by conduction through the wall. This type is referred to as an "ordinary heat exchanger," as compared to the other two types classified as "regenerators" and "cooling towers."

An ordinary heat exchanger is single-phase or two-phase. In a single-phase heat exchanger, both of the fluids (cooled and heated) remain in their initial gaseous or liquid states. In two-phase exchangers, either of the fluids may change its phase during the heat exchange process. The steam generator and main condenser of nuclear facilities are of the two-phase, ordinary heat exchanger classification. [2]

Single-phase heat exchangers are usually of the tube-and-shell type; that is, the exchanger consists of a set of tubes in a container called a shell. At the ends of the heat exchanger, the tube-side fluid is separated from the shell-side fluid by a tube

sheet. The design of two-phase exchangers is essentially the same as that of single-phase exchangers. [2]

Applications of heat exchangers may be classified as either regenerative or non-regenerative. The non-regenerative application is the most frequent and involves two separate fluids. One fluid cools or heats the other with no interconnection between the two fluids. Heat that is removed from the hotter fluid is usually rejected to the environment or some other heat sink

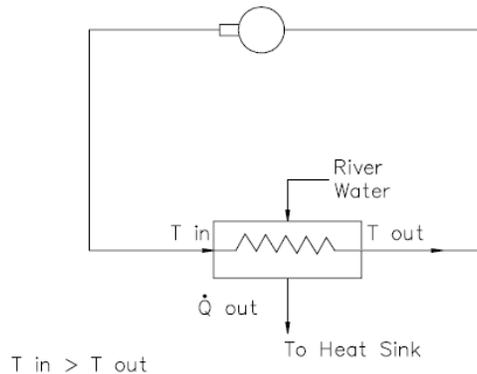


Figure 1: Non-regenerative heat exchanger schema [2]

A regenerative heat exchanger typically uses the fluid from a different area of the same system for both the hot and cold fluids. An example of both regenerative and non-regenerative heat exchangers working in conjunction is commonly found in the purification system of a reactor facility. The primary coolant to be purified is drawn out of the primary system, passed through a regenerative heat exchanger, non-regenerative heat exchanger, demineralizer, back through the regenerative heat exchanger, and returned to the primary system.

In the regenerative heat exchanger, the water returning to the primary system is pre-heated by the water entering the purification system. This accomplishes two objectives. The first is to minimize the thermal stress in the primary system piping due to the cold temperature of the purified coolant being returned to the primary system.

The second is to reduce the temperature of the water entering the purification system prior to reaching the non-regenerative heat exchanger, allowing use of a smaller heat exchanger to achieve the desired temperature for purification. The primary advantage of a regenerative heat exchanger application is conservation of system energy (that is, less loss of system energy due to the cooling of the fluid).

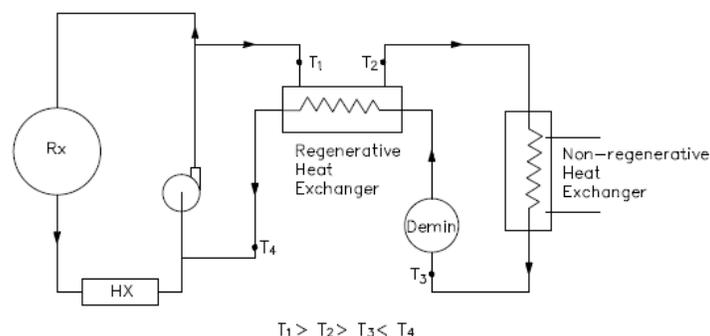


Figure 2: Regenerative Heat Exchanger Schema [2]

### 3.3. Fluid Dynamics

Fluid flow is an important part of most industrial processes; especially those involving the transfer of heat. Frequently, when it is desired to remove heat from the point at which it is generated, some type of fluid is involved in the heat transfer process. Examples of this are the cooling water circulated through a gasoline or diesel engine, the air flow past the windings of a motor, and the flow of water through the core of a nuclear reactor. Fluid flow systems are also commonly used to provide lubrication.

Even though a detailed analysis of fluid flow can be extremely difficult, the basic concepts involved in fluid flow problems are fairly straightforward. These basic concepts can be applied in solving fluid flow problems through the use of simplifying assumptions and average values, where appropriate. Even though this type of analysis would not be sufficient in the engineering design of systems, it is very useful in understanding the operation of systems and predicting the approximate response of fluid systems to changes in operating parameters.

#### 3.3.1. Properties of fluids

A *fluid* is any substance which flows because its particles are not rigidly attached to one another. This includes liquids, gases and even some materials which are normally considered solids, such as glass. Essentially, fluids are materials which have no repeating crystalline structure.

Some of several properties of fluids are temperature, pressure, mass, specific volume and density. *Temperature* was defined as the relative measure of how hot or cold a material is. It can be used to predict the direction that heat will be transferred. *Pressure* was defined as the force per unit area. Common units for pressure are pounds force per square inch (psi). *Mass* was defined as the quantity of matter contained in a body and is to be distinguished from weight, which is measured by the pull of gravity on a body. The *specific volume* of a substance is the volume per unit mass of the substance. Typical units are  $\text{m}^3/\text{kg}$ . *Density*, on the other hand, is the mass of a substance per unit volume. Typical units are  $\text{kg}/\text{m}^3$ . Density and specific volume are the inverse of one another. Both density and specific volume are dependant on the temperature and somewhat on the pressure of the fluid. As the temperature of the fluid increases, the density decreases and the specific volume increases. Since liquids are considered incompressible, an increase in pressure will result in no change in density or specific volume of the liquid. In actuality, liquids can be slightly compressed at high pressures, resulting in a slight increase in density and a slight decrease in specific volume of the liquid. [3]

*Compressibility* is the measure of the change in volume a substance undergoes when a pressure is exerted on the substance. Liquids are generally considered to be incompressible. For instance, a pressure of 16,400 psig will cause a given volume of water to decrease by only 5% from its volume at atmospheric pressure. Gases on the other hand, are very compressible. The volume of a gas can be readily changed by exerting an external pressure on the gas. [3]

### 3.3.2. Flow regimes

All fluid flow is classified into one of two major categories or regimes. These two flow regimes are laminar flow and turbulent flow. The flow regime, whether laminar or turbulent, is important in the design and operation of any fluid system. The amount of fluid friction, which determines the amount of energy required to maintain the desired flow, depends upon the mode of flow. This is also an important consideration in certain applications that involve heat transfer to the fluid.

*Laminar flow* is also referred to as streamline or viscous flow. These terms are descriptive of the flow because, in laminar flow, layers of water flowing over one another at different speeds with virtually no mixing between layers, fluid particles move in definite and observable paths or streamlines, and the flow is characteristic of viscous (thick) fluid or is one in which viscosity of the fluid plays a significant part.

Turbulent flow is characterized by the irregular movement of particles of the fluid. There is no definite frequency as there is in wave motion. The particles travel in irregular paths with no observable pattern and no definite layers.

The flow regime (either laminar or turbulent) is determined by evaluating the Reynolds number of the flow. The *Reynolds number*, based on studies of Osborn Reynolds, is a dimensionless number comprised of the physical characteristics of the flow.

$$N_R = \rho v D / \mu g_c$$

where:

$N_R$  = Reynolds number (unitless)

$v$  = average velocity (m/sec)

$D$  = diameter of pipe (m)

$\mu$  = absolute viscosity of fluid (kg.sec/m<sup>2</sup>)

$\rho$  = fluid mass density (kg/m<sup>3</sup>)

$g_c$  = gravitational constant (9.8 m/sec<sup>2</sup>)

## 4. Geometry, physical data, boundary conditions

In this section, the readers can find the most important information such as geometry, physical problem properties and boundary conditions of the considered case. As mentioned above, the system consists of two parts.

**4.1. Main part:** This part is square prismatic (shown at the picture below). It is the place where the water flows and the heating is realized. It is also long enough to let heated water mix and to have a homogeneous temperature at the output.

**Dimensions:** H=410 mm W= 410 mm L=2500mm

**4.2. Heating tubes:** This part consists of 13 small cylindrical tubes which form 5 rows. The number of cylinders in each row is 3-2-3-2-3. This aim of the design is to increase heating surface and to heat water as homogeneous as possible. The temperature of the pipes can be changed by controller in order to have desired output temperature. (All heating pipes have the same temperature)

**Dimensions:** r=35 mm H= 410 mm

The first line of tubes start from the distance of 660 mm from inlet and there exists a 100 mm distance between each line. Moreover, the distance between two tube centers in the same line is 140 mm.

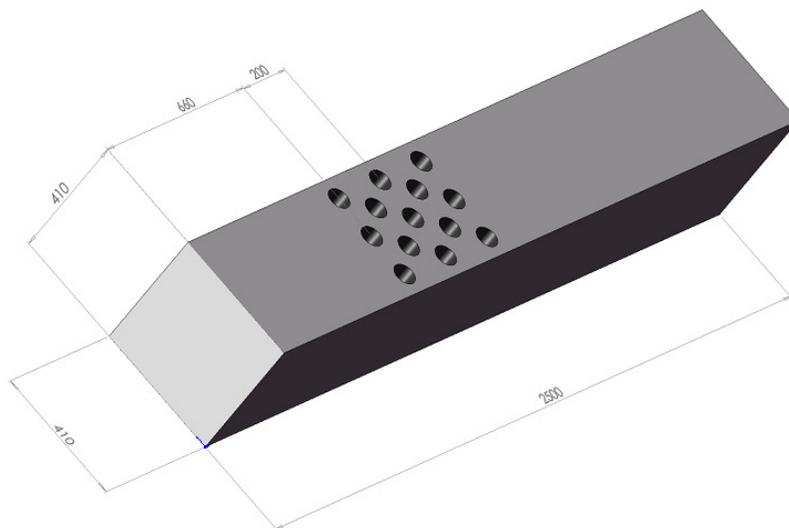


Figure 3: SolidWorks drawing and dimensioning of the heat exchanger

First the geometry was drawn in Solidworks as can be seen Figure 4, then this drawing was imported into ANSYS Workbench as in Figure 5 where meshes were created.

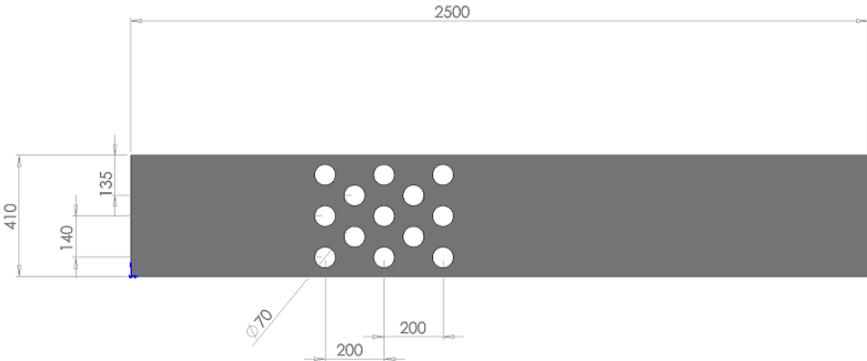


Figure 4: SolidWorks drawing and dimensioning of the heat exchanger, top view

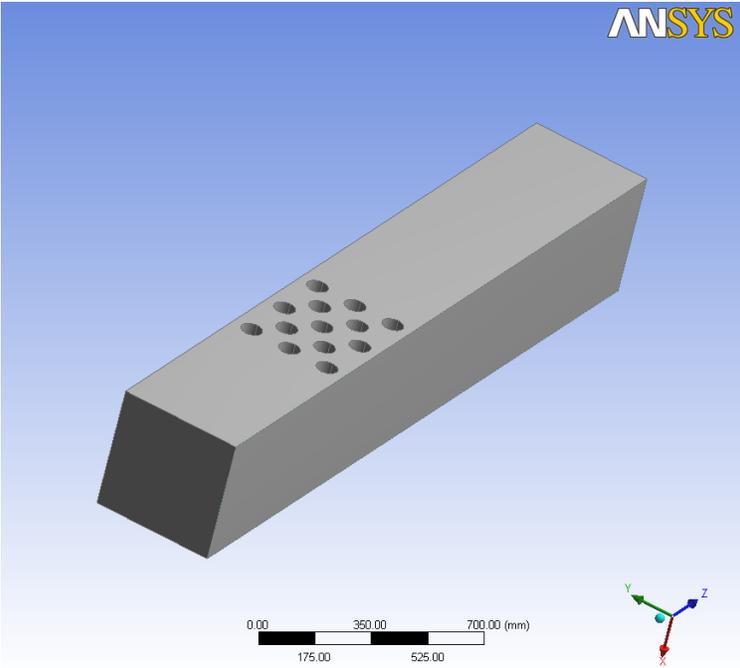


Figure 5: Ansys drawing of the heat exchanger

The inlet temperature of the water was set to 20 °C and constant velocity of 100 mm/s. Then it flows through the heating pipes and is heated up. At the end of the pipes, the water has a non-homogeneous temperature distribution. But it continues to flow through main part. At the same time, heat transfer occurs between hot and cold water.

This allows water to have a more homogeneous temperature distribution at the end of the main part (output).

In the project, it is accepted that, the walls of the main part is insulated and there is no heat transfer between water and the outside of the system. ( $dT/dn = 0$ , adiabatic).

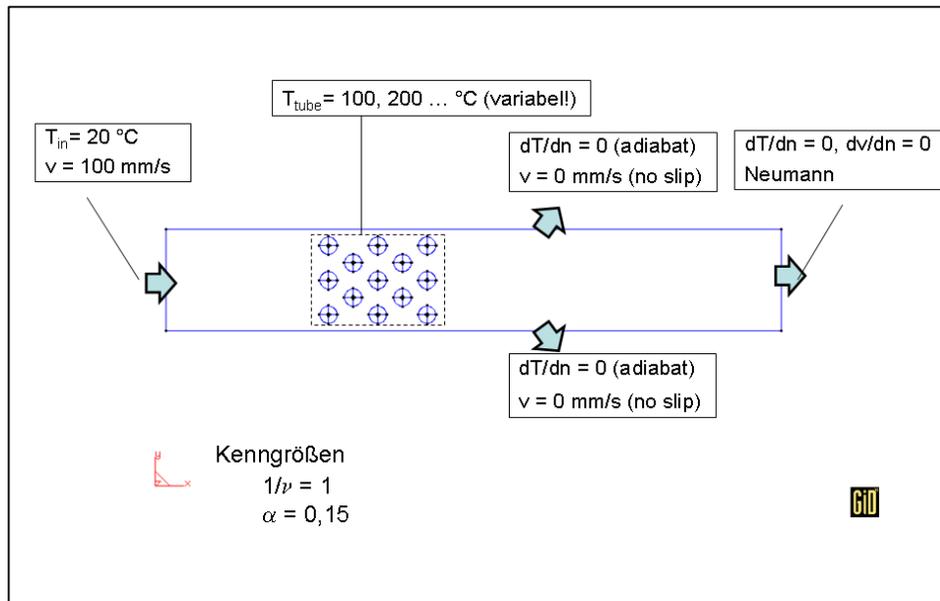


Figure 6: Heat exchanger scheme

Figure 6 shows a scheme of the considered heat exchanger from the simulation point of view. Also the used boundary conditions for the momentum and heat balance equation are depicted.

## 5. Prestudies

In order to be able to achieve the desired output temperature, we have carried out simulation experiments with different tube temperatures and noticed the steady state output temperature of each input value. The aim of this was finding out the exact tube temperature that must be applied to get 80° C.

### 5.1. Different steady states

In Table 1 the different steady states according with the tube temperature are shown. It seems clear that an input temperature between 110° C and 125° C will give the output we are looking for.

Tube temperature (°C)	Output temperature (°C)
100	69,84
110	75,77
125	82,13
140	85,45
150	100,99

Table 1 Steady states with different input temperature.

The idea is that in order to obtain good estimates for the parameters of a given linear model one should use more observations than there are parameters to be determined. So we used the linear least squares method to find out the exact tube temperature required to get the desired output. Figure 7 depicts the plotting of the five tube temperatures used as well as the outputs. Then the linear least squares gives us the linear-like plot which equation is

$$T_{\text{out}} = 0.5435 \cdot T_{\text{tube}} + 14.899 \text{ [K]}$$

where,

$T_{\text{tube}}$  is the tube temperature and  
 $T_{\text{out}}$  is the output temperature

Then it is easy to calculate the tube temperature out of this equation, putting  $y=80$ :

$$T_{\text{tube}} = (80 - 14.899) / 0.5435 = \mathbf{119.71}$$

This tube temperature will give us the desired steady state with an output temperature of 80°, but we want to have a more robust model in which the tube temperature is a function of the output temperature. How could the output temperature change if we already know the steady state temperature required to maintain the output temperature constant? Well, for instance, the inlet flow takes a while to reach first the tubes and then the outlet, where the temperature is measured; or maybe a tube fails and the others have to overcome the situation by heating up their selves.

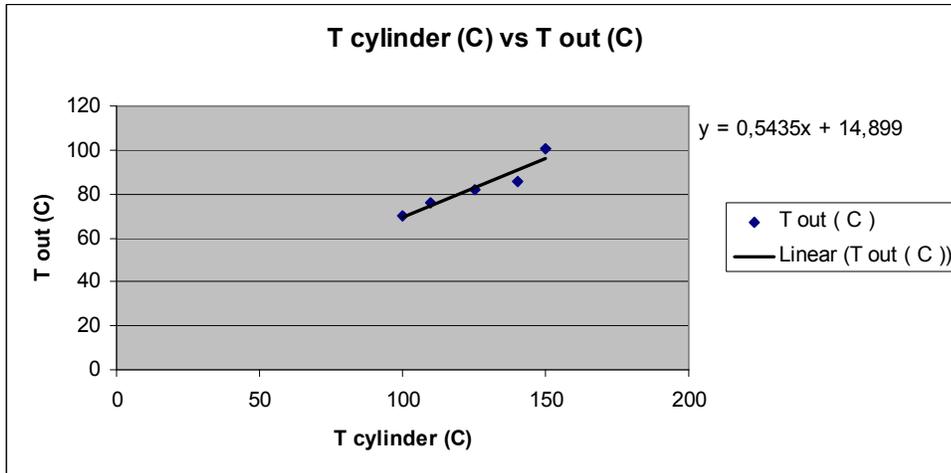


Figure 7: Linear least squares calculation

## 5.2. Step calculation

As stated in the previous section, the tube temperature has to be a function of the output temperature, for this we have designed a proportional integral (PI) controller which will help us to reach the desired output temperature with few oscillations and fast response.

The Proportional value determines the reaction to the current error; the Integral determines the reaction based on the sum of recent errors. The weighted sum of these two actions is used to adjust the process via a control element such as the tube temperature.

When increasing the tube temperature also the output temperature should increase. This is the step we want to know, how much the increased tube temperature affects the output temperature. Figure 8 shows how this calculation should be made.

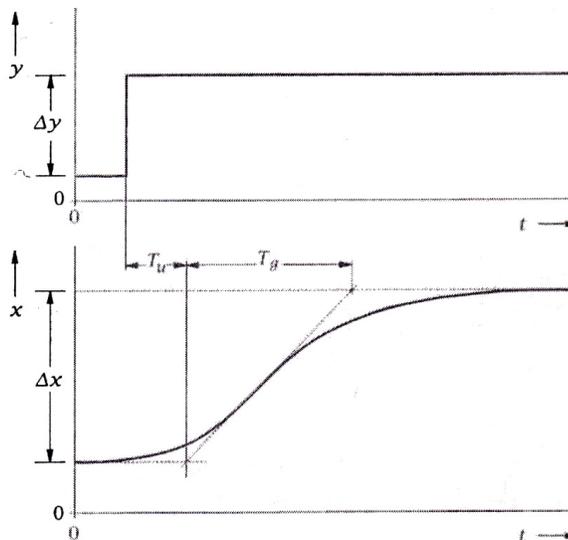


Figure 8: The step response parameters to be calculated  $\Delta x$ ,  $\Delta y$ ,  $T_u$  and  $T_g$ .

$\Delta y$  corresponds to change of operation condition (tube temperature),  $\Delta x$  corresponds the response of the system according to the change in operating conditions.

We conducted an experiment with our model. We set the tube temperature to 100° C, then after 1 second we increased it to 120° C. The step can be shown in Figure 9.

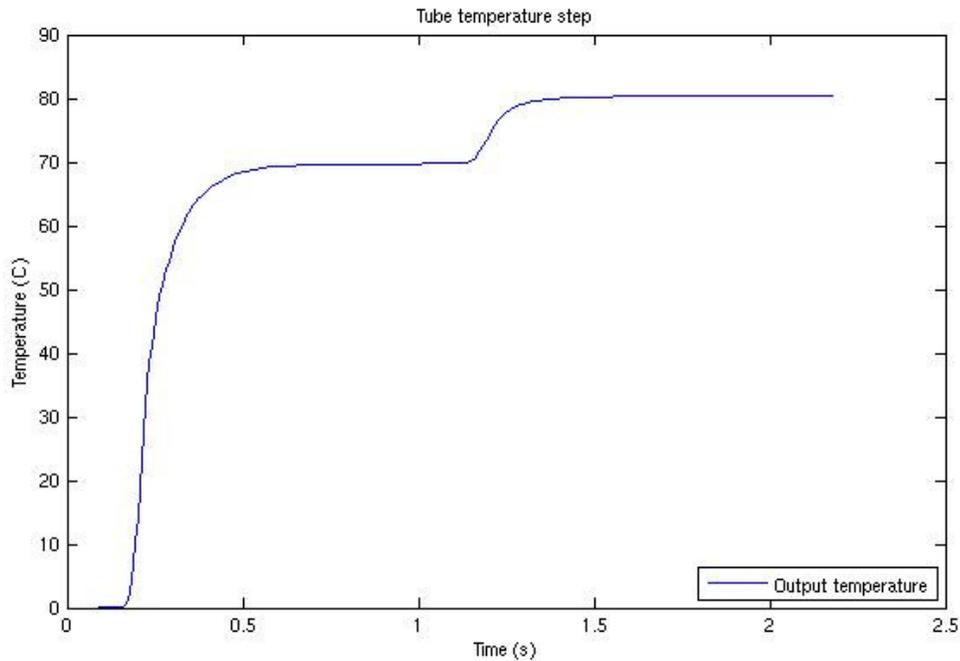


Figure 9: Step response of an increment of tube temperature from 100° C to 120° C.

We used the Schwarze method in order to find an appropriate controller. As shown in the Figure 8, delay time  $T_u$  and settling time  $T_g$ , which describe static and dynamic behavior of the controlled system with the transmission factor  $K_s = \Delta x / \Delta y$ , are found with the help of constructed tangent lines. Recommendations for the controller settings depend on the calculated specific values for a favorable behavior. Parameter values in the Table 2 are advised for  $T_u / T_g < 1/3$  as suitable setting values. The restriction  $T_u / T_g < 1/3$  states that the values calculated with these formulas are less suitable for the controlled system with distinctive dead time behavior. Based on the Schwarze method, the formulas for the controller parameters seem as follows;

$$K_p = \frac{0.6 T_g}{K_s T_u}$$

$$K_i = \frac{K_p}{T_n} = \frac{K_p}{4T_u}$$

We calculated the  $T_u$  and  $T_g$  parameters needed for the automation control (PI controller), as shown in Figure 10. The results showed that  $T_u$  is approximately equal to 0.15 and  $T_g$  is approximately equal to 0.13. Then  $\Delta x$  and  $\Delta y$  are 10 and 20 respectively. The controller parameters  $K_p$  and  $K_i$  are calculated below according to Schwarze method;

$$K_S = \frac{\Delta x}{\Delta y} = \frac{10}{20} = 0.5$$

$$K_P = \frac{0.6 T_g}{K_S T_u} = \frac{0.6 \cdot 0.13}{0.5 \cdot 0.15} \cong 1$$

$$K_I = \frac{K_P}{T_n} = \frac{K_P}{4T_u} = \frac{1}{0.6} \cong 1.66$$

Regler		Aperiodischer Regelverlauf		Regelverlauf mit 20% Überschwingen	
		Störung	Führung	Störung	Führung
P	$K_R$	$\frac{0.3 \cdot T_g}{K_S \cdot T_u}$	$\frac{0.3 \cdot T_g}{K_S \cdot T_u}$	$\frac{0.7 \cdot T_g}{K_S \cdot T_u}$	$\frac{0.7 \cdot T_g}{K_S \cdot T_u}$
		$\frac{0.6 \cdot T_g}{K_S \cdot T_u}$	$\frac{0.35 \cdot T_g}{K_S \cdot T_u}$	$\frac{0.7 \cdot T_g}{K_S \cdot T_u}$	$\frac{0.6 \cdot T_g}{K_S \cdot T_u}$
PI	$T_n$	$4 T_u$	$1.2 T_g$	$2.3 T_u$	$1 T_g$
	$K_R$	$\frac{0.95 \cdot T_g}{K_S \cdot T_u}$	$\frac{0.6 \cdot T_g}{K_S \cdot T_u}$	$\frac{1.2 \cdot T_g}{K_S \cdot T_u}$	$\frac{0.95 \cdot T_g}{K_S \cdot T_u}$
PID	$T_n$	$2.4 T_u$	$1 T_g$	$2 T_u$	$1.35 T_g$
	$T_v$	$0.42 T_u$	$0.5 T_u$	$0.42 T_u$	$0.47 T_u$

Table 2: Calculation of PI controller parameters [6]

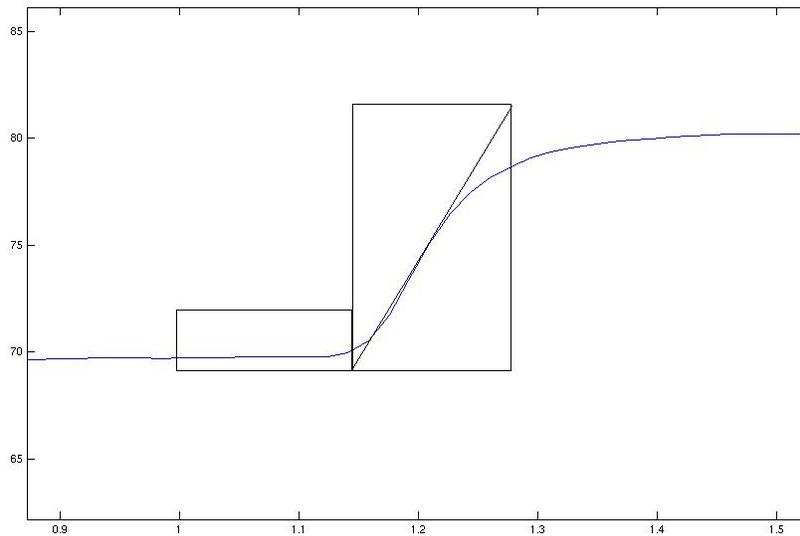


Figure 10: Calculation of  $T_u$  and  $T_g$

In our case  $T_u/T_g \approx 1$ , so that we expect a less reliable, robust controller behavior.

## 6. Implementation of mathematical models for automation and control

### 6.1. 1<sup>st</sup> approach

#### System 1.1

To begin with our mathematical method implementation for controlling our process, a simple method is applied initially to observe the effects and requirement of developing advanced control mechanisms.

The algorithm of our initial simple system is as follows;

1. 100 °C degrees of temperature is applied initially to tubes,
2. at each step, the output error which is obtained by subtracting the mean output temperature from the goal temperature of 80 °C degrees, is calculated,
3. if the error is greater than 0, the tube temperature is increased by 0.25 °C degrees; if the error is less than 0, the tube temperature is decreased by 0.25 °C degrees,
4. the tube temperature values are always kept in the range of 50 C -130 C degrees.

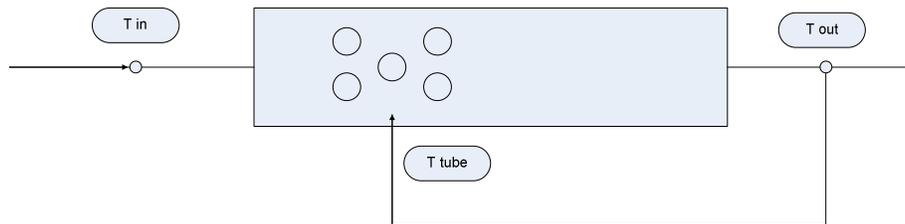


Figure 11: Schematic implementation of control system

After the computational simulations and calculations the following mean output graph is obtained.

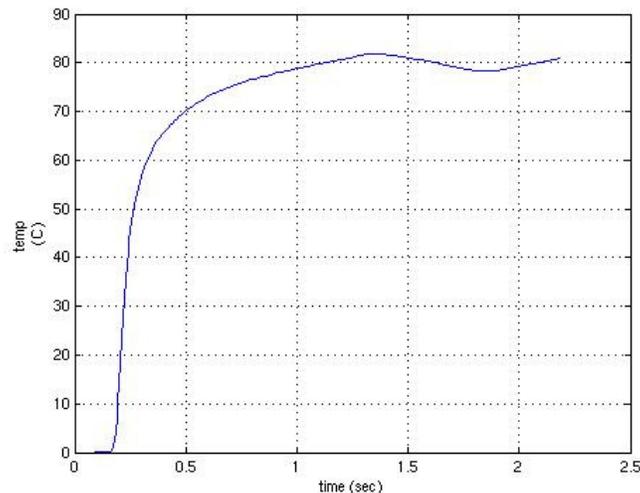


Figure 12: Output temperature of 1<sup>st</sup> approach

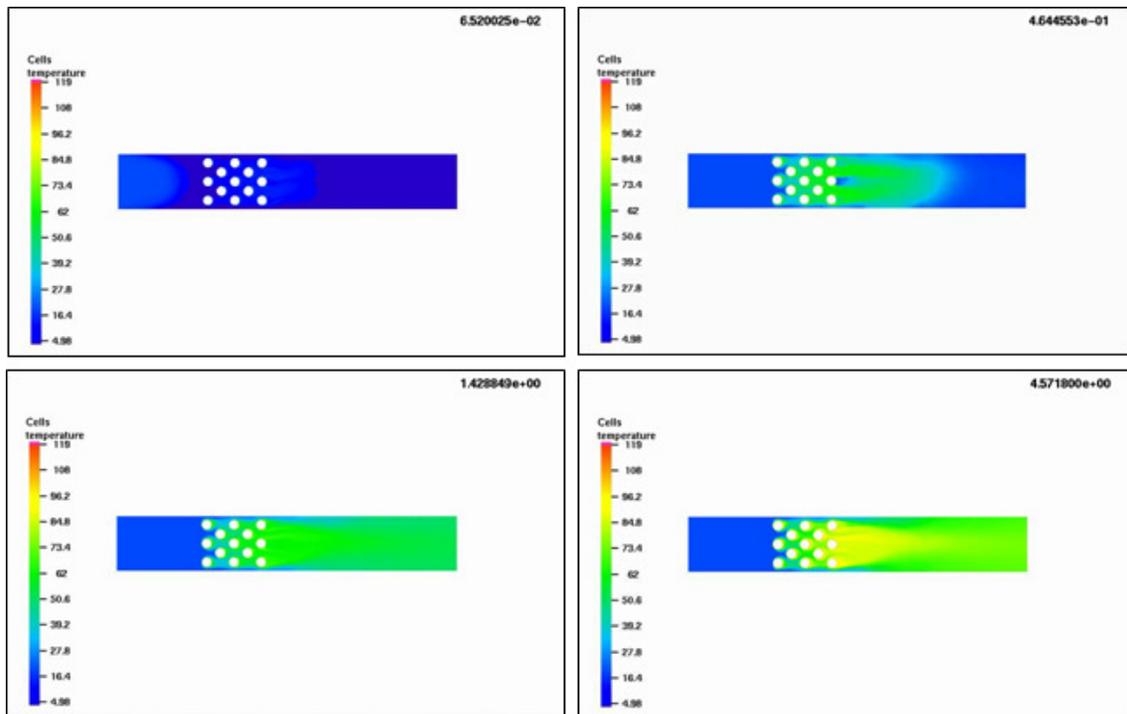
So, it is obvious that the process is too slow and even if the mean output temperature value reaches to goal temperature, the stability cannot be obtained, and the output value oscillates around 80 °C degrees, at least for a long time period. Therefore, in order to satisfy the needs of our project, two different methods are designed and outcomes of the two different approaches are presented.

## 6.2 2<sup>nd</sup> approach:

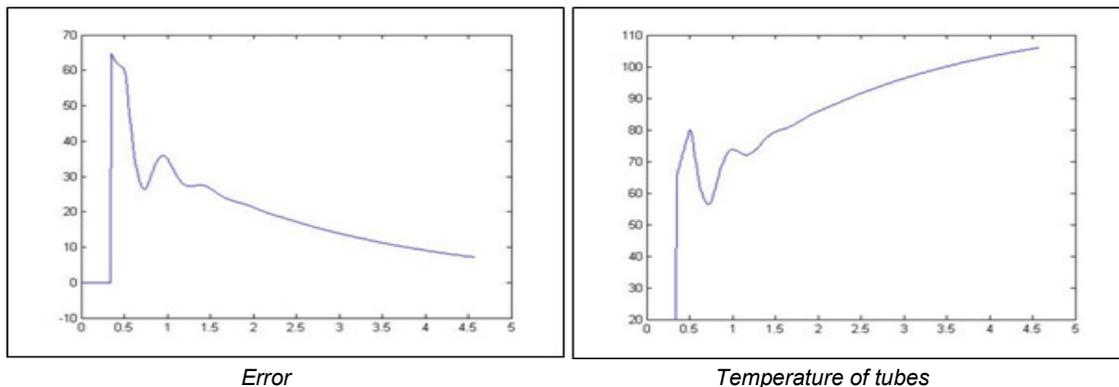
### 6.2.1. System 2.1

After achieving a mathematical model for the system using Schwarze method we designed several controllers for regulating the system within the desired output. First we used  $K_i=1.66$  and  $K_p=1$  for the parameters of PI controller and obtained the results below.

$$\text{Error} = T_{\text{out}} - T_{\text{out}}^* \text{ where } T_{\text{out}}^* = 80^\circ\text{C}$$



Samples of the system with  $K_i=1.66$   $K_p=1$



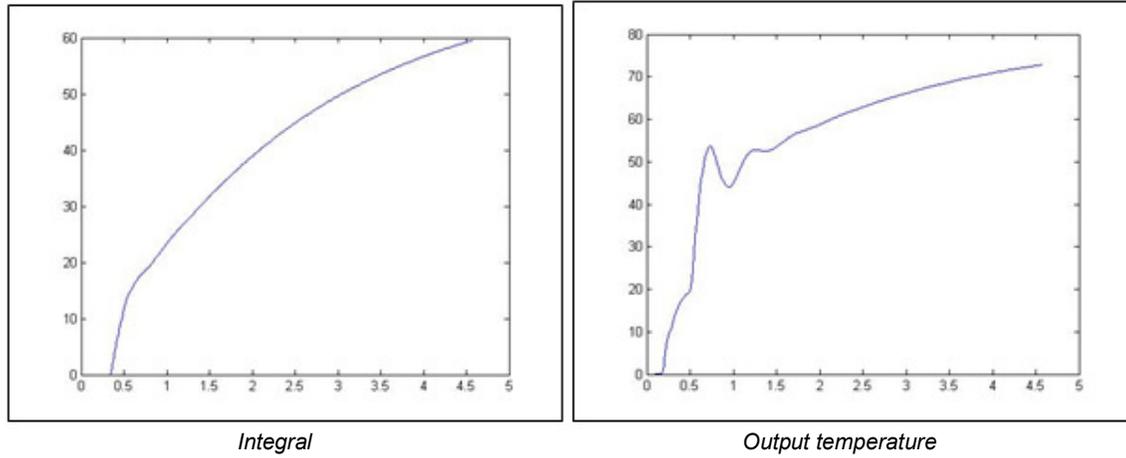
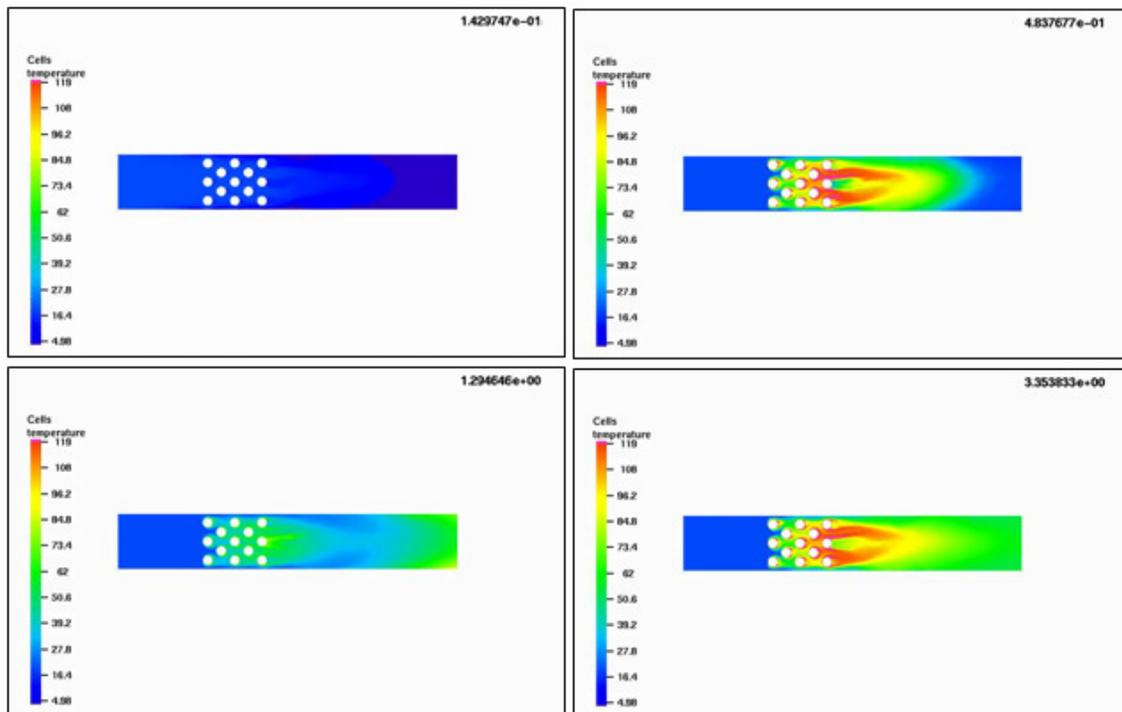


Figure 13: Results of system 2.1.

### 6.2.2. System 2.2

After getting these results we tried to enforce the system with a constant of 2 to speed up. Therefore we multiplied the input temperature of the tubes by 2. But the result was worse and the system was unstable.



Samples of the system with  $K_i=1.66$   $K_p=1$  and enforced by 2

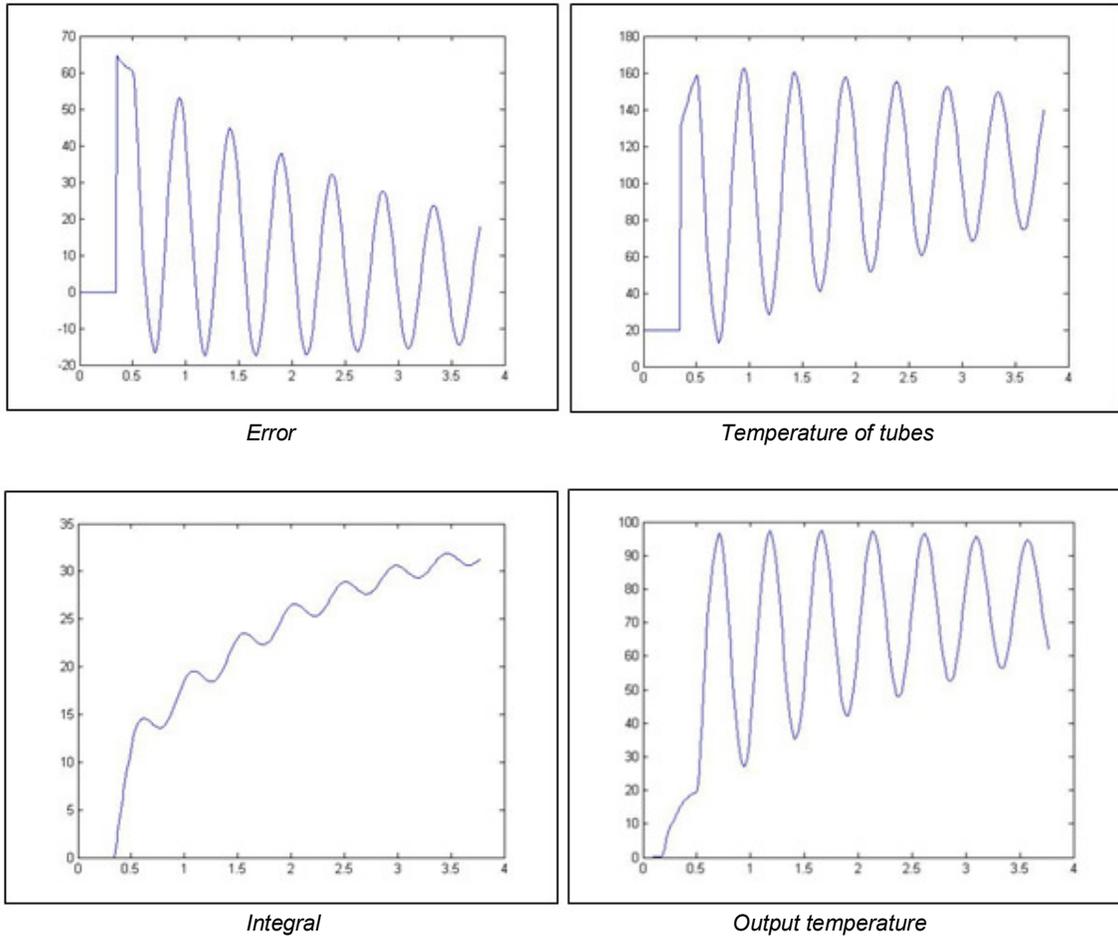
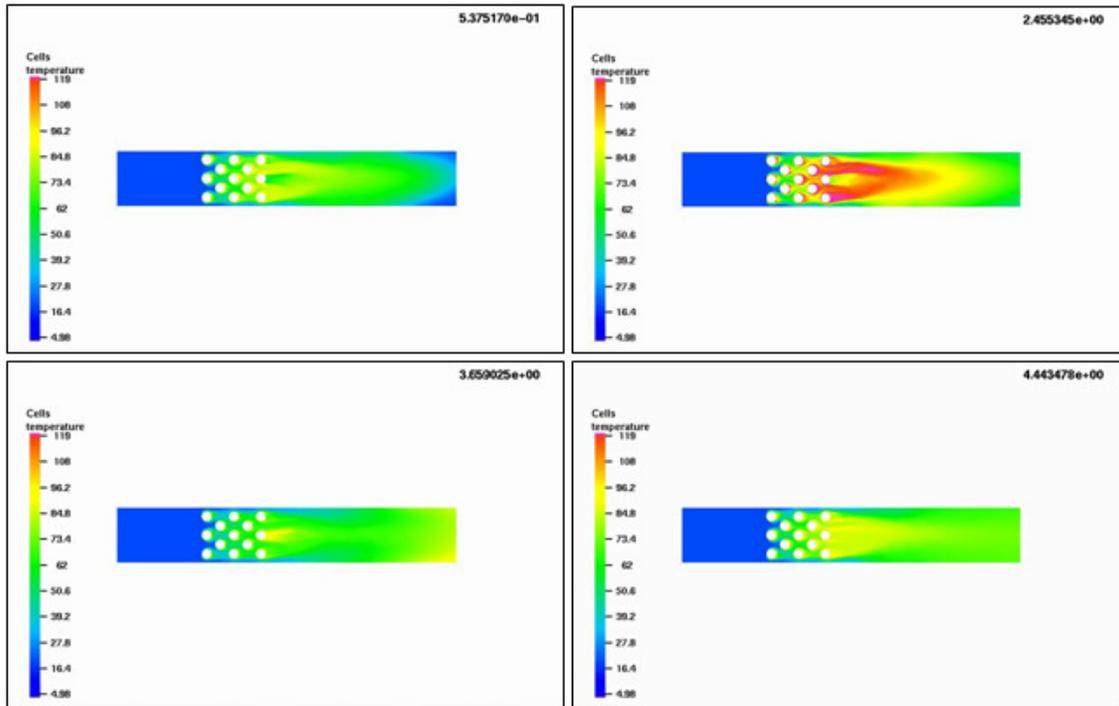


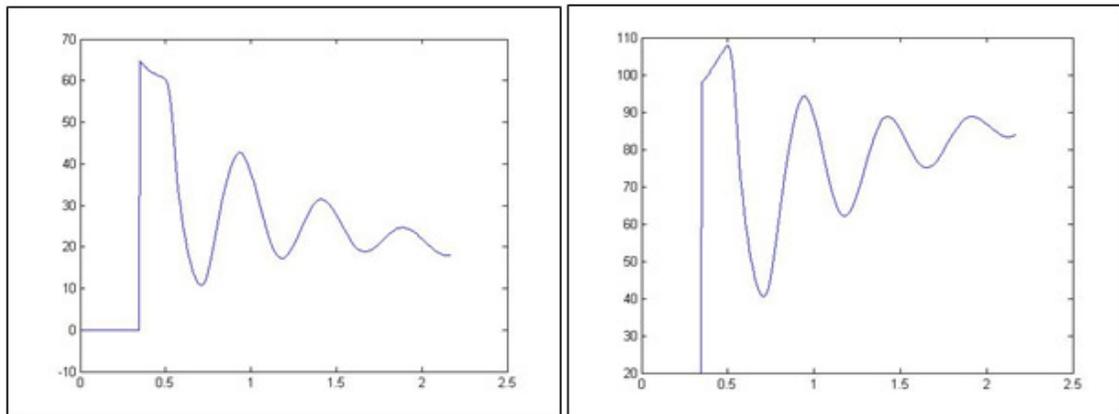
Figure 14: Results of system 2.2.

### 6.2.3. System 2.3

Getting a worse result by emphasizing the tube temperature, we just tuned the controller parameters such as  $K_i=K_p=1.5$ . Output temperature still wasn't like it should be.



Samples of the system with  $K_p=K_i=1.5$



Error

Temperature of tubes

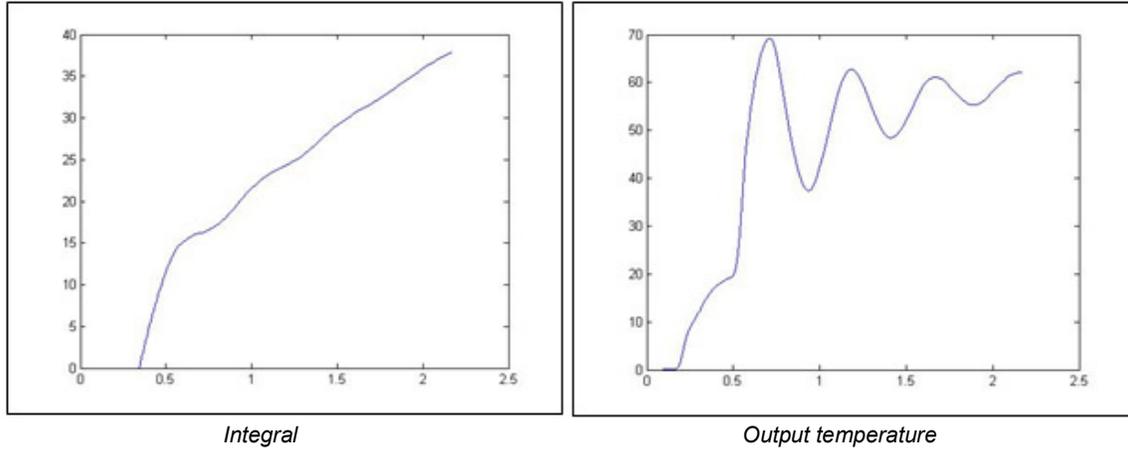
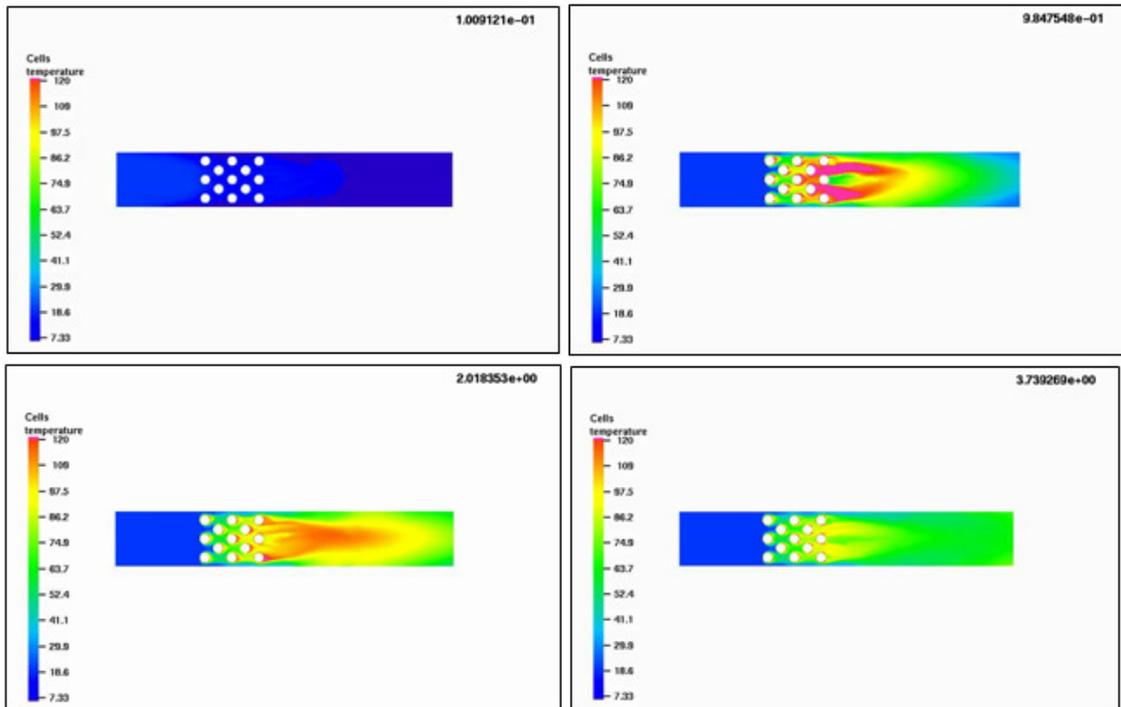


Figure 15: Results of system 2.3.

### 6.2.4. System 2.4

We went on increasing the controller parameters as  $K_i=3.5$  and  $K_p=2$ , but the system went on behaving unstable.



Samples of the system with  $K_i=3.5$   $K_p=2$

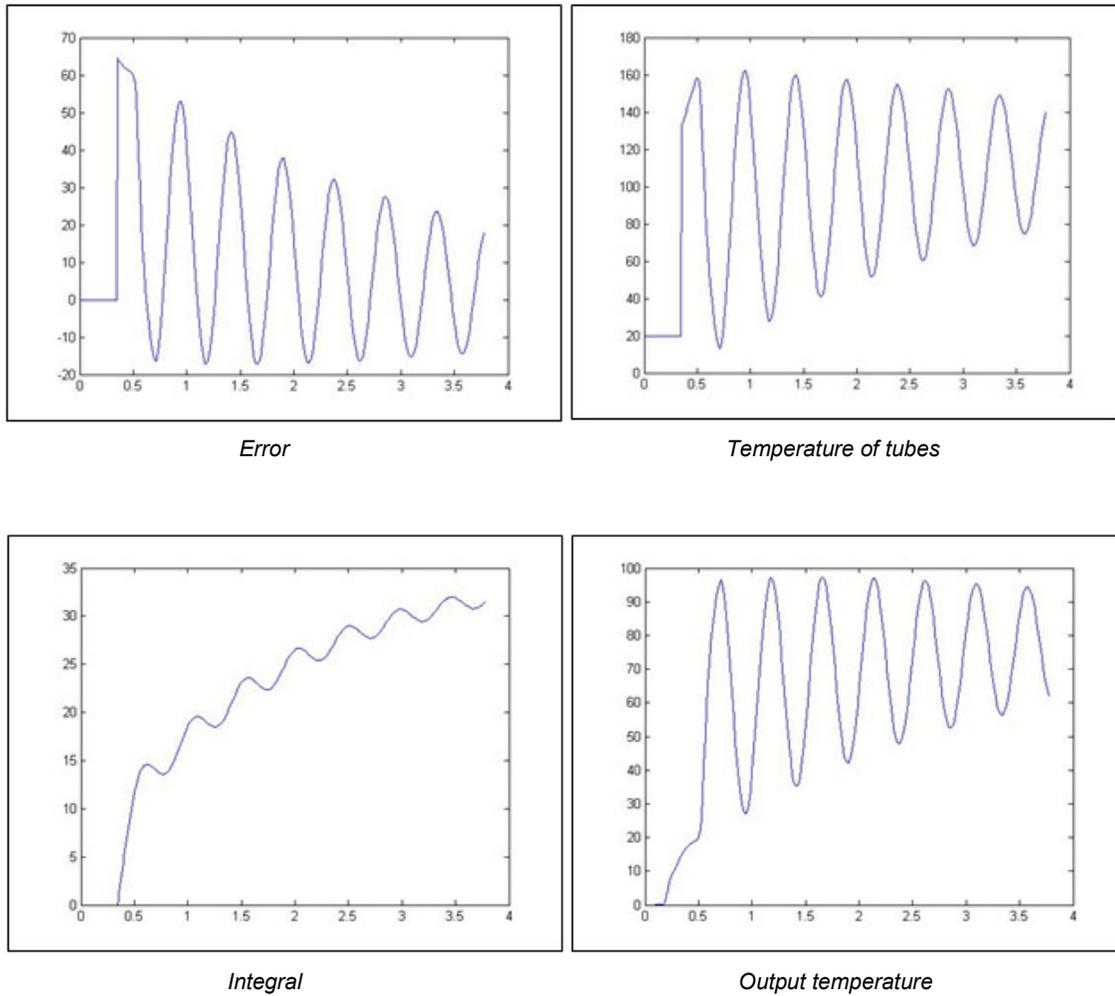
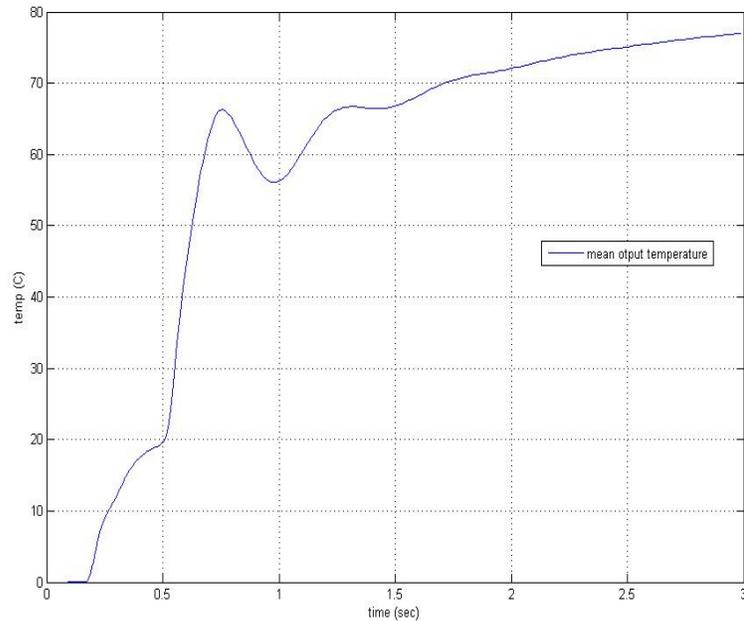


Figure 16. Results of system 2.4.

### 6.2.5. System 2.5

These dissatisfying results led us to decrease  $K_p$  value to obtain stability. Thus we kept the  $K_i$  value constant at  $K_i=3.5$  and changed the  $K_p$  value to 1 in order to see the effect of decreasing  $K_p$  value to stability



Output of the system with  $K_p=1$ ,  $K_i=3.5$

The graph clearly shows that the mean output temperature stabilizes when the  $K_p$  value is decreased to 1, however the process is slow and there will be a steady state error. To have a better understanding of the integral and proportion contributions to the system the next graph is introduced.

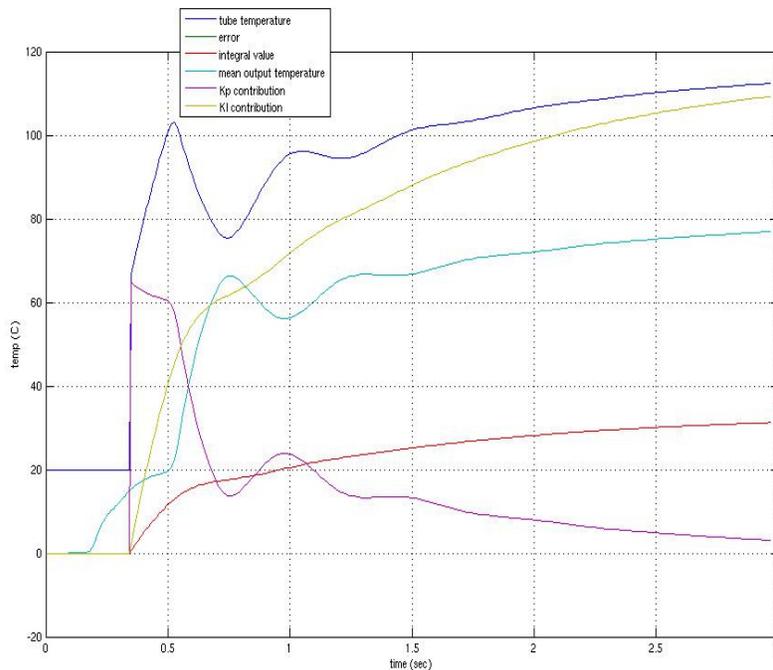
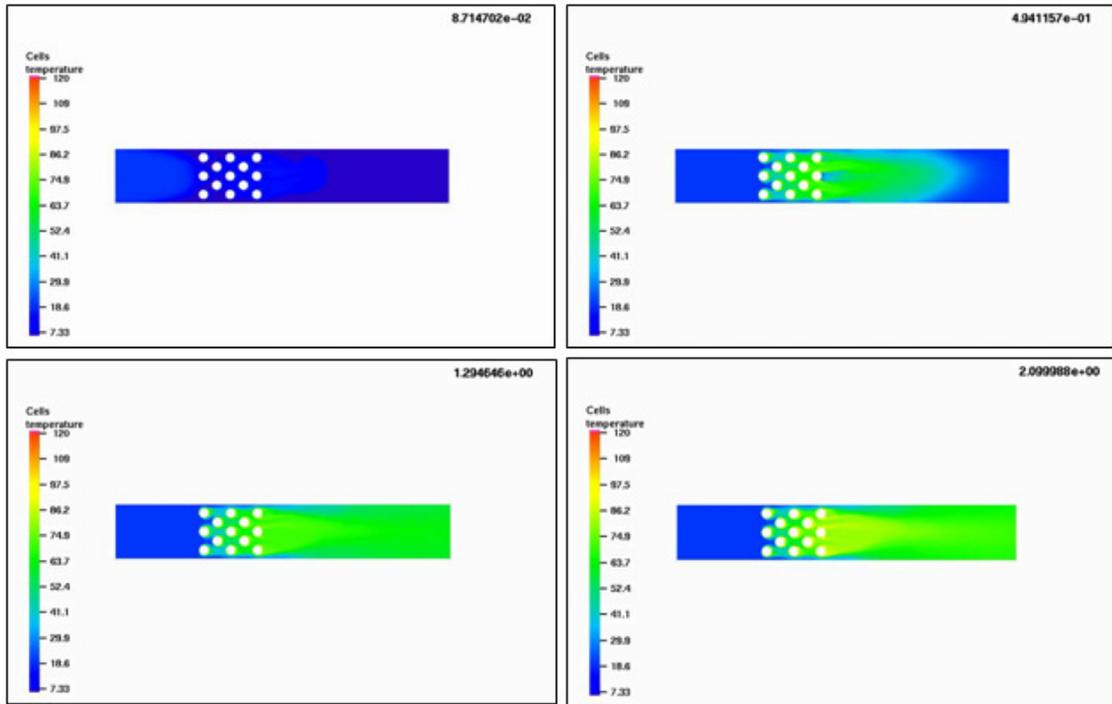


Figure 17: Results of system 2.5.

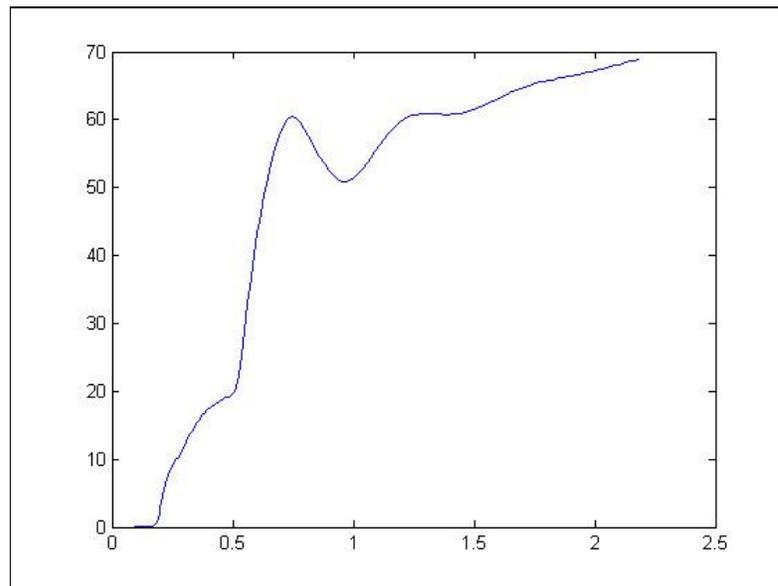
As can clearly be seen, the contribution to the tube temperature values of integral part is increasing with the time, whereas the proportion contribution is decreasing. Now the next step should be the increase in the speed of the reaction of the system, while keeping the stability constant.

### 6.2.6. System 2.6

In order to see the effects of changing the integral term, we decreased the controller parameters to  $K_i=2.66$   $K_p=1$



*Samples of the system with  $K_i=2.66$   $K_p=1$*

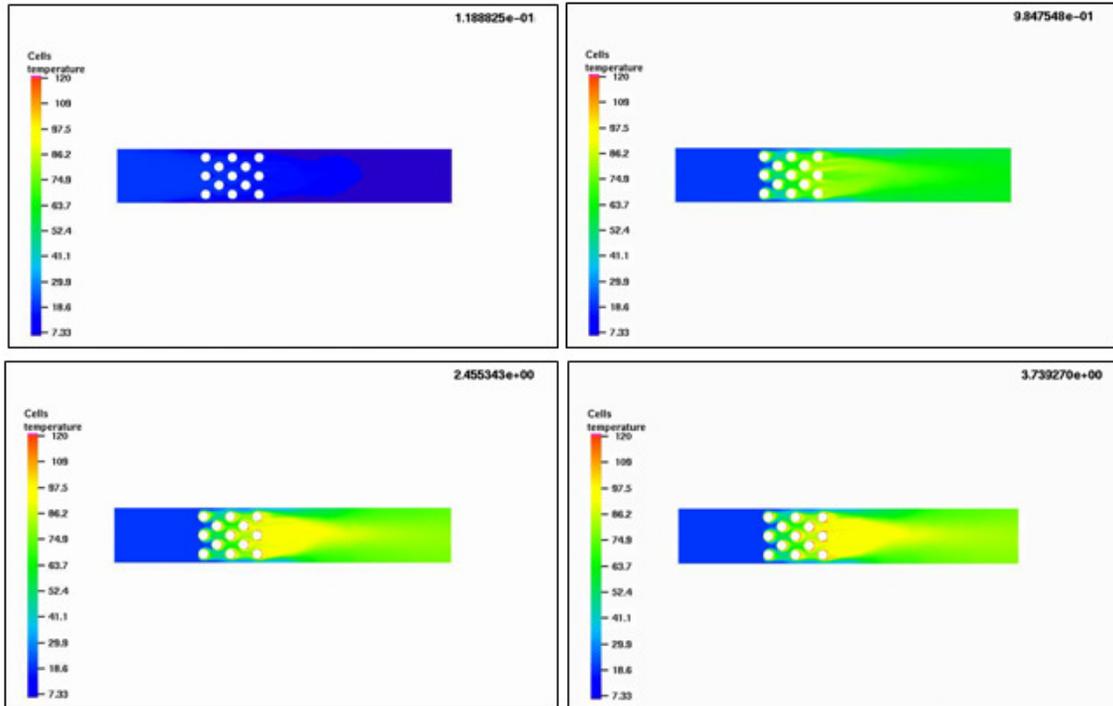


*Output of the system*

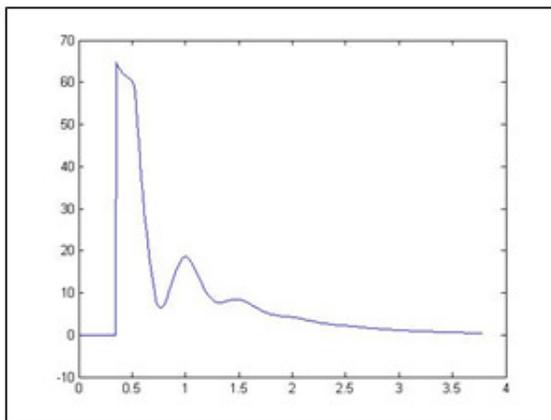
Figure 18: Results of system 2.6.

### 6.2.7. System 2.7

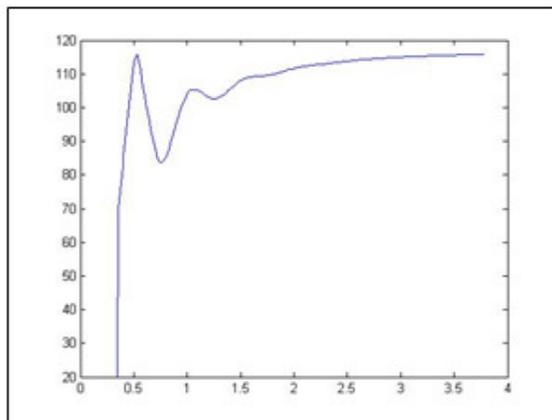
By decreasing the integral term, the system became stable but still slow. Therefore we increased integral term to  $K_i=4.5$  but proportional term stayed as  $K_p=1$ .



Samples of the system with  $K_i= 4.5$   $K_p=1$



Error



Temperature of tubes

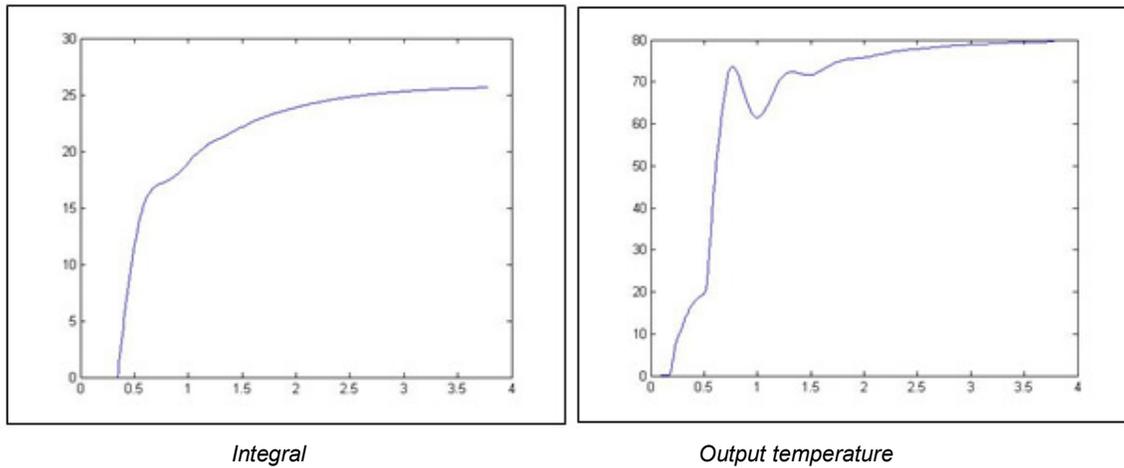
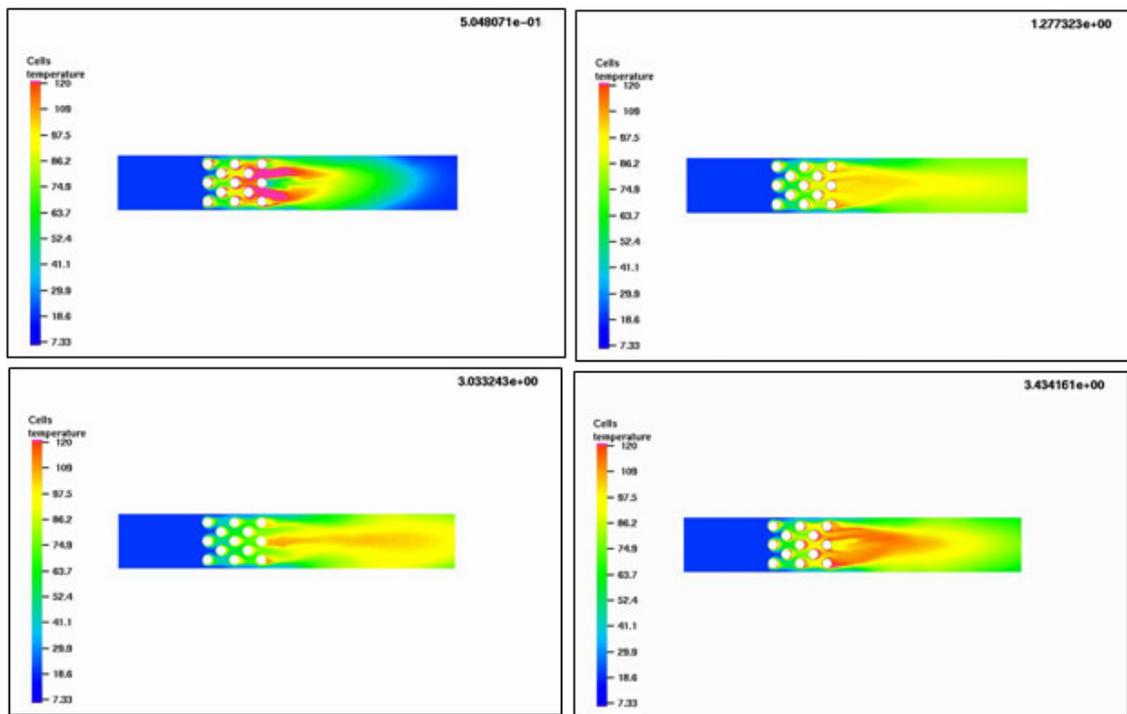


Figure 19: Results of system 2.7.

### 6.2.8. System 2.8

Control of the system seems better but it should be faster. Therefore we increased the integral term as  $K_i=10$  while fixing the proportional term to  $K_p=1$ .



Samples of the system with  $K_i=10$   $K_p= 1$

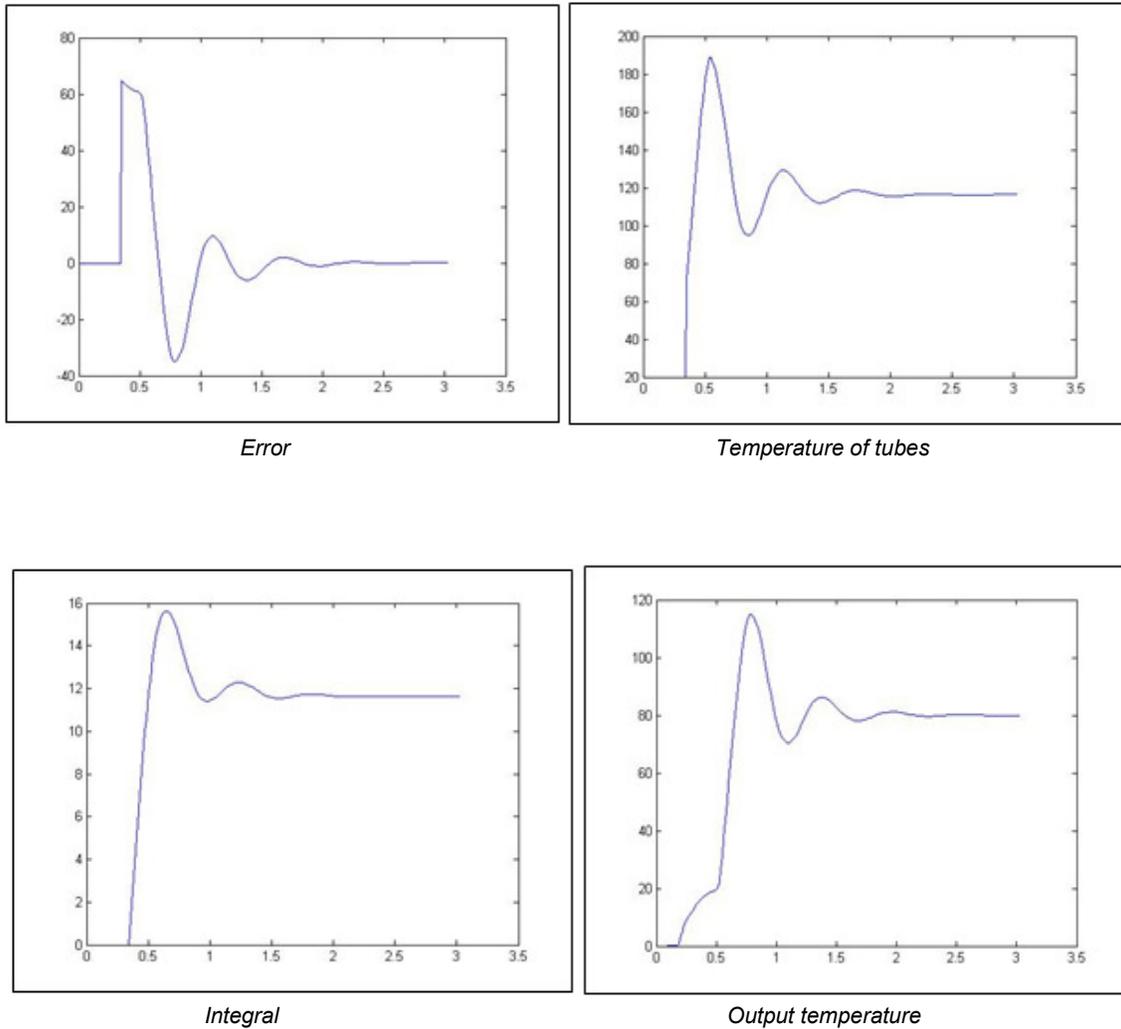
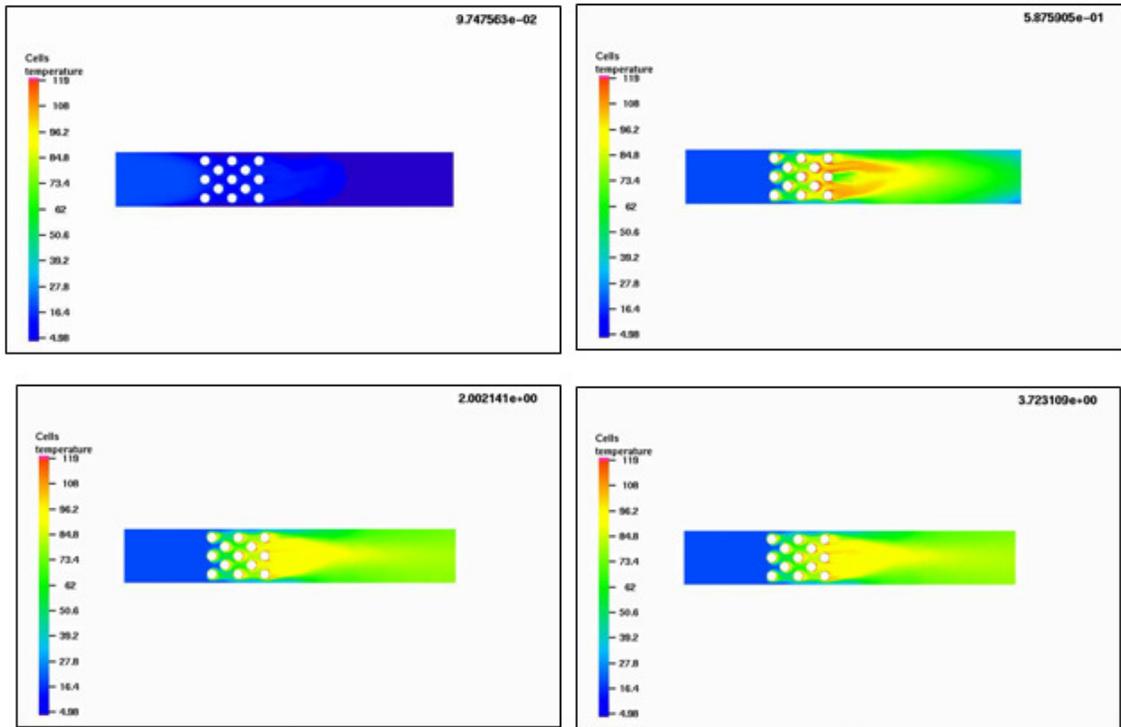


Figure 20: Results of system 2.8.

### 6.2.9. System 2.9

By increasing the integral term steady state error converged to zero but we still had overshoot around %50. Therefore, we needed to decrease the integral term. After some calculations we agreed to have an integral term of  $K_i=6$  and fix proportional term of  $K_p=1$ . The result was satisfying with a 6% of overshoot and settling time of around 2 seconds which the system output stays in a band of 3%.



Samples of the system with  $K_i=6$   $K_p=1$

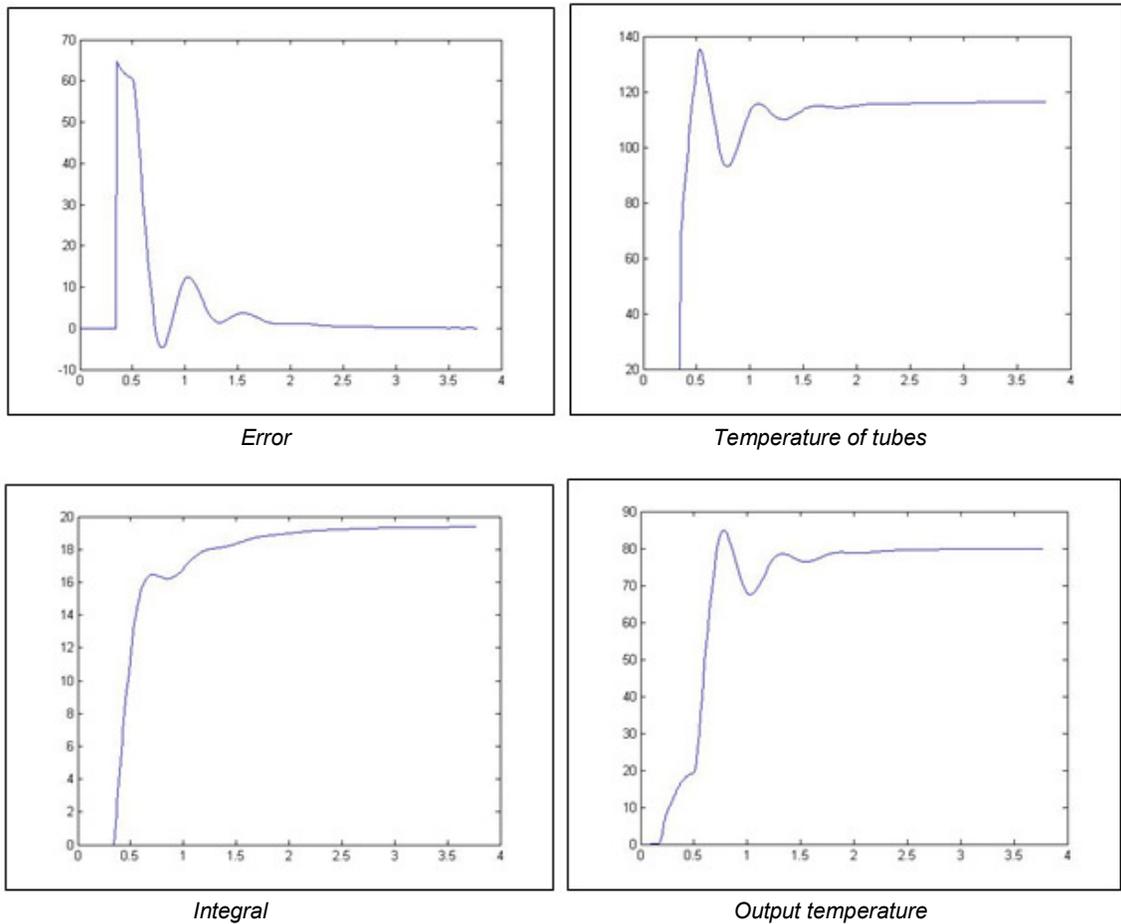


Figure 21: Results of system 2.9.

### 6.3. 3<sup>rd</sup> approach:

The third approach which is applied for the solution of our problem is adding a constant temperature value of 100 °C degrees to the tube temperature value at each control step i.e.

$$e(\tau) = T_{out} - T_{out}^* \text{ where } T_{out}^* = 80^\circ\text{C}$$

$$T_{tube} = 100 + K_p * e(t) + K_i * \int_0^t e(\tau) d\tau$$

#### 6.3.1. System 3.1

After the decision of the approach we set the initial control values of  $K_p=1.5$  and  $K_i=1.66$  which we found by step calculation earlier in part 3.

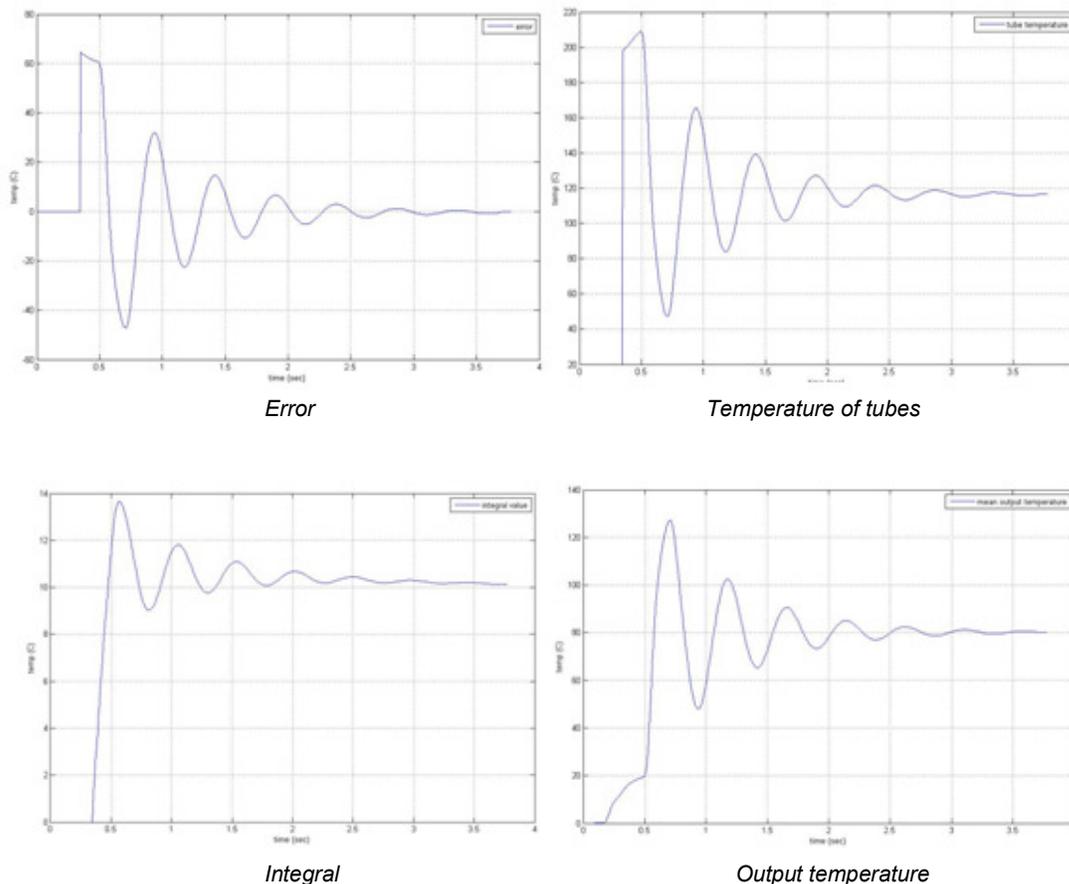


Figure 22: Results of system 3.1.

When the preceding graphs are investigated, the controller we designed and the controller constants  $K_p$  and  $K_i$  result in oscillating output values, great overshoot, and long stabilizing time period. The goal output temperature value of 80 °C degrees is reached after almost 3.5 seconds and the output temperature reaches 130 °C during the process, which means more than %60 overshoot.

### 6.3.2. System 3.2

By obtaining the previous dissatisfying results, it is decided to tune the proportion value ( $K_p$ ) in order to reach more stable controller and system. Considering the experience and knowledge obtained from previous approach decreasing  $K_p$  value would give more satisfying results. In order to prove it, 2.0 is assigned to the proportional gain  $K_p$ .

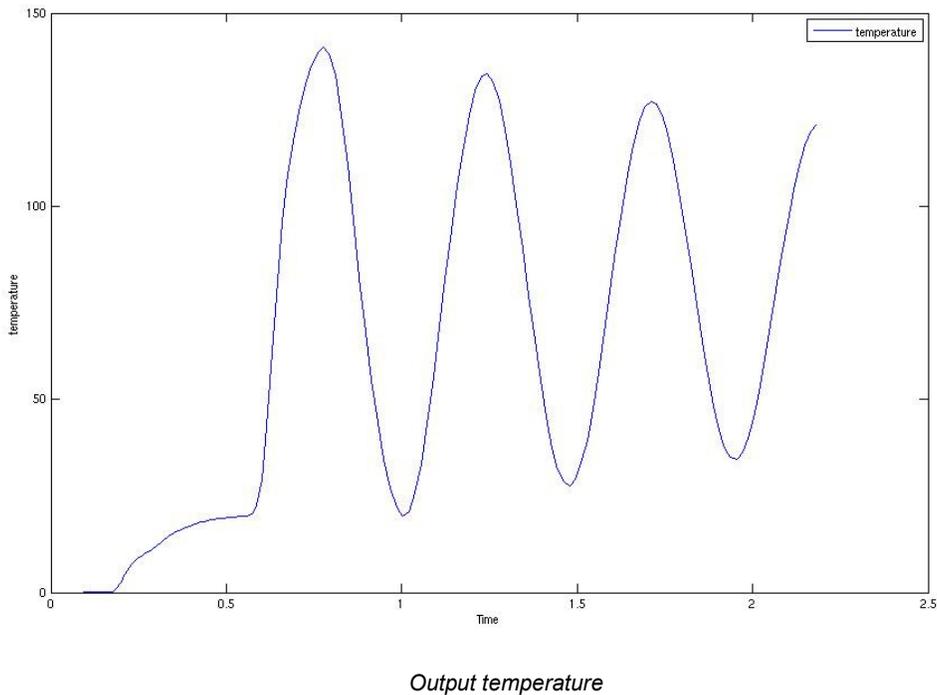
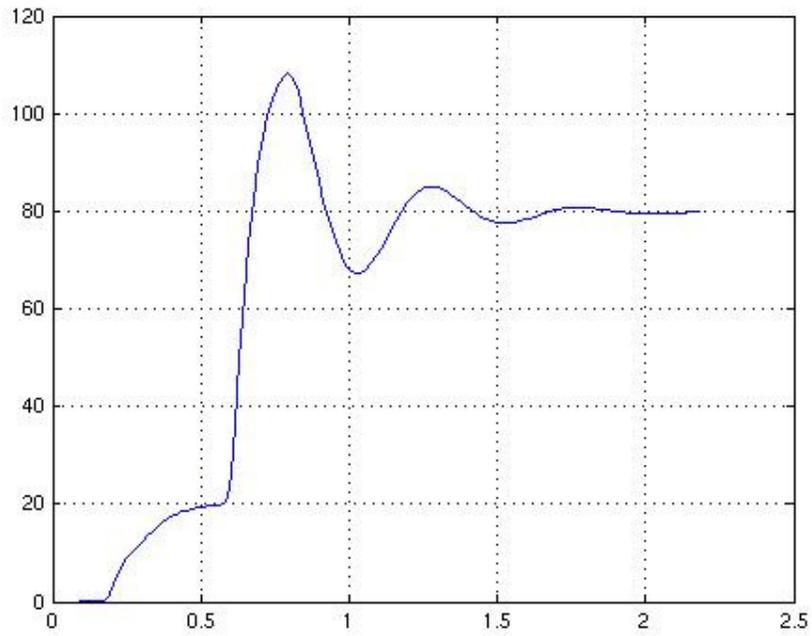


Figure 23: Results of system 3.2.1

As shown in the Figure 23, the process oscillated around the set point  $80\text{ }^{\circ}\text{C}$  at varying amplitude. It is observed practically too that the system goes to unstable mode if we keep to increase the  $K_p$  value.

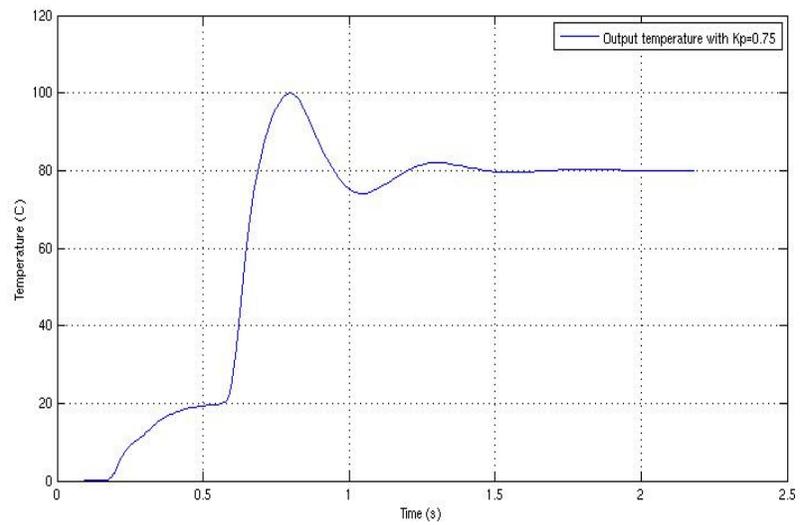
Secondly, we assigned different values between 0.5 and 1.0 to the proportional gain  $K_p$  while we were keeping  $K_i$  constant at 1.66. The following results are obtained and examined later on.

**Kp=1**



*Output temperature with Kp=1*  
Figure 24: Results of system 3.2.2

**Kp=0.75**



*Output temperature with Kp=0.75*  
Figure 25: Results of system 3.2.3

**Kp=0.5**

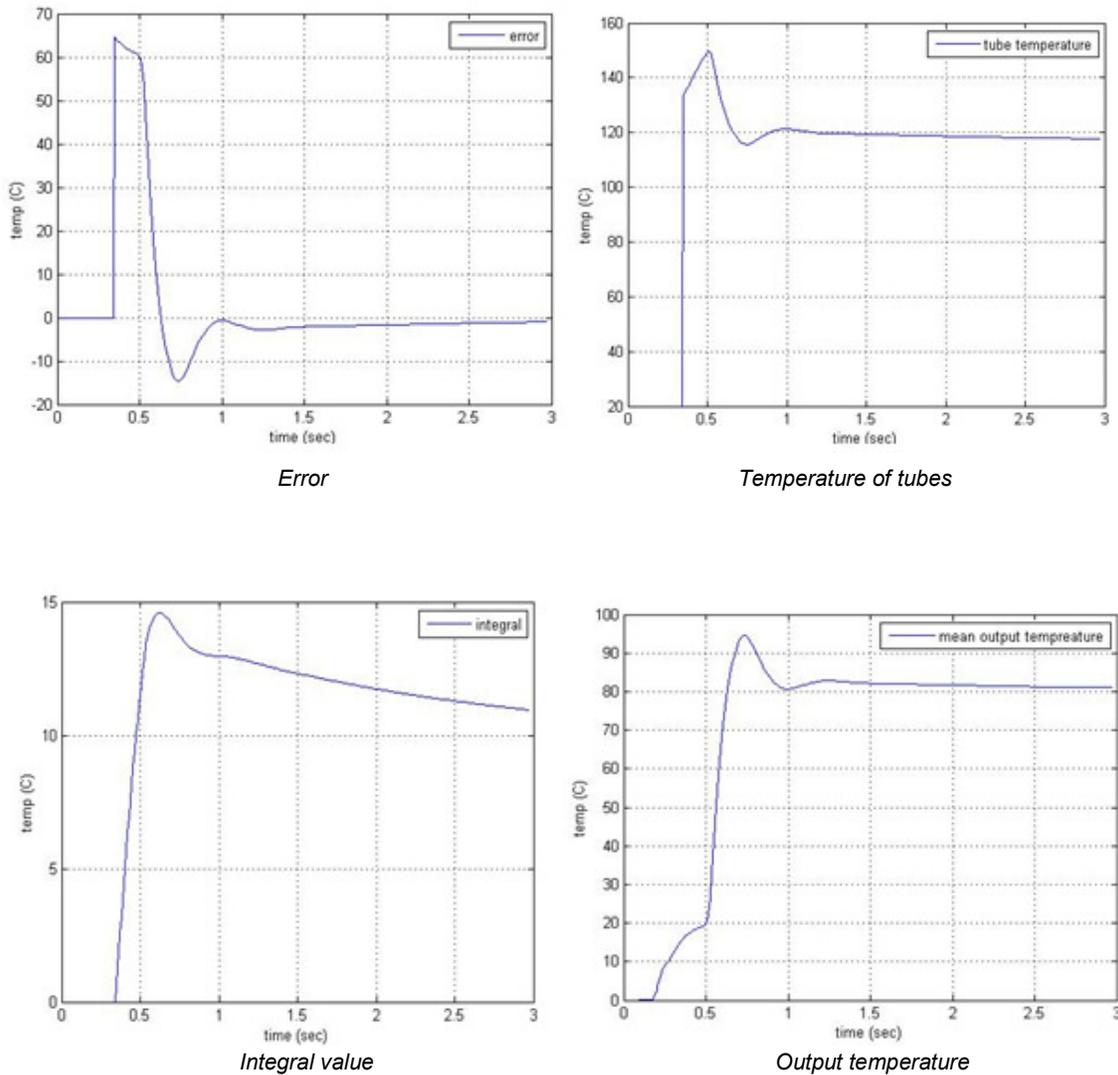


Figure 26: Results of system 3.2.4

As expected, decreasing  $K_p$  value has resulted in providing drastic change in our system such that the stability is increased, overshoot is decreased, and the response time is decreased. The only drawback, which is observed on the third and last try ( $K_p=0.5$ ), is the steady state error. The controller could not settle at its target value  $80^\circ\text{C}$ . The new controller, in many ways, satisfies our needs better than the previous try. Still there can be applied some improvements on the controller to speed up the process a little bit and decrease overshoot and steady state errors even further.

## 7. Conclusion

The aim of this project is to optimise the system response by focusing on two effects of controllers:

- 1- Settling time (the time for reaching bandwidth of 3%)
- 2- Overshoot

After testing different controllers, it was observed that these two effects contradict each other. This means, trying to decrease settling time results with an increase in overshoot or trying to decrease overshoot results with an increase in settling time. For this reason, it was decided to find optimum controller which satisfies both properties as much as possible.

It is known that the proportional term makes a change to the output that is proportional to the current error value. The proportional response can be adjusted by multiplying the error by a constant  $K_p$ , called the proportional gain.

$$P_{\text{out}} = K_p e(t)$$

A high proportional gain results in a large change in the output for a given change in the error. It decreases rise time, however, if the proportional gain is too high, the system can become unstable. In contrary, a small gain results in a small output response to a large input error, and a less responsive (or sensitive) controller. It also causes large error at the beginning (before settling of output).

A controller with only proportional gain is not enough to eliminate steady state errors. For this reason integral term is added to controller. The contribution from the integral term is proportional to both the magnitude and the duration of the error. Summing the instantaneous error over time (integrating the error) gives the accumulated offset that should have been corrected previously. The accumulated error is then multiplied by the integral gain and added to the controller output. The magnitude of the contribution of the integral term to the overall control action is determined by the integral gain,  $K_i$ .

$$I_{\text{out}} = K_i \int_0^t e(\tau) d\tau$$

The integral term accelerates the movement of the process towards set point and eliminates the residual steady-state error that occurs with a proportional only controller. However, since the integral term is responding to accumulated errors from the past, a high integral constant can cause the present value to overshoot the set point value.

After deciding to use a PI controller, different PI controllers were implemented and obtained some results which all can be seen in Table 3. Firstly, Schwarze method was applied in order to find PI controller parameters. Schwarze method offered a controller which is suitable but not the best. After that, it is decided to change controller parameters manually in order to have better system response. All the effects are recorded and showed in different graphs to see the effects of controller parts (**P**roportional or **I**ntegral parts) better.

	Ki	Kp	overshoot	% overshoot	settling time (s)	steady state error
System 1.1	-	-	1,84	0,023%	0,897	0,87
System 2.1	1,66	1,00	-	-	> 4,62	7,14
System 2.2	1,66	1,00	16,65	20,813%	> 3,77	17,81
System 2.3	1,50	1,50	-	-	> 2,5	17,96
System 2.4	3,50	2,00	16,53	20,663%	> 3,77	17,88
System 2.5	3,50	1,00	-	-	> 2.98	2,35
System 2.6	2,66	1,00	-	-	> 2,181	11,13
System 2.7	4,50	1,00	-	-	3,77	0,40
System 2.8	10,00	1,00	35,00	43,750%	1,50	0,03
System 2.9	6,00	1,00	4,82	6,025%	1,70	0,02
System 3.1	1,66	1,50	49,63	62,038%	2,65	0,02
System 3.2.1	1,66	2,00	62,30	77,875%	> 2,34	38,60
System 3.2.2	1,66	1,00	28,20	35,250%	1,544	0,13
System 3.2.3	1,66	0,75	19,78	24,725%	1,41	0,05
System 3.2.4	1,66	0,50	14,57	18,213%	1,379	0,01

Table 3: All systems with parameters and results

The effects of changing parameters can be seen well in Table 4.

Effects of increasing parameters				
Parameter	Rise Time	Overshoot	Settling Time	S-State Error
$K_p$	Decrease	Increase	Small Change	Decrease
$K_i$	Decrease	Increase	Increase	Eliminate

Table 4: Effects of increasing parameters

### Suggestions:

In order to have a better controller, derivative term could be added to controller. It can help to decrease overshoot; however it makes system more sensitive to disturbances.

Another important parameter, which is not considered in the project, is dead time of the process. It is the time of water passing from the heating tubes until the end of the main tube (output). The effects of the controller can not be seen before the heated water comes out from the main tube. For this reason, a dead time PI or PID controller could have better results for this experiment.

## 8. References

- [1] DOE Fundamentals Handbook, Thermodynamics, Heat Transfer, and Fluid Flow Volume 1 of 3
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- [5] <http://www.featflow.de>
- [6] Regelungstechnik 1 Praktikum, Fachhochschule Gießen-Friedberg, Prof. Dr. Ing Peter Schmitz