technische universität
dortmund

Radiation Effects From an Isothermal Vertical Wavy Cone With Variable Fluid Properties

Sadia Siddiqa, Naheed Begum, Md. Anwar Hossain

Institute of Applied Mathematics (LS III), TU Dortmund University, Vogelpothsweg 87, D-44227 Dortmund, Germany

Oberseminar 2017

Summer Term, 08 June 2017

fakultät für mathematik

Layout of presentation

- Problem Statement
- Governing Equations
- Boundary Layer Equations
- Primitive Variable Formulation
- Solution Methodology
- Physical Quantities
- Comparison with Previous Results
- Graphical Results

Assumptions

- Steady, 2D laminar natural convection boundary layer flow
- Density obeys the Boussinesq approximation
- The surface of the cone is maintained at uniform temperature T_w, such that $T_w >> T_\infty$.

Challenges

- Imposing the Boundary conditions at wavy surface of shape, $\hat{y}_w = \hat{\sigma}(\hat{x}) = \hat{a}\sin\left(\frac{\pi\hat{x}}{L}\right)$ with local radius $\hat{r} = \hat{x}\sin\phi$
- Fluid viscosity and thermal conductivity as function of temperature [1], i.e.,
  $\mu = \mu_\infty\left(1 + \frac{\epsilon(T-T_\infty)}{T_w - T_\infty}\right)$, $\kappa = \kappa_\infty\left(1 + \frac{\gamma(T-T_\infty)}{T_w - T_\infty}\right)$
- Presence of non-linear thermal radiation effects in the fluid in terms of q_r.

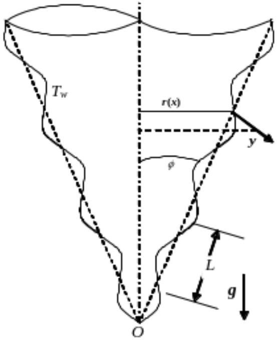

Figure: Physical Model

Conservation of mass, momentum and energy

$$\begin{cases} \dfrac{\partial(\hat{r}\hat{u})}{\partial\hat{x}} + \dfrac{\partial(\hat{r}\hat{v})}{\partial\hat{y}} = 0 \\ \rho\left(\hat{u}\dfrac{\partial\hat{u}}{\partial\hat{x}} + \hat{v}\dfrac{\partial\hat{u}}{\partial\hat{y}}\right) = -\dfrac{\partial\hat{p}}{\partial\hat{x}} + \nabla\cdot(\mu\nabla\hat{u}) + \rho g\beta\,(T - T_\infty)\cos\phi \\ \rho\left(\hat{u}\dfrac{\partial\hat{v}}{\partial\hat{x}} + \hat{v}\dfrac{\partial\hat{v}}{\partial\hat{y}}\right) = -\dfrac{\partial\hat{p}}{\partial\hat{y}} + \nabla\cdot(\mu\nabla\hat{v}) - \rho g\beta\,(T - T_\infty)\sin\phi \\ \rho c_p\left(\hat{u}\dfrac{\partial T}{\partial\hat{x}} + \hat{v}\dfrac{\partial T}{\partial\hat{y}}\right) = \nabla\cdot(\kappa\nabla T) - \nabla.\boldsymbol{q_r} \end{cases}$$

where the radiative heat flux [3]:

$$\boldsymbol{q_r} = -\frac{4\sigma^*}{3\kappa\,(\alpha_r + \sigma_s)}\nabla T^4$$

along with the boundary conditions:

$$\begin{cases} \hat{u}(\hat{x},\hat{y}_w) = \hat{v}(\hat{x},\hat{y}_w) = T(\hat{x},\hat{y}_w) - T_w = 0 \\ \hat{u}(\hat{x},\infty) = T(\hat{x},\infty) - T_\infty = 0 \end{cases}$$

Transformations

$$\begin{cases} u = \frac{L}{\nu_\infty} Gr^{-1/2} \hat{u},\ v = \frac{L}{\nu_\infty} Gr^{-1/4} \left(\hat{v} - \sigma_x \hat{u}\right),\ x = \frac{\hat{x}}{L},\ r = \frac{\hat{r}}{L},\ a = \frac{\hat{a}}{L} \\ y = \frac{\hat{y} - \sigma\left(\hat{x}\right)}{L} Gr^{1/4},\ \theta = \frac{T - T_\infty}{T_w - T_\infty},\ p = \frac{L^2}{\rho \nu_\infty^2 Gr} \hat{p},\ \sigma(x) = \frac{\hat{\sigma}(\hat{x})}{L} \end{cases}$$

Dimensionless system of equations

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \sigma_x Gr^{1/4}\frac{\partial p}{\partial y} + \left(1 + \sigma_x^2\right)\left((1 + \epsilon\theta)\frac{\partial^2 u}{\partial y^2} + \epsilon\frac{\partial u}{\partial y}\frac{\partial \theta}{\partial y}\right) + \theta$$

$$\sigma_x\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) + \sigma_{xx}u^2 = -Gr^{1/4}\frac{\partial p}{\partial y} + \sigma_x\left(1 + \sigma_x^2\right)\left((1 + \epsilon\theta)\frac{\partial^2 u}{\partial y^2} + \epsilon\frac{\partial u}{\partial y}\frac{\partial \theta}{\partial y}\right) - \tan\phi\theta$$

$$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\frac{\left(1+\sigma_x^2\right)}{\mathrm{Pr}}\left[(1+\gamma\theta)\frac{\partial}{\partial y}\left(1+\left(\frac{4}{3}R_d\,(1+\delta\theta)^3\right)\frac{\partial\theta}{\partial y}\right)+\gamma\left(\frac{\partial\theta}{\partial y}\right)^2\right]$$

subject to the transformed boundary conditions:

$$\begin{cases} u(x,0)=v(x,0)=0,\theta(x,0)-1=0 \\ u(x,\infty)=0,\ \theta(x,\infty)=0 \end{cases}$$

where the dimensionless variables are:

$$\begin{cases} Gr=\dfrac{g\beta(T_w-T_\infty)\cos\phi L^3}{\nu^2},\ \theta_w=\dfrac{T_w}{T_\infty},\ \delta=\theta_w-1, \\ R_d=\dfrac{4\sigma^* T_\infty^3}{\kappa\left(\alpha_r+\sigma_s\right)},\quad \mathrm{Pr}=\dfrac{\mu c_p}{\kappa} \end{cases}$$

- **Next step**

  Dimensionless system $\Rightarrow$ Tri-diagonal solver for numerical treatment

Transformations

$$x = X, \quad y = x^{\frac{1}{4}} Y, \quad u = x^{\frac{1}{2}} U, \quad v = x^{-\frac{1}{4}} V, \quad \theta = \Theta, \quad r = x \sin\phi$$

Parabolic system of PDEs

$$\frac{3}{2} U + X \frac{\partial U}{\partial X} - \frac{1}{4} Y \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial Y} = 0$$

$$\left(\frac{1}{2} + \frac{X \sigma_X \sigma_{XX}}{\left(1 + \sigma_X^2\right)} \right) U^2 + XU \frac{\partial U}{\partial X} + \left(V - \frac{Y}{4} U \right) \frac{\partial U}{\partial Y} = \left(1 + \sigma_X^2\right) \left((1 + \epsilon\Theta) \frac{\partial^2 U}{\partial Y^2} + \epsilon \frac{\partial U}{\partial Y} \frac{\partial \Theta}{\partial Y} \right) + \frac{1 - \sigma_X \tan\phi}{\left(1 + \sigma_X^2\right)} \Theta$$

$$XU \frac{\partial \Theta}{\partial X} + \left(V - \frac{Y}{4} U \right) \frac{\partial \Theta}{\partial Y} = \frac{\left(1 + \sigma_X^2\right)}{\mathrm{Pr}} \left[(1 + \gamma\Theta) \frac{\partial}{\partial Y} \left(1 + \left(\frac{4}{3} R_d \left(1 + \delta\Theta\right)^3 \right) \frac{\partial \Theta}{\partial Y} \right) + \gamma \left(\frac{\partial \Theta}{\partial Y} \right)^2 \right]$$

$$\begin{cases} U(X,0) = V(X,0) = 0, \Theta(X,0) - 1 = 0 \\ U(X,\infty) = 0, \ \Theta(X,\infty) = 0 \end{cases}$$

- Discretization Procedure

$$\left(\frac{\partial \Omega}{\partial X}\right)_{i,j} = \frac{\Omega_{i,j} - \Omega_{i-1,j}}{\Delta X}, \quad \left(\frac{\partial \Omega}{\partial Y}\right)_{i,j} = \frac{\Omega_{i,j+1} - \Omega_{i,j-1}}{2\Delta Y}$$

$$\Omega_{i,j} = \Omega(X_i, Y_j) \quad Y_j = (j-1)\Delta Y \quad \text{for} \quad j = 1, 2, 3 \ldots N,$$

$$Y_\infty = Y_N \quad X_i = (i-1)\Delta X \quad \text{for} \quad i = 1, 2, 3 \ldots M$$

Here Ω denotes the dependent variable U and Θ.

- Tridiagonal matrix equation

$$A_{i,j}\Omega_{i,j-1} + B_{i,j}\Omega_{i,j} + C_{i,j}\Omega_{i,j+1} = D_{i,j}$$

$$\text{where} \quad \Omega_{i,j} = \begin{bmatrix} U \\ \Theta \end{bmatrix} \quad A_{i,j} = \begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \end{bmatrix} \quad B_{i,j} = \begin{bmatrix} B_{11} & B_{12} \\ 0 & B_{22} \end{bmatrix}$$

$$C_{i,j} = \begin{bmatrix} C_{11} & 0 \\ 0 & C_{22} \end{bmatrix} \quad D_{i,j} = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \quad \Omega_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \Omega_N = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- An algorithm to find $\Omega_{i,j}$ is adopted as:

$$\Omega_{i,j} = -E_j \Omega_{i,j+1} + F_j \quad 1 \leq j \leq N-1$$

$$\text{where} \quad E_1 = E_N = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad F_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad F_N = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$E_j = (B_j - A_j E_{j-1})^{-1} C_j \quad 2 \leqslant j \leqslant N-1$$

$$F_j = (B_j - A_j E_{j-1})^{-1} (D_j - A_j F_{j-1}) \quad 2 \leqslant j \leqslant N-1$$

- Based on the information available at the i^{th} nodal point, the dependent variables Ω_j are predicted at $i+1^{th}$ stage.
- Use of continuity equation to obtain normal velocity component V

$$\begin{cases} V_{i,j} = V_{i-1,j} + \dfrac{Y\left(U_{i,j} - U_{i-1,j}\right)}{4} - \dfrac{X \triangle Y \left(U_{i,j} - U_{i,j-1} + U_{i-1,j} - U_{i-1,j-1}\right)}{2\triangle X} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad - \dfrac{3}{2} \triangle Y U_{i,j} \end{cases}$$

Next step $\Rightarrow$ Calculation of physical quantities

- Physical quantities of interest

$$\begin{cases} C_f = \dfrac{\tau_w}{\rho_\infty \left(\nu_\infty / L\right)^2}, \quad Nu = \dfrac{L Q_w}{\kappa_\infty \left(T_w - T_\infty\right)} \\ \text{where} \quad \tau_w = \mu \left(\hat{\mathbf{n}}.\nabla \hat{u}\right)_{\hat{y}=0}, \quad Q_w = -\kappa \left(\hat{\mathbf{n}}.\nabla T\right)_{\hat{y}=0} - \dfrac{4\sigma^*}{3\left(\alpha_r + \sigma_s\right)} \left(\nabla T^4\right)_{\hat{y}=0} \end{cases}$$

- Here $\hat{\mathbf{n}}$ is the unit vector normal to the wavy surface defined as:

$$\hat{\mathbf{n}} = \left(-\frac{\sigma_X}{\sqrt{1+\sigma_X^2}}, \frac{1}{\sqrt{1+\sigma_X^2}} \right)$$

- Expressions for skin friction and rate of heat transfer

$$\begin{cases} C_f \left(\dfrac{Gr^{-3}}{X}\right)^{1/4} = (1+\epsilon)\sqrt{1+\sigma_X^2} \left(\dfrac{\partial U}{\partial Y}\right)_{Y=0} \\ Nu \left(\dfrac{Gr}{X}\right)^{-1/4} = -(1+\gamma)\sqrt{1+\sigma_X^2} \left(1 + \dfrac{4}{3} R_d \theta_w^3\right) \left(\dfrac{\partial \Theta}{\partial Y}\right)_{Y=0} \end{cases}$$

Next step $\Rightarrow$ Validation of results

- Tabular comparison with previous results

Table: Values of rate of heat transfer and skin friction coefficient for $a = 0.3$, $R_d = 0.0$, $\theta_w = 3.1$, $\gamma = \epsilon = 5.0$ and $\phi = \pi/6$.

Pr	$-\Theta'(X,0)$		$U'(X,0)$	
	Ref.[2]	Present	Ref.[2]	Present
0.1	0.09730	0.09731	0.01488	0.01477
0.7	0.09783	0.09686	0.01487	0.01438
1.0	0.09809	0.09724	0.01486	0.01420
7.0	0.10344	0.10447	0.01467	0.01472

Present results with PVF and tri-diagonal solver!

Ref.[2] results with **SFF** and Keller box!

- Graphical comparison

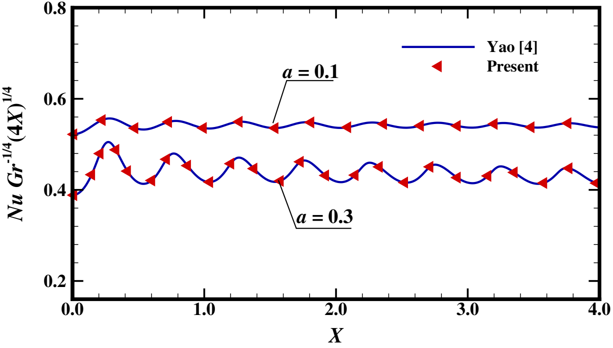

Figure: Local Nusselt number coefficient for $a = 0.1, 0.3$ while $\epsilon = \gamma = R_d = 0.0$, $\mathrm{Pr} = 1.0$, $\theta_w = 1.1$ and $\phi = 0$.

- Effect of viscosity variation parameter on skin friction coefficient

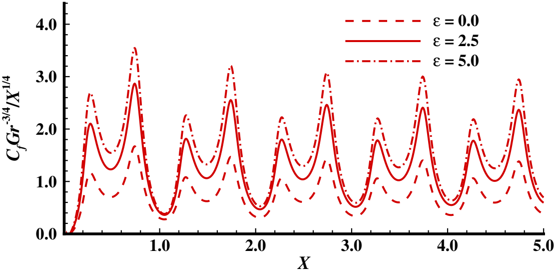

Figure: Skin friction coefficient for $\epsilon = 0.0, 2.5, 5.0$, $R_d = 10.0$, $\Pr = 0.7$, $\theta_w = 3.1$, $\gamma = 5.0$, $a = 0.3$ and $\phi = \pi/6$.

- Effect of thermal conductivity variation parameter on rate of heat transfer

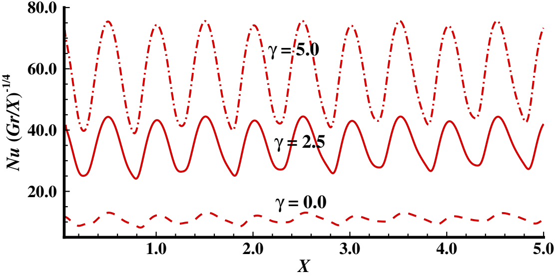

Figure: Nusselt number coefficient for $\gamma = 0.0, 2.5, 5.0$, $R_d = 10.0$, $\mathrm{Pr} = 0.7$, $\theta_w = 3.1$, $\epsilon = 5.0$, $a = 0.3$ and $\phi = \pi/6$.

- Effect of thermal radiation parameter

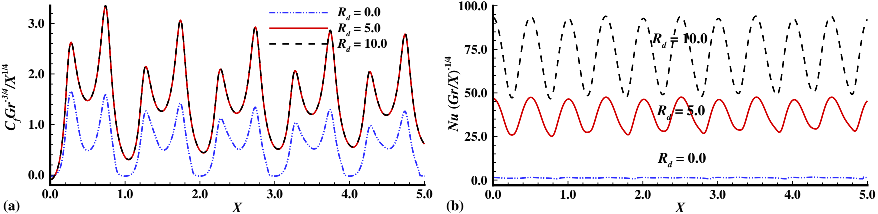

Figure: (a) Skin friction coefficient and (b) Nusselt number coefficient for $R_d = 0.05.0, 10.0$, $\gamma = \epsilon = 5.0$, $\Pr = 0.7$, $\theta_w = 3.1$, $a = 0.3$ and $\phi = \pi/6$.

- Effect of surface heating parameter

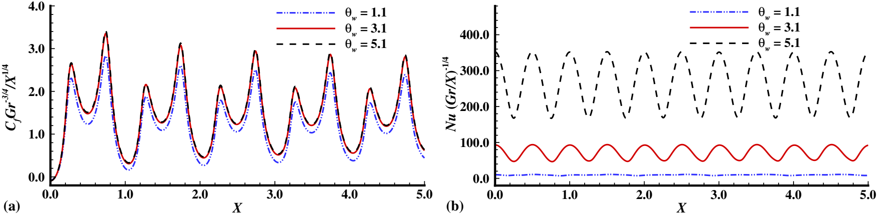

Figure: (a) Skin friction coefficient and (b) Nusselt number coefficient for $\theta_w = 1.1, 3.1, 5.1$, $R_d = 10.0$, $\gamma = \epsilon = 5.0$, $\Pr = 0.7$, $a = 0.3$ and $\phi = \pi/6$.

- Effect of thermal radiation on streamlines

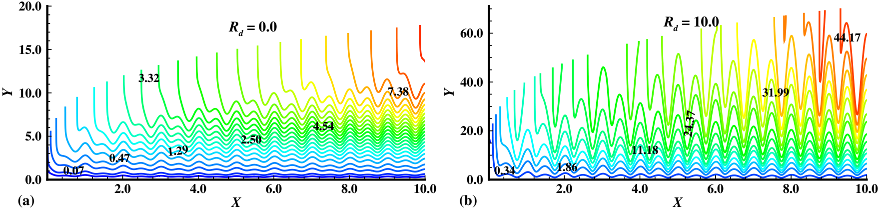

Figure: Streamlines for (a)$R_d = 0.0$, (b)$R_d = 10.0$ while $\mathrm{Pr} = 0.7$, $\theta_w = 3.1$, $\gamma = \epsilon = 2.0$, $a = 0.3$ and $\phi = \pi/6$.

- Effect of thermal radiation on isotherms

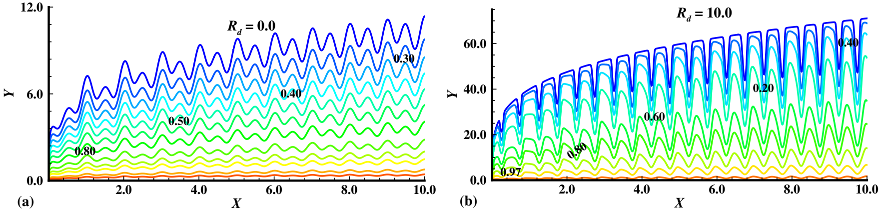

Figure: Isotherms for (a)$R_d = 0.0$, (b)$R_d = 10.0$ while $\Pr = 0.7$, $\theta_w = 3.1$, $\gamma = \epsilon = 2.0$, $a = 0.3$ and $\phi = \pi/6$.

From the graphical results, it can be concluded that:

- Local skin friction coefficient enhances about 55.04% as viscosity variation parameter increases from 0.0 to 5.0, which is quite a significant figure.
- Thermal radiation and thermal conductivity variation parameter extensively promotes the heat transfer coefficient near the surface of the wavy cone.
- The skin friction coefficient exhibits the asymptotic behavior by intensifying the value of radiation parameter.

Summary

technische universität dortmund

Numerical investigations of Navier-Stokes equations w.r.t.

- temperature-dependent viscosity and thermal conductivity
- thermal radiation in participating fluid
- sinusoidal wavy cone geometry
- mass conservation
- tri-diagonal solver for parabolic system of PDEs

In addition, I also obtained the results for dusty fluid flow, bioconvection nanofluid flow, Marangoni convection in context of different geometries.

My future aims relates to two categories:

1. Numerical investigations of FEM for Navier-Stokes equations for concrete flow in geometry of rectangular channel.
2. Finite difference simulation of non-Newtonain fluids in confined geometries like channel, cavities etc.

Jacques Charraudeau.
Influence de gradients de proprieties physiques en convection force – application au cas du tube.
International Journal of Heat and Mass Transfer, 18(1):87 – 95, 1975.

Md.Anwar Hossain, Md.Sazzad Munir, and Ioan Pop.
Natural convection with variable viscosity and thermal conductivity from a vertical wavy cone.
International Journal of Thermal Sciences, 40(5):437 – 443, 2001.

M. N. Osizik.
Radiative Transfer and Interactions with Conduction and Convection.
John Wiley & Sons, 1973.

L. S. Yao.
Natural convection along a vertical wavy surface.
ASME Journal of Heat Transfer, 105(3):465 – 468, 1983.