Monolithic Finite Element Methods for the simulation of thixo-viscoplastic flows

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fakultät für mathematik

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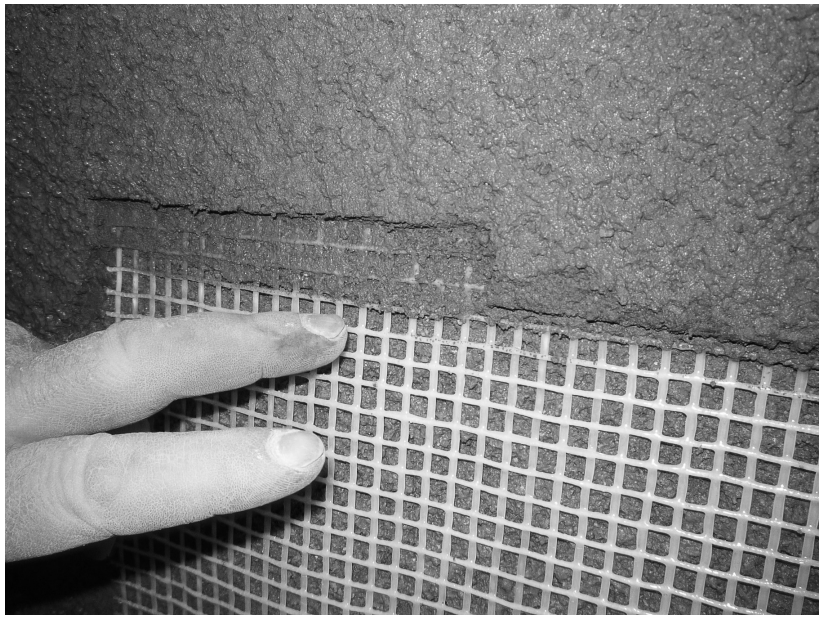
Motivation

Why "Thixotropic materials?

- **Processing of thixotropic materials relevant for industrial applications**

➡ **Lubrication, asphalt, self-compacting concrete…**

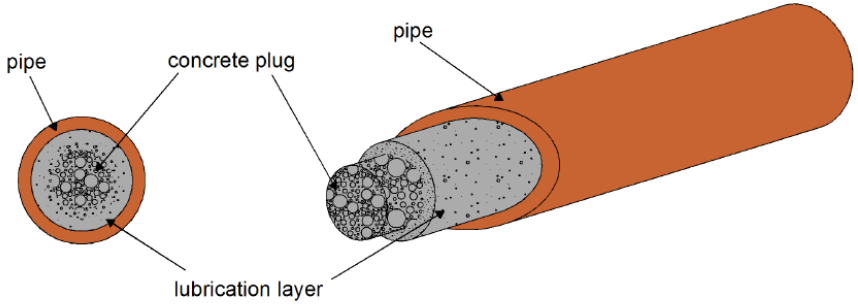
- **Physically fascinating due to improved mechanical properties**



Goal:

- **Modern CFD methods with high accuracy, robustness and efficiency for thixotropic materials**

➡ **Saving time, money and resources**



Investigation of solid/liquid and liquid/solid transitions based on micro-structure

Introduction

- **Thixotropy means**
  - ➢ **combination of two greek words**
  - ➢ **Thixis: shaking/stirring**
  - ➢ **trepo: turning/changing**

- **Thixotropy concept**
  - ➢ **Based on viscosity**
  - ➢ **Flow induced by time-dependent decrease of viscosity**
  - ➢ **The phenomena is reversible**

- **Rejuvenation / Breakdown**
  - ➢ **"Faster" flow: fluid rejuvenates**
    Decreases of viscosity with acceleration of the flow

- **Aging / Build-up**
  - ➢ **At rest or under slow flow: fluid ages**
    Increases of the viscosity in time

Realization in FeatFlow

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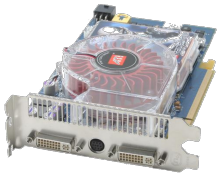
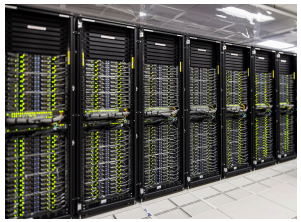
HPC features:
- Moderately parallel
- GPU computing
- Open source





Hardware -oriented Numerics

Numerical features:
- Higher order FEM in space & (semi-) Implicit FD/FEM in time
- Semi-(un)structured meshes with dynamic adaptive grid deformation
- Fictitious Boundary (FBM) methods
- Newton-Multigrid-type solvers

Non-Newtonian flow module:
- generalized Newtonian model (Power-law, Carreau,Houska,…)
- viscoelastic differential model (Giesekus, FENE, Oldroyd,...)

Multiphase flow module (resolved interfaces):
- level set – interface capturing (Level Set)
- fbm – interface tracking (FBM)
- fbm/level set – combination of level set and fbm

Engineering aspects:
- Geometrical design
- Modulation strategy
- Optimization

Here: FEM-based tools for the accurate simulation of (thixotropic) flow problems, particularly with complex rheology

For details, please visit: www.featflow.de

Governing equations

➡**starting point: Generalized Navier-Stokes equations (+initial and boundary conditions)**

$$\rho\left(\frac{\partial}{\partial t}+\boldsymbol{u}\cdot\nabla\right)\boldsymbol{u}-\nabla\cdot\boldsymbol{\sigma}+\nabla p=\rho f,$$

$$\nabla\cdot\boldsymbol{u}=0,$$

- **velocity- and pressure field $\boldsymbol{u}$ and p**
- **stress tensor $\boldsymbol{\sigma}$**
- **linear material behaviour - Newtonian fluids**

$$\boldsymbol{\sigma}=2\eta_s\boldsymbol{D}(\boldsymbol{u})$$ **: η_s is constant viscosity**

- **non-linear material behaviour- structurally viscous / viscoplastic**

$$\boldsymbol{\sigma}=2\eta_s(D_{\mathrm{II}},p,\Theta,\lambda)\boldsymbol{D}(\boldsymbol{u}),\quad D_{\mathrm{II}}=\mathrm{tr}\left(\frac{1}{2}\boldsymbol{D}(\boldsymbol{u})^2\right)$$

↓

- **Power-law, Carreau, Bingham, Herschel-Bulkley, Houska, …**

- **structure paremeter λ**

Rheological Models

- **Archetypical thixotropic viscoplastic (TVP) models**

$$\begin{cases} \boldsymbol{\sigma} = 2\,\eta(D_{II},\lambda)\boldsymbol{D}(\boldsymbol{u}) + \sqrt{2}\,\tau(\lambda)\dfrac{\boldsymbol{D}(\boldsymbol{u})}{\sqrt{D_{II}}} & \text{if } D_{II} \neq 0 \\ \sigma_{II} \leq \tau(\lambda) & \text{if } D_{II} = 0 \end{cases}$$

- **Relations between rheological parameters and structural parameter**

	$\eta(D_{II},\lambda)$	$\tau(\lambda)$
Worrall and Tulliani[1]	$\lambda\eta_0$	τ_0
Coussot *et al.*[2]	$\lambda^a\eta_0$	–
Houska[3]	$(\eta_0+\eta_1\lambda)\,D_{II}^{\frac{(n-1)}{2}}$	$(\tau_0+\tau_1\lambda)$
Mujumbar *et al.*[4]	$(\eta_0+\eta_1\lambda)\,D_{II}^{\frac{(n-1)}{2}}$	$\lambda^{a+1}\mathrm{G}_0{\Lambda_c}^*$
Burgos *et al.*[5]	η_0	$\lambda\tau_0$
Dullaert & Mewis[6]	$\lambda\eta_0$	$\lambda\mathrm{G}_0\left(\lambda D_{II}^{\frac{1}{2}}\right)\Lambda_c^*$

*Λ_c is a constant/variable elastic strain.

Rheological Models

- **General format of evolution equation for structural parameter:**

$$\frac{\partial \lambda}{\partial t} + u \cdot \nabla \lambda = F_{buildup} - F_{breakdown}$$

- **Expressions for different thixotropic models:**

	$F_{buildup}$	$F_{breakdown}$
Worrall and Tulliani[1]	$c_1(1-\lambda)D_{II}^{\frac{1}{2}}$	$c_2\lambda D_{II}^{\frac{1}{2}}$
Coussot *et al.*[2]	c_1	$c_2\lambda D_{II}^{\frac{1}{2}}$
Houska[3]	$c_1(1-\lambda)$	$c_2\lambda^m D_{II}^{\frac{1}{2}}$
Mujumbar *et al.*[4]	$c_1(1-\lambda)$	$c_2\lambda D_{II}^{\frac{1}{2}}$
Burgos *et al.*[5]	$c_1(1-\lambda)$	$c_2\lambda D_{II}^{\frac{1}{2}}\exp(aD_{II}^{\frac{1}{2}})$
Dullaert & Mewis[6]	$(c_1 + c_3 D_{II}^{\frac{1}{2}})(1-\lambda)t^{-b}$	$c_2\lambda D_{II}^{\frac{1}{2}}t^{-b}$

Thixo-viscoplastic model

- **Viscoplastic (VP) flow**

$$\begin{cases} \boldsymbol{\sigma} = 2\,\eta_0 \boldsymbol{D}(\boldsymbol{u}) + \sqrt{2}\,\tau_0 \dfrac{\boldsymbol{D}(\boldsymbol{u})}{\sqrt{D_{\text{II}}}} & \text{if } D_{\text{II}} \neq 0 \\ \sigma_{\text{II}} \leq \tau_0 & \text{if } D_{\text{II}} = 0 \end{cases}$$

- **Thixo-viscoplastic (TVP) flow**

$$\begin{cases} \boldsymbol{\sigma} = 2\,\eta(D_{\text{II}}, \lambda) \boldsymbol{D}(\boldsymbol{u}) + \sqrt{2}\,\tau(\lambda) \dfrac{\boldsymbol{D}(\boldsymbol{u})}{\sqrt{D_{\text{II}}}} & \text{if } D_{\text{II}} \neq 0 \\ \sigma_{\text{II}} \leq \tau(\lambda) & \text{if } D_{\text{II}} = 0 \end{cases}$$

  - **Affine functions**

$$\begin{cases} \eta(\lambda) = \eta_0 + \eta_1 \lambda \\ \tau(\lambda) = \tau_0 + \tau_1 \lambda \end{cases}$$

  - **Structure evolution equation**

$$\frac{\partial \lambda}{\partial t} + u \cdot \nabla \lambda = a(1-\lambda) - b\lambda D_{\text{II}}^{\frac{1}{2}}$$

$$(a, b \text{ are structure parameters})$$

Quasi-Newtonian TVP model

- **Viscosity model for TVP flow i.e. extended viscosity defined on all domains s.t.**

$$\begin{cases} I. & \eta_s(D_{\text{II}}, \lambda) = \eta(\lambda) + \dfrac{\sqrt{2}}{2}\tau(\lambda)\dfrac{1}{\sqrt{(D_{\text{II}} + (k^{-1})^2)}} \\ II. & \eta_s(D_{\text{II}}, \lambda) = \eta(\lambda) + \dfrac{\sqrt{2}}{2}\tau(\lambda)\dfrac{1}{D_{\text{II}}^{\frac{1}{2}}}(1 - e^{-kD_{\text{II}}^{\frac{1}{2}}}) \end{cases}$$

$(k:$ regularization parameter$)$

- **Full set of equations**

$$\begin{cases} \left(\dfrac{\partial}{\partial t} + u \cdot \nabla\right) u - \nabla \cdot \left(2\eta_s(D_{\text{II}}, \lambda) D(u)\right) + \nabla p = 0 & \text{in } \Omega \\ \nabla \cdot u = 0 & \text{in } \Omega \\ \dfrac{\partial \lambda}{\partial t} + u \cdot \nabla \lambda - a(1-\lambda) + b\lambda D_{\text{II}}^{\frac{1}{2}} = 0 & \text{in } \Omega \end{cases}$$

Newton's method

Let $\mathcal{U} = (\lambda, \boldsymbol{u}, p)$, and $\mathcal{R}_{\mathcal{U}}(\mathcal{U})$ be the continuous or the discrete corresponding system's residum.

- **Update of the nonlinear iteration with the correction $\delta\mathcal{U}$ i.e.**

$$\mathcal{U}^N = \mathcal{U} + \delta\mathcal{U}$$

- **The linearization of the residual provides**

$$\begin{aligned}\mathcal{R}_{\mathcal{U}}\left(\mathcal{U}^N\right) &= \mathcal{R}_{\mathcal{U}}\left(\mathcal{U} + \delta\mathcal{U}\right) \\ &= \mathcal{R}_{\mathcal{U}}\left(\mathcal{U}\right) + \mathcal{J}\left(\mathcal{U}\right) \cdot \delta\mathcal{U}\end{aligned}$$

- **The Newton's method assuming invertible Jacobian**

$$\mathcal{U}^N = \mathcal{U} - \mathcal{J}^{-1}(\mathcal{U})\,\mathcal{R}_{\mathcal{U}}(\mathcal{U})$$

Adaptive Newton's method

Jacobian calculations

$$\mathcal{J}\left(\mathcal{U}\right) = \left(\frac{\partial \mathcal{R}_{\mathcal{U}}\left(\mathcal{U}\right)}{\partial \mathcal{U}}\right)$$

- **Continuous Adaptive Newton based on a priori study of Jacobian's properties and decompositions**

$$\mathcal{J}\left(\mathcal{U}\right) = \left(\frac{\partial \hat{\mathcal{R}}_{\mathcal{U}}\left(\mathcal{U}\right)}{\partial \mathcal{U}}\right) + \delta\left(\frac{\partial \tilde{\mathcal{R}}_{\mathcal{U}}\left(\mathcal{U}\right)}{\partial \mathcal{U}}\right)$$

- **Discrete Adaptive Newton based on the rate of residum's convergence**

$$\left(\frac{\partial \mathcal{R}}{\partial \mathcal{U}}\right)_{ij} \approx \left(\frac{\mathcal{R}_i\left(\mathcal{U} + \epsilon e_j\right) - \mathcal{R}_i\left(\mathcal{U} - \epsilon e_j\right)}{2\epsilon}\right)$$

Continuous thixotropic problem

- **Flow variables** $(\lambda, \boldsymbol{u}, p)$
  - **Set** $\mathbb{T} := L^2(\Omega), \mathbb{V} := \left[H_0^1(\Omega)\right]^2, \mathbb{Q} := L_0^2(\Omega)$
  - **Set** $\tilde{\boldsymbol{u}} := (\lambda, \boldsymbol{u})$
  - **Find** $(\lambda, \boldsymbol{u}, p) \in \left(\mathbb{T} \cap H^1(\Omega)\right) \times \mathbb{V} \times \mathbb{Q}$ **s.t.**

$$\left\langle \mathcal{K}(\lambda, \boldsymbol{u}, p), (\xi, \boldsymbol{v}, q) \right\rangle = \left\langle \mathcal{L}, (\xi, \boldsymbol{v}, q) \right\rangle, \quad \forall (\xi, \boldsymbol{v}, q) \in \mathbb{T} \times \mathbb{V} \times \mathbb{Q}$$

$$\mathcal{K} = \begin{bmatrix} \mathcal{A}_{\tilde{\boldsymbol{u}}} & \mathcal{B}^{\mathrm{T}} \\ \mathcal{B} & 0 \end{bmatrix}$$

  - **Compatibily constraints**

$$\sup_{\boldsymbol{v} \in \mathbb{V}} \frac{\langle \mathcal{B}\boldsymbol{v}, q \rangle}{\|\boldsymbol{v}\|_{\mathbb{V}}} \geq \beta \|q\|_{\mathbb{Q}/Ker\mathcal{B}^{\mathrm{T}}}, \quad \forall q \in \mathbb{Q}$$

Numerical challenges

- **Discretizations have to handle the following challenges**
  - ➢ **Stable FEM spaces**
  - ➢ **Non-symmetric, non-coercive and ill-posedness**
  - ➢ **Convection and positivity preserving**
  - ➢ **Locally adatpted meshes for steep gradients**

- **Solvers have to deal with**
  - ➢ **Different source of nonlinearities**
  - ➢ **Strong coupling of equations**
  - ➢ **Robustness and efficiency**

Approximated problem

- **Conforming approximations**

$$\mathbb{T}_h \subset \mathbb{T}, \quad \mathbb{V}_h \subset \mathbb{V}, \quad \mathbb{Q}_h \subset \mathbb{Q}$$
$$\mathcal{A}_{\tilde{\boldsymbol{u}}_h} = \mathcal{A}_{\tilde{\boldsymbol{u}}}, \; \mathcal{B}_h = \mathcal{B}$$

- **Discrete inf-sup condition**

$$\sup_{\boldsymbol{v}_h \in \mathbb{V}_h} \frac{\left\langle \mathcal{B}_h \boldsymbol{v}_h, q_h \right\rangle}{\left\| \boldsymbol{v}_h \right\|_{\tilde{\mathbb{V}}}} \geq \beta_h \left\| q_h \right\|_{\mathbb{Q}/Ker\mathcal{B}_h^{\mathrm{T}}}, \qquad \forall q_h \in \mathbb{Q}_h$$

The family of conforming FEM $Q_r/Q_r/P_{r-1}^{\text{disc}},\ r \geq 2$ for $(\lambda, \boldsymbol{u}, p)$ with stabilization

$$J_u(u_h, v_h) = \gamma_u \sum_{e \in \mathcal{E}_h} h^2 \int_e [\nabla u_h] : [\nabla v_h]\, d\Omega$$

$$J_\lambda(\lambda_h, \xi_h) = \gamma_\lambda \sum_{e \in \mathcal{E}_h} h \int_e [\nabla \lambda_h] : [\nabla \xi_h]\, d\Omega$$

- Inf-sup conditions is satisfied
- Discontinuous pressure
  - Good for the solver
  - Element-wise mass conservation
- Discrete problem is well-posed
- Highly consistent and symmetric stabilization
- Robust solver w.r.t. the monolitic approach
- Efficient solver w.r.t. multigird solver

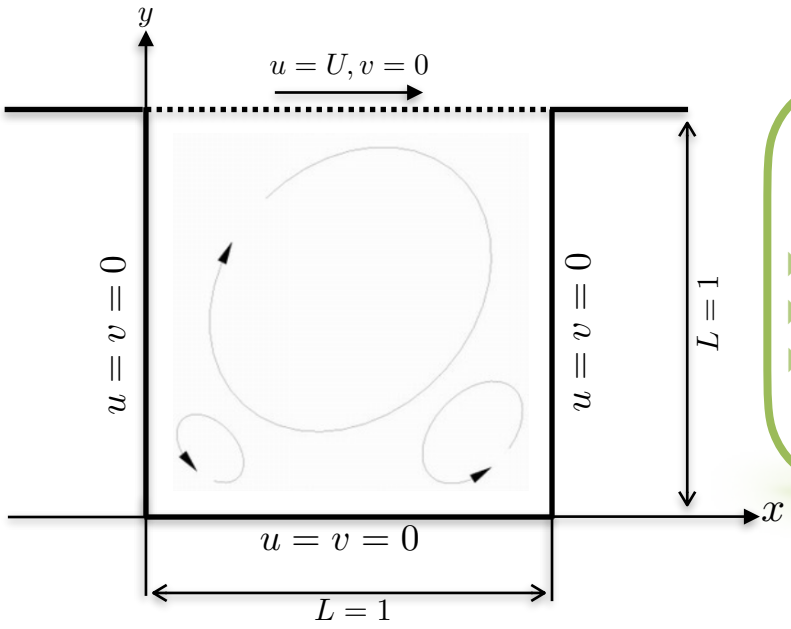
Monolitic-multigrid linear solver

- **Standard geometric multigrid solver for linearized system**
- **Full Q_r and P_{r-1}^{disc} restriction and prolongation**
- **Local Multilevel Pressure Schur Complement via Vanka-like smoother**

$$\begin{pmatrix} \lambda^{l+1} \\ u^{l+1} \\ p^{l+1} \end{pmatrix} = \begin{pmatrix} \lambda^{l} \\ u^{l} \\ p^{l} \end{pmatrix} + \omega^l \sum_{T \in \mathcal{T}_h} \left(\begin{pmatrix} \mathcal{K}_h + \mathcal{J} \end{pmatrix}_{|T} \right)^{-1} \begin{pmatrix} \mathcal{R}_{\lambda^l} \\ \mathcal{R}_{u^l} \\ \mathcal{R}_{p^l} \end{pmatrix}_{|T}$$

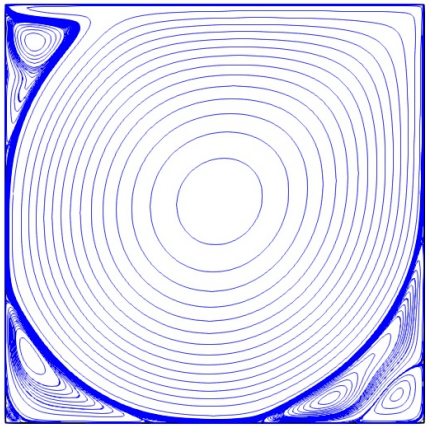
Coupled Monolithic Multigrid Solver !

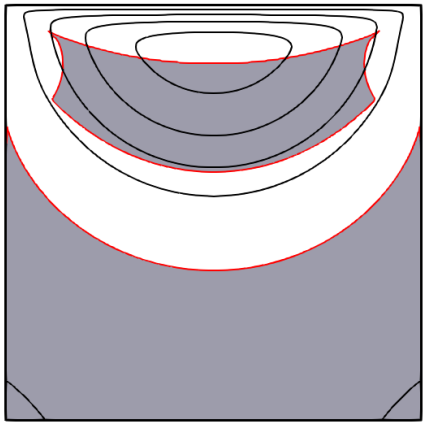
Thixo-viscoplastic flow in Lid-driven cavity

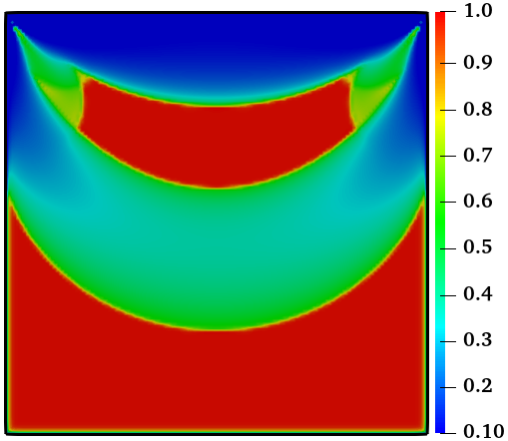


- **Starting point: consider flow in a cavity with unit height**
- **Steady, incompressible flow**
- **Constant speed at upper lid**
- **No-slip Dirichlet boundary conditions**

➡ **Newtonian, Viscoplastic, and Thixo-viscoplastic (TVP)**

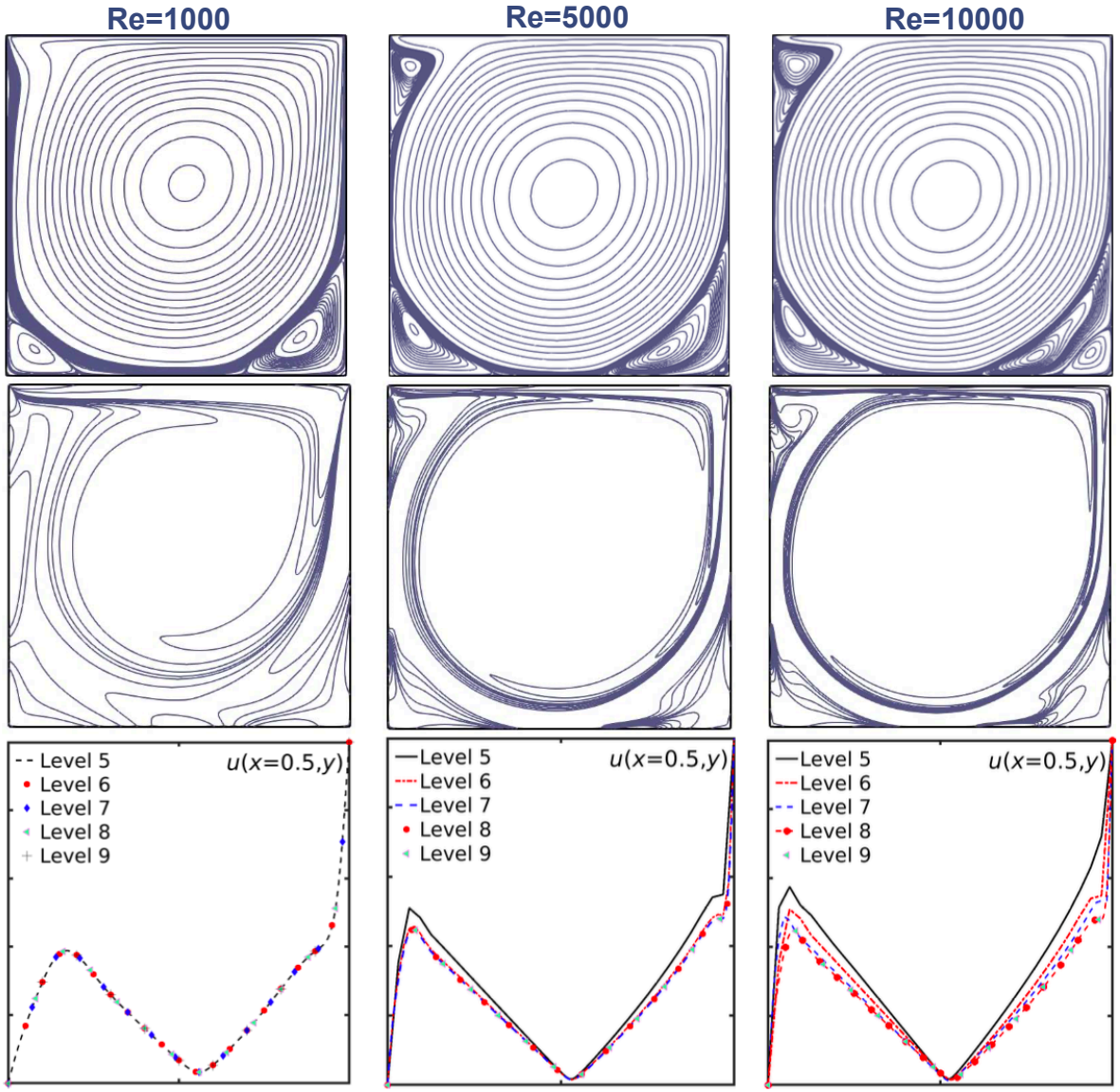






Newtonian Lid-driven cavity flow

- **Point-wise convergence for Newtonian flow**



Newtonian Lid-driven cavity flow

- **Global and point-wise quantities and solver behaviour**

Re		1000		5000		10000	
Level	cells	Energy$\times 10^2$	N/M	Energy$\times 10^2$	N/M	Energy$\times 10^2$	N/M
5	1024	4.541506	5/1	6.082524	6/1	7.940472	7/1
6	4096	4.458877	5/1	4.955858	5/1	5.369527	6/1
7	16384	4.452357	3/1	4.768669	4/1	4.868399	5/1
8	65536	4.451904	3/1	4.744815	3/2	4.783917	4/2
9	262144	4.451846	3/1	4.742921	3/1	4.773500	3/2
10	1048576	4.451834	2/1	4.742815	3/1	4.772692	3/1
$Ref.values \approx$:		4.45		4.74		4.77	

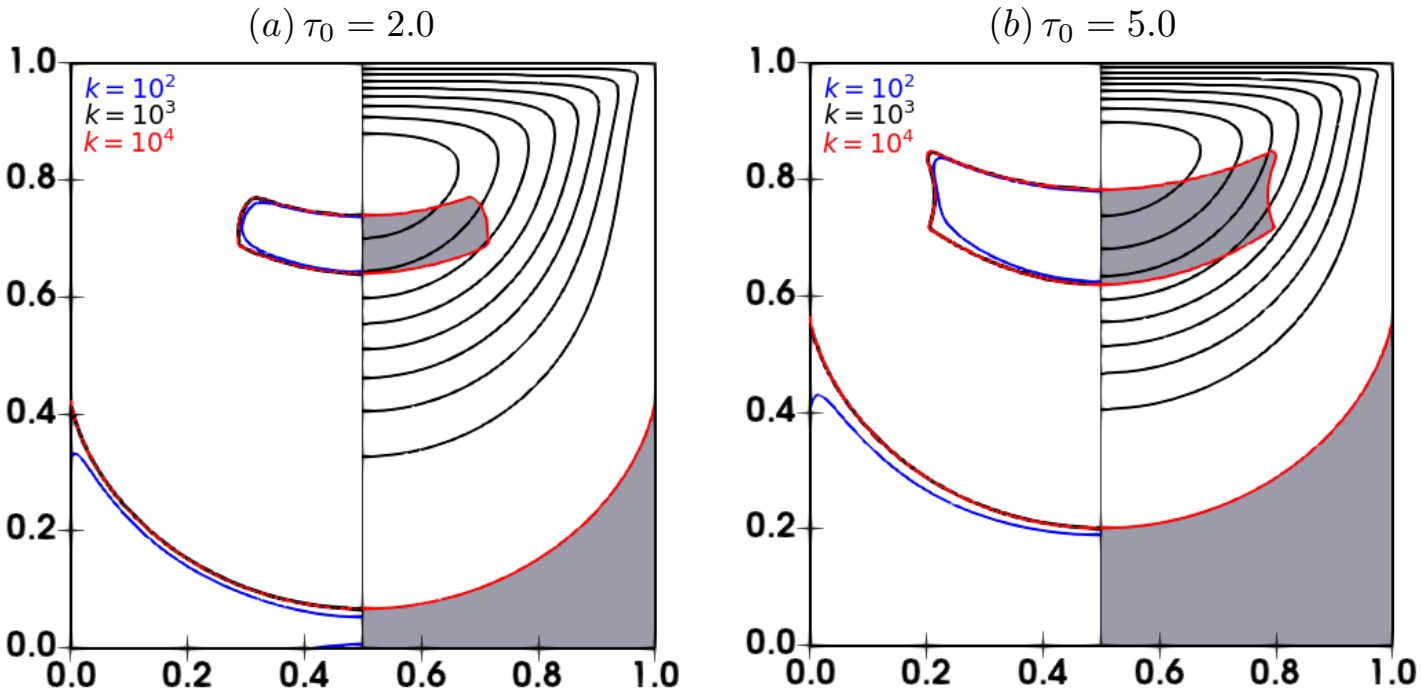
Re	1000		5000		10000	
Level	ψ_{max}	$\psi_{min} \times 10^3$	ψ_{max}	$\psi_{min} \times 10^3$	ψ_{max}	$\psi_{min} \times 10^3$
6	0.1190073	−1.72813	0.1249471	−3.145666	0.1586626	−5.7535749
7	0.1189360	−1.72649	0.1225439	−3.077555	0.1236127	−3.2070181
8	0.1189361	−1.72851	0.1222499	−3.072411	0.1225210	−3.1831353
9	0.1189362	−1.72963	0.1222269	−3.073524	0.1224097	−3.1910101
10	0.1189366	−1.72965	0.1222259	−3.073589	0.1223892	−3.1797390
Ref.	0.11892	−1.7292	0.12216	−3.0706		
	Brunrau	Bruneau	Kupperman	Bruneau		

- ✓ **Mesh convergence of the solutions irrespective of Re number**
- ✓ **Efficient non-linear solver**
- ✓ **Mesh independent linear solver**

Accurate, robust and efficient Monolithic Multigrid Solver

Viscoplastic flow in Lid-driven cavity

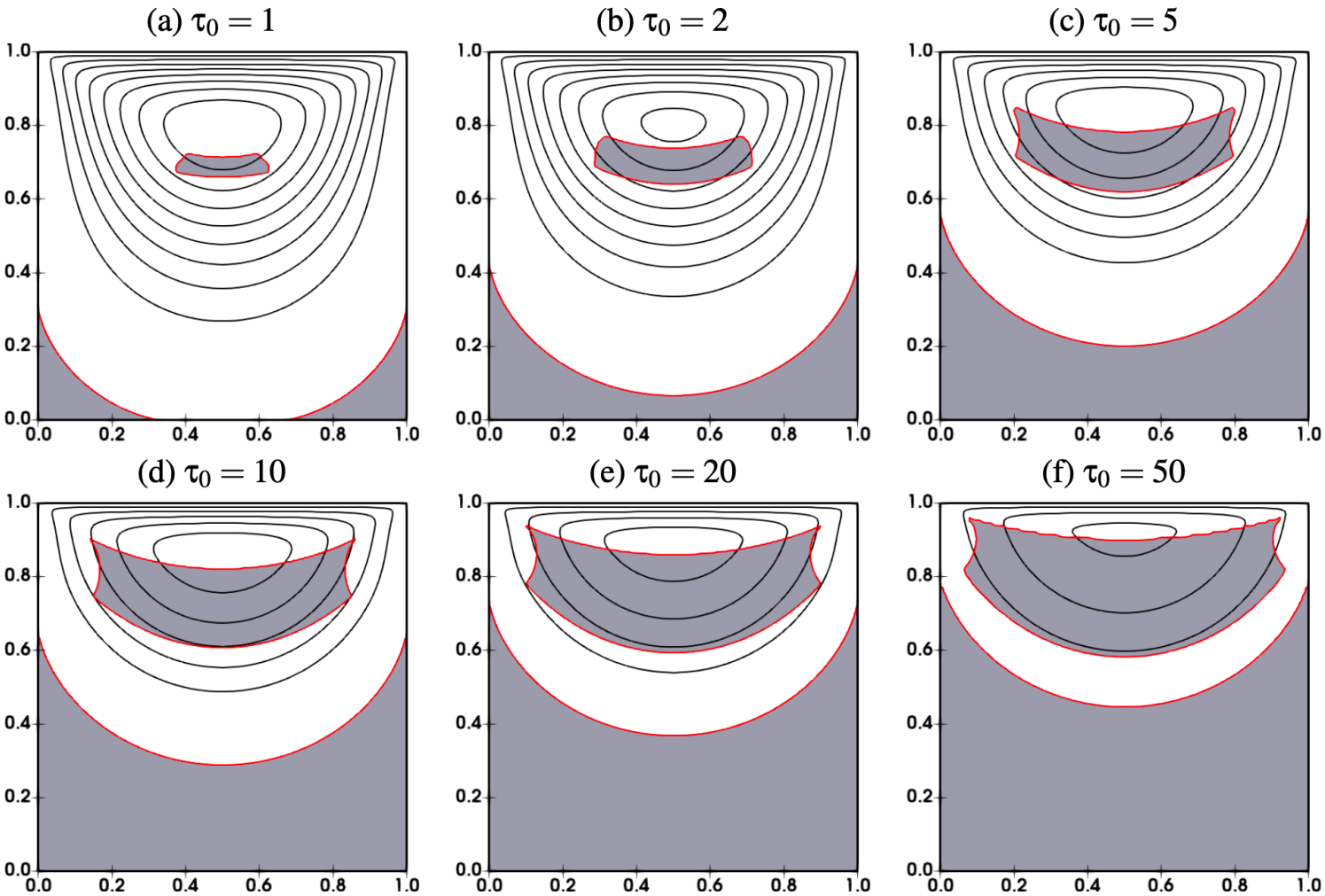
- **Boundary limit for rigid-zone w.r.t regularization k**



- **Accurate track of interface requires**
  - ✓ **larger k solutions**
  - ✓ **finer mesh refinement**
- **Existence of pair (k, L) beyond which no further improvement in solutions is expected**

Viscoplastic flow in Lid-driven cavity

- **progressive growth of unyielded zones for non-thixotropic (Bingham Plastic) flow**



✓ **Unyielded zones's shape and extent is in agreement with Ref. Results**

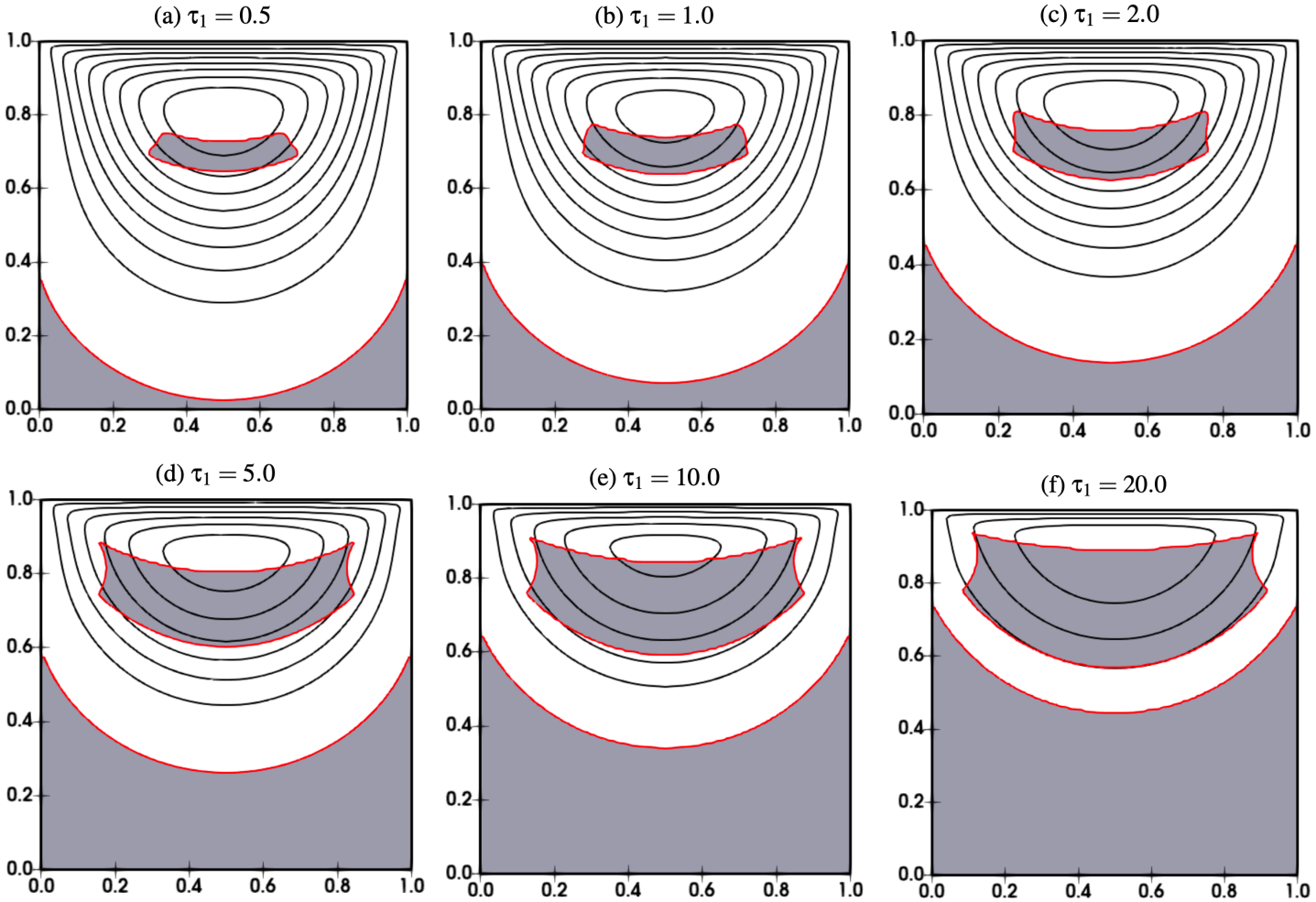
Viscoplastic flow in Lid-driven cavity

- **Solver behaviour w.r.t Regulariztaion and mesh refinement**

$k\backslash L$	5	6	7	5	6	7	5	6	7
	$\tau_0 = 1$			$\tau_0 = 2$			$\tau_0 = 5$		
1×10^1	3/1	3/1	3/1	3/1	3/1	3/1	4/1	4/1	4/1
5×10^1	2/1	2/1	2/1	2/1	2/1	2/1	3/1	3/1	3/1
1×10^2	3/1	3/1	3/1	3/1	3/1	3/1	4/1	4/1	4/1
5×10^2	3/1	2/1	2/1	3/1	2/1	3/1	3/2	3/2	3/1
1×10^3	2/2	3/2	3/1	3/1	3/1	4/1	4/1	5/2	5/2
5×10^3	2/1	2/1	4/1	3/1	3/2	6/2	4/1	8/2	6/1
1×10^4	2/1	2/2	5/1	3/1	3/1	6/1	4/1	5/4	6/3
	$\tau_0 = 10$			$\tau_0 = 20$			$\tau_0 = 50$		
1×10^1	5/1	5/1	5/1	6/1	6/1	6/1	5/1	7/1	7/1
5×10^1	4/1	3/1	3/1	4/1	4/1	3/2	5/4	4/2	4/2
1×10^2	5/2	4/1	4/1	5/2	5/2	5/1	6/5	5/4	5/1
5×10^2	5/3	3/2	3/1	4/4	3/4	4/3	5/4	4/2	4/3
1×10^3	5/2	7/4	9/1	5/5	7/2	8/1	5/5	9/2	9/2
5×10^3	5/1	7/3	8/2	6/3	6/4	6/4	6/4	7/2	8/2
1×10^4	6/1	7/2	8/3	6/3	5/5	7/3	6/3	7/3	8/2

- ✓ **Efficient non-linear solver**
- ✓ **Mesh independent linear solver**
- ✓ **Solutions are obtained with continuation strategy w.r.t. k**
- ➡ **Integration of continuation strategy w.r.t. k in the solver**

- **Impact of thixotropic yield stress on morphology of unyielded zones in TVP flow**



✓ **Main rheological characteristics of materials with yield stress is preserved**

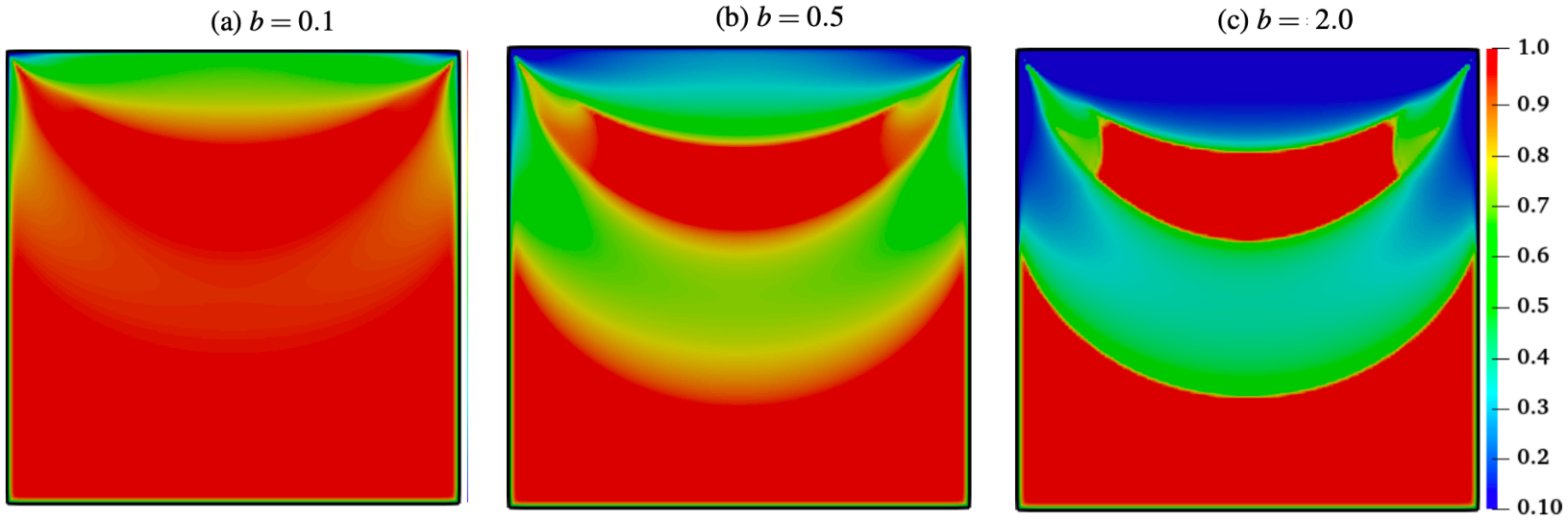
- **Solver behaviour w.r.t Regulariztaion and mesh refinement**

$k\backslash L$	5	6	7	5	6	7	5	6	7
	$\tau_1 = 0.5$			$\tau_1 = 1$			$\tau_1 = 2$		
1×10^1	5/2	5/3	6/2	5/2	5/2	9/1	5/2	5/2	9/1
5×10^1	4/2	4/2	4/2	3/2	3/3	7/1	3/2	3/3	8/1
1×10^2	4/1	4/2	5/1	4/1	4/2	7/1	4/2	4/2	8/1
5×10^2	4/1	4/1	5/1	3/1	4/1	6/1	4/2	4/2	8/1
1×10^3	4/1	4/1	4/1	4/2	4/2	8/1	4/4	6/1	7/1
5×10^3	4/1	4/1	3/2	7/1	9/1	5/1	6/1	9/1	8/1
1×10^4	4/1	4/2	4/2	5/1	7/1	4/1	7/1	10/1	8/2
	$\tau_1 = 5.0$			$\tau_1 = 10.0$			$\tau_1 = 20.0$		
1×10^1	6/2	6/2	10/1	11/1	8/2	11/1	10/1	9/2	11/1
5×10^1	4/2	3/2	11/1	11/1	4/2	7/1	12/1	5/3	9/1
1×10^2	4/2	5/2	11/1	10/1	5/3	8/1	12/1	6/3	10/1
5×10^2	5/2	4/2	10/1	9/1	5/3	5/1	8/1	5/5	11/1
1×10^3	5/2	9/1	10/1	10/1	9/1	7/1	8/2	9/1	9/2
5×10^3	5/1	5/1	5/1	8/1	8/2	6/1	8/1	7/1	11/1
1×10^4	5/1	5/2	5/1	8/3	7/1	5/1	8/2	7/1	9/1

- ✓ **Efficient non-linear solver**
- ✓ **Mesh independent linear solver**
- ✓ **Solutions are obtained with continuation strategy w.r.t. k**
- ➡ **Integration of continuation strategy w.r.t. k in the solver**

Thixo-viscoplastic flow in Lid-driven cavity

- **Material micro-structural level w.r.t. breakdown parameter**



- **Interplay of yield stress and thixotropy**

  - ✓ **Structuring level is predicting shape and extent of rigid zones**
  - ✓ **Induction of more breakdown layers**
  - ✓ **Shear localization**
  - ✓ **Shear band**

Thixo-viscoplastic flow in Couette

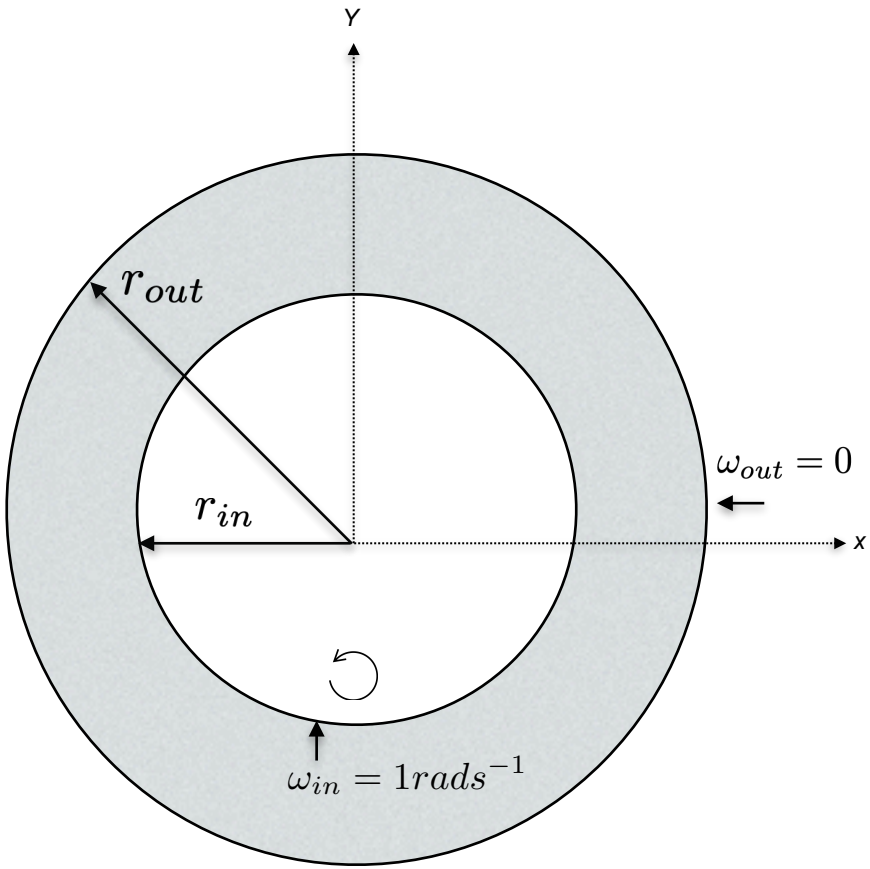
Continuous axial-Flow Couette device:

The material is sheared in the annulus between the interior and exterior cylinder shells of radii r_{in} and r_{out} respectively.

- **Concentric cylinders**
- **Rotating inner cylinder with $\omega_{in} = 1 rads^{-1}$**
- **Stationary outer cylinder**
- **Vertical flow super-imposed in radial direction**

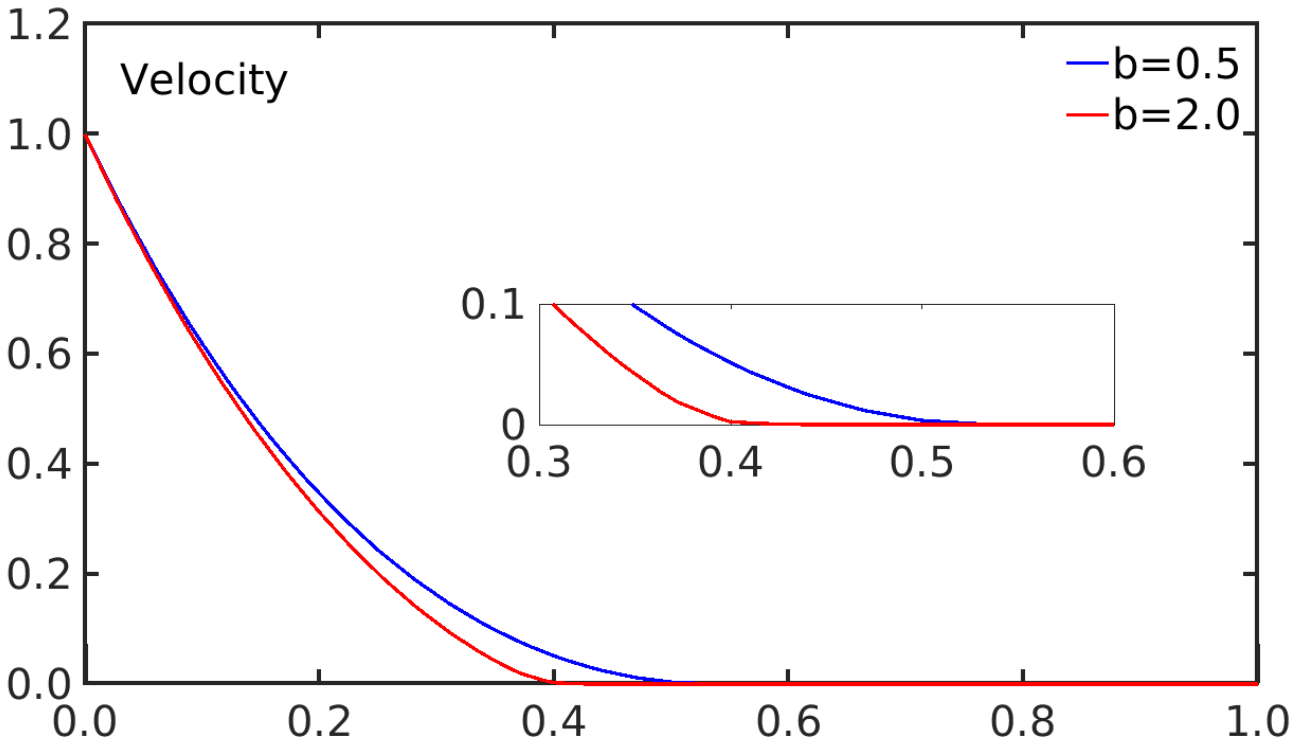
Investigations of thixo-viscoplastic phenomena

- **Shear localization**
- **Shear banding**
- **Consistent transition points between velocity and structure**



Thixo-viscoplastic flow in Couette

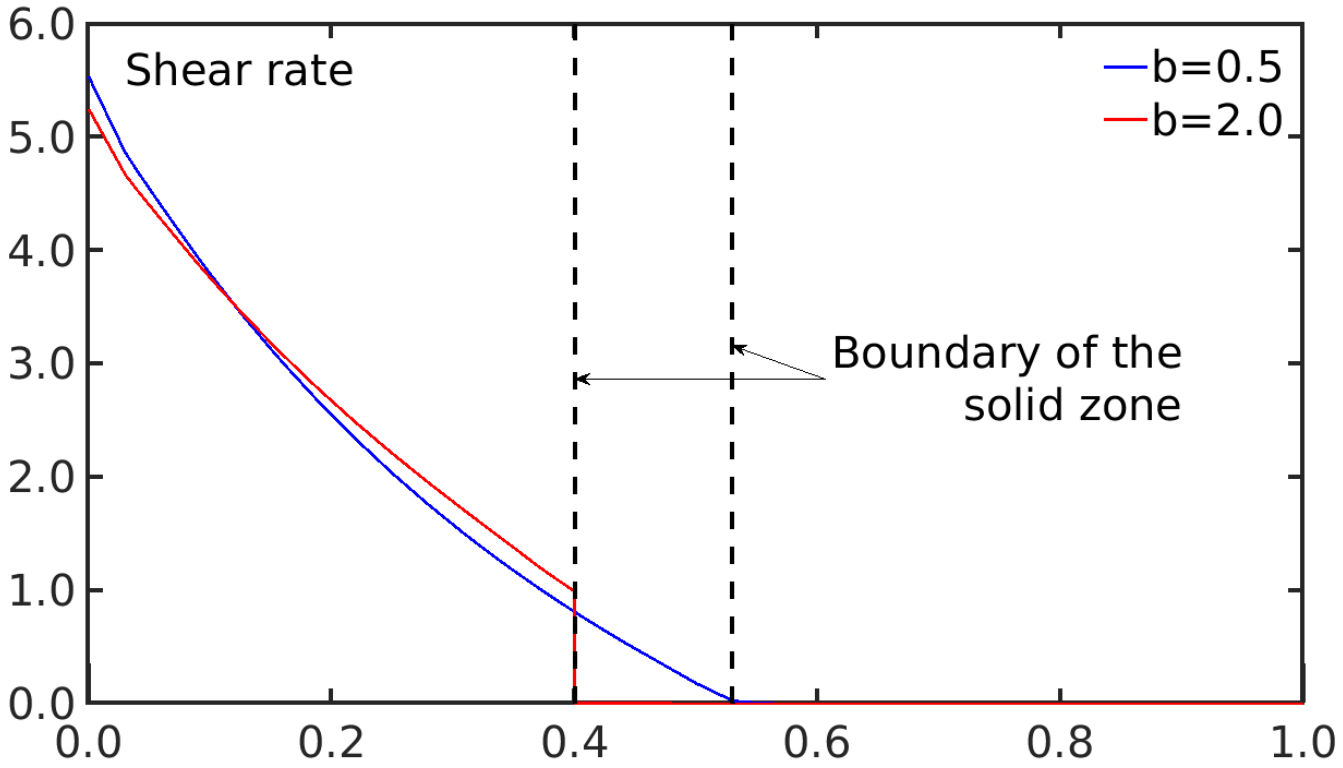
- **Velocity profile at cut-line positions $c; c \in [0, 2\pi]$ in a Couette device w.r.t breakdown parameter**



- ✓ **Localization**
- ✓ **Shear banding**

Thixo-viscoplastic flow in Couette

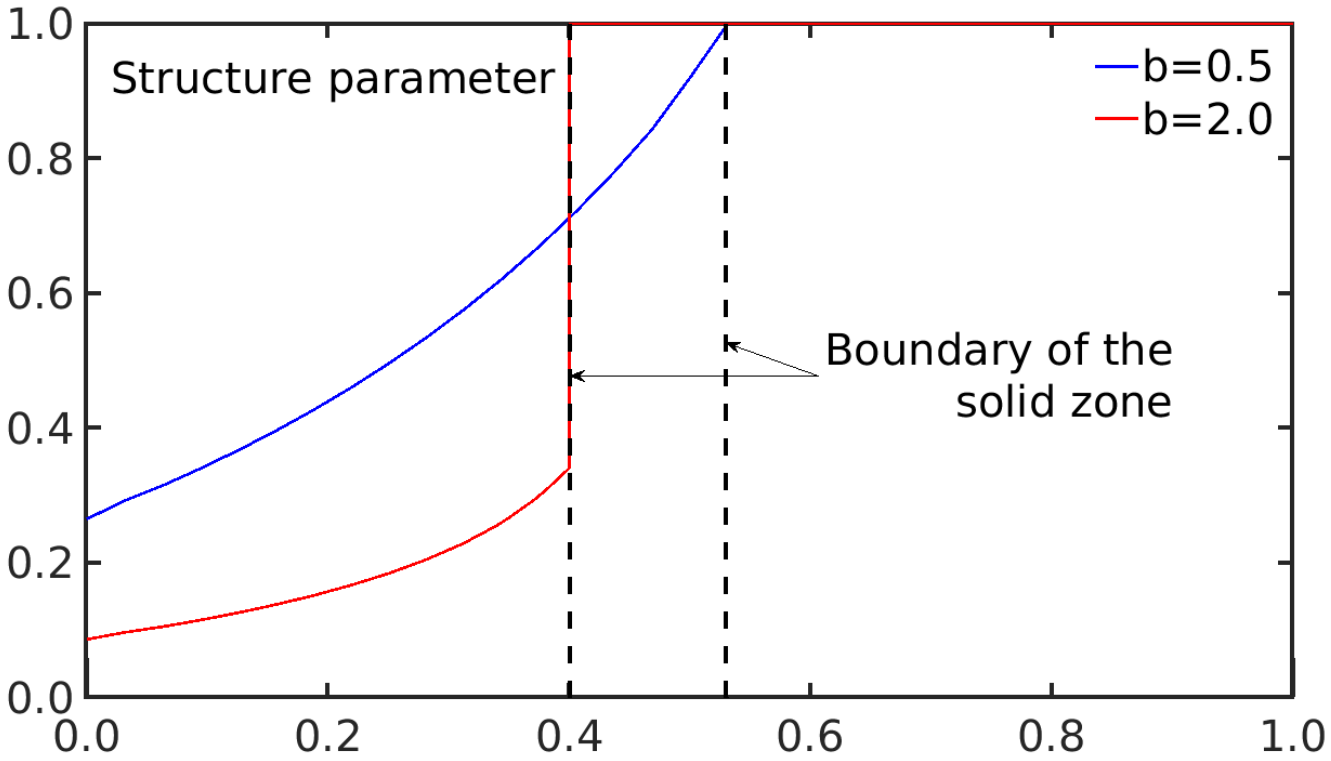
- **Shear rate at cut-line positions $c; c \in [0, 2\pi]$ in a Couette device w.r.t breakdown parameter**



- ✓ **Smooth and sharp transition are possible**
- ✓ **Transition point matches with the velocity**

Thixo-viscoplastic flow in Couette

- **Structure parameter at cut-line positions $c; c \in [0, 2\pi]$ in a couette w.r.t. breakdown parameter**



- ✓ **Transition point matches with the velocity**
- ✓ **Structuring level is predicting shape and extent of rigid zones**

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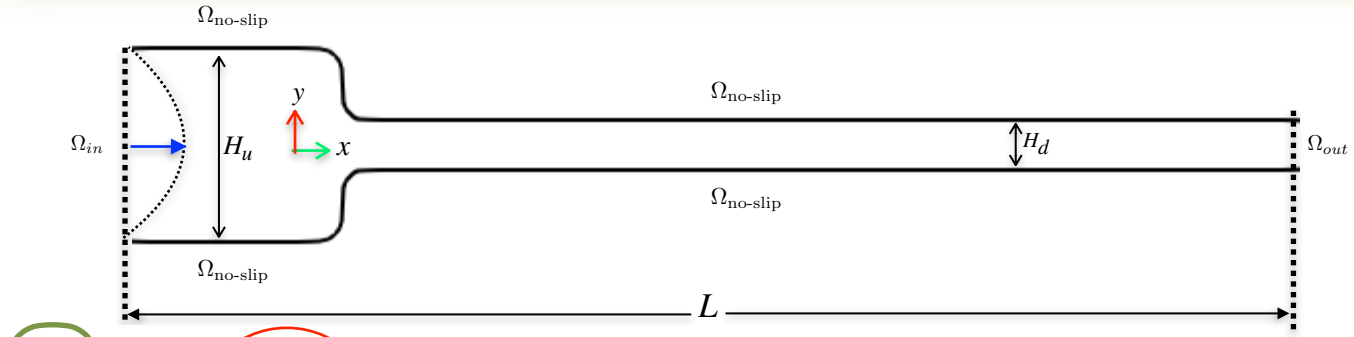
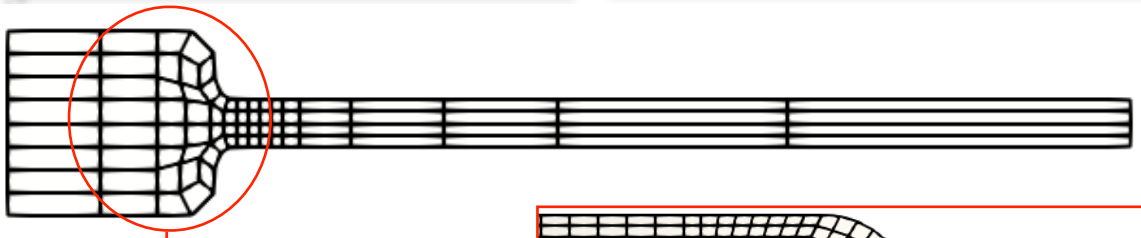
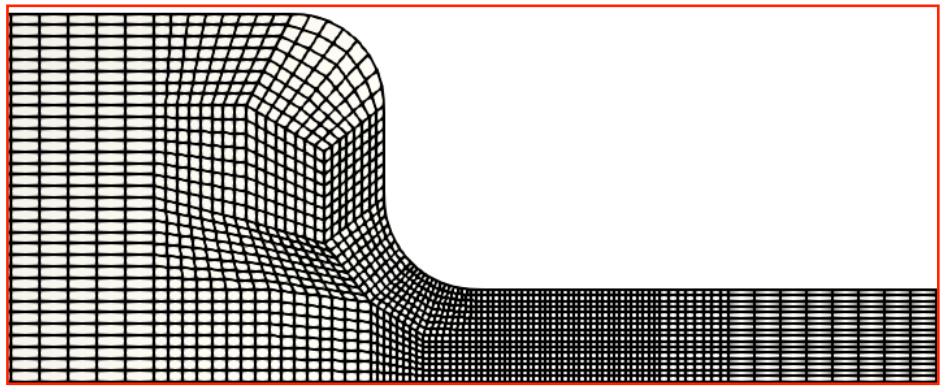
- **2D-FEM simulation results for thixo-viscoplastic flow- validation of 1D tool**
- **Specifying the "unidirectional profiles as boundary Data" in 2D for contraction**



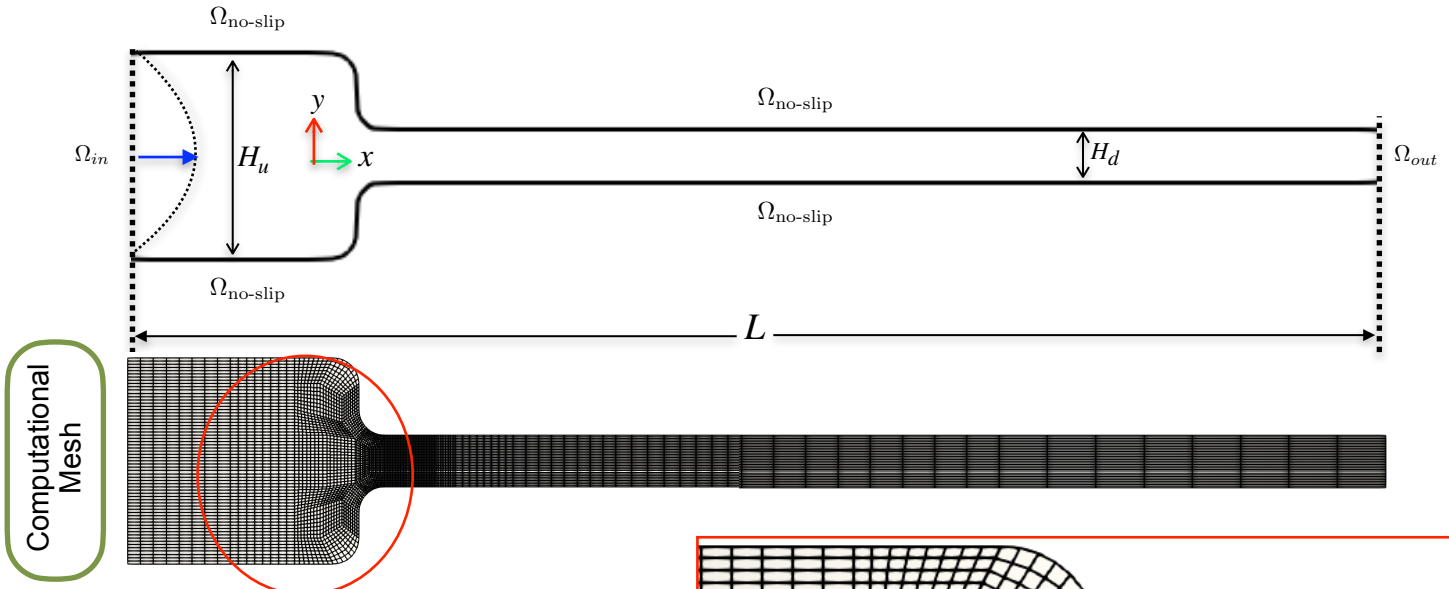
Coarse Mesh

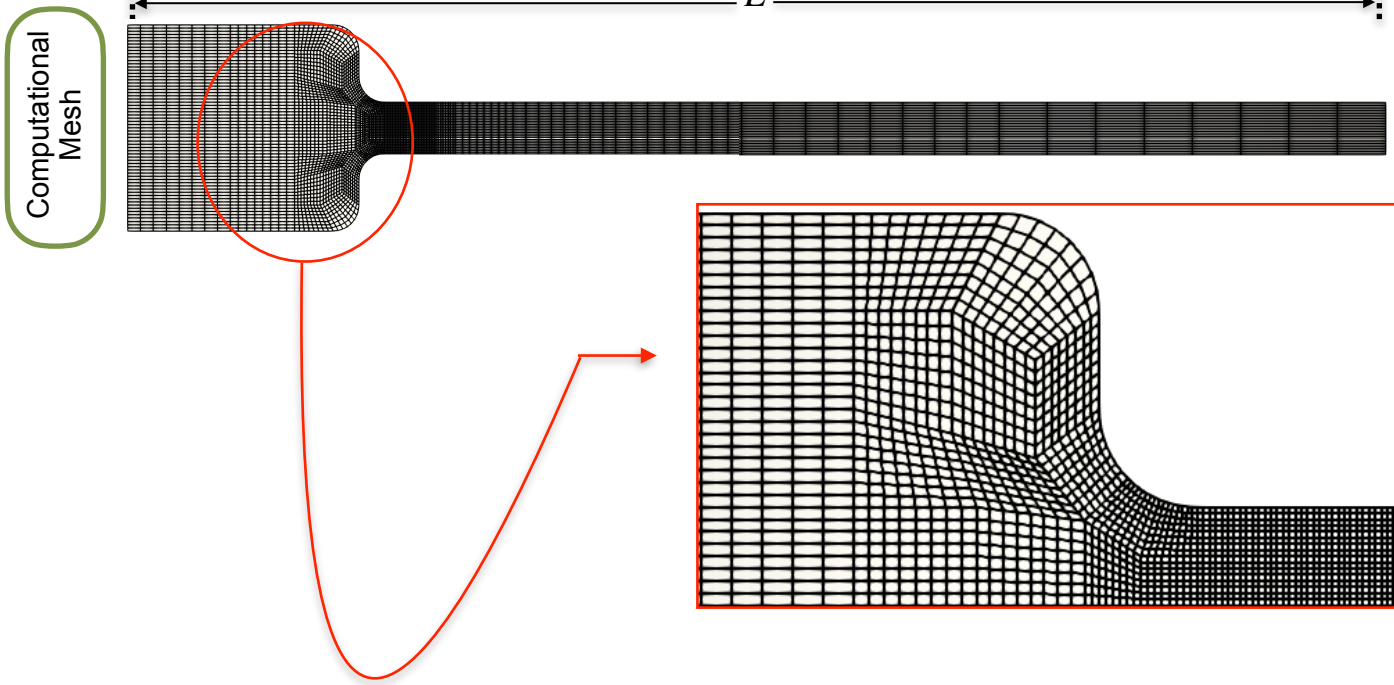




Thixo-viscoplastic flow in curved contractions

- **2D-FEM simulation results for thixotrpic flow- validation of 1D tool**
- **Specifying the "1D-profiles as boundary Data" in 2D simulations for contraction domain**





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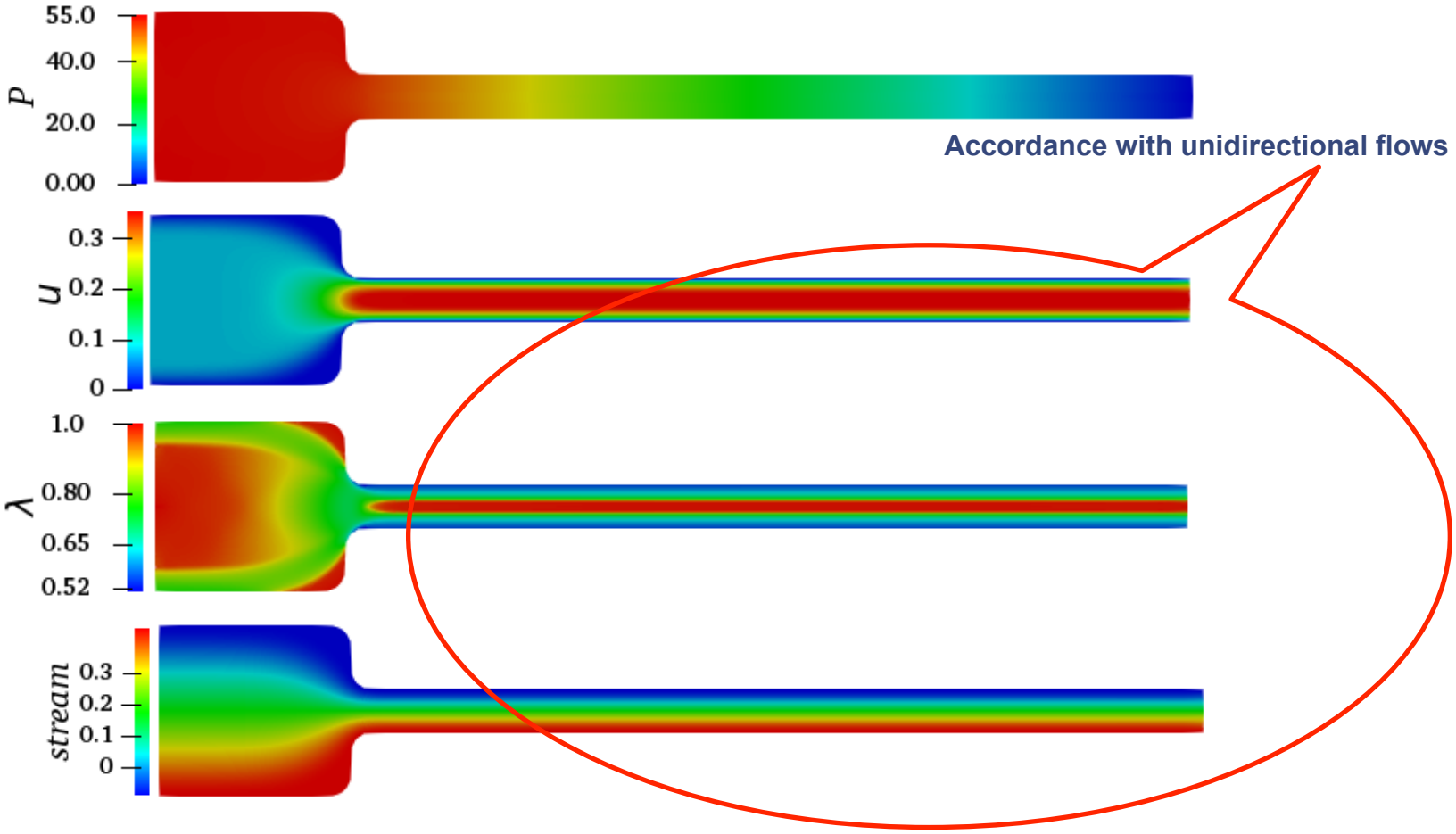
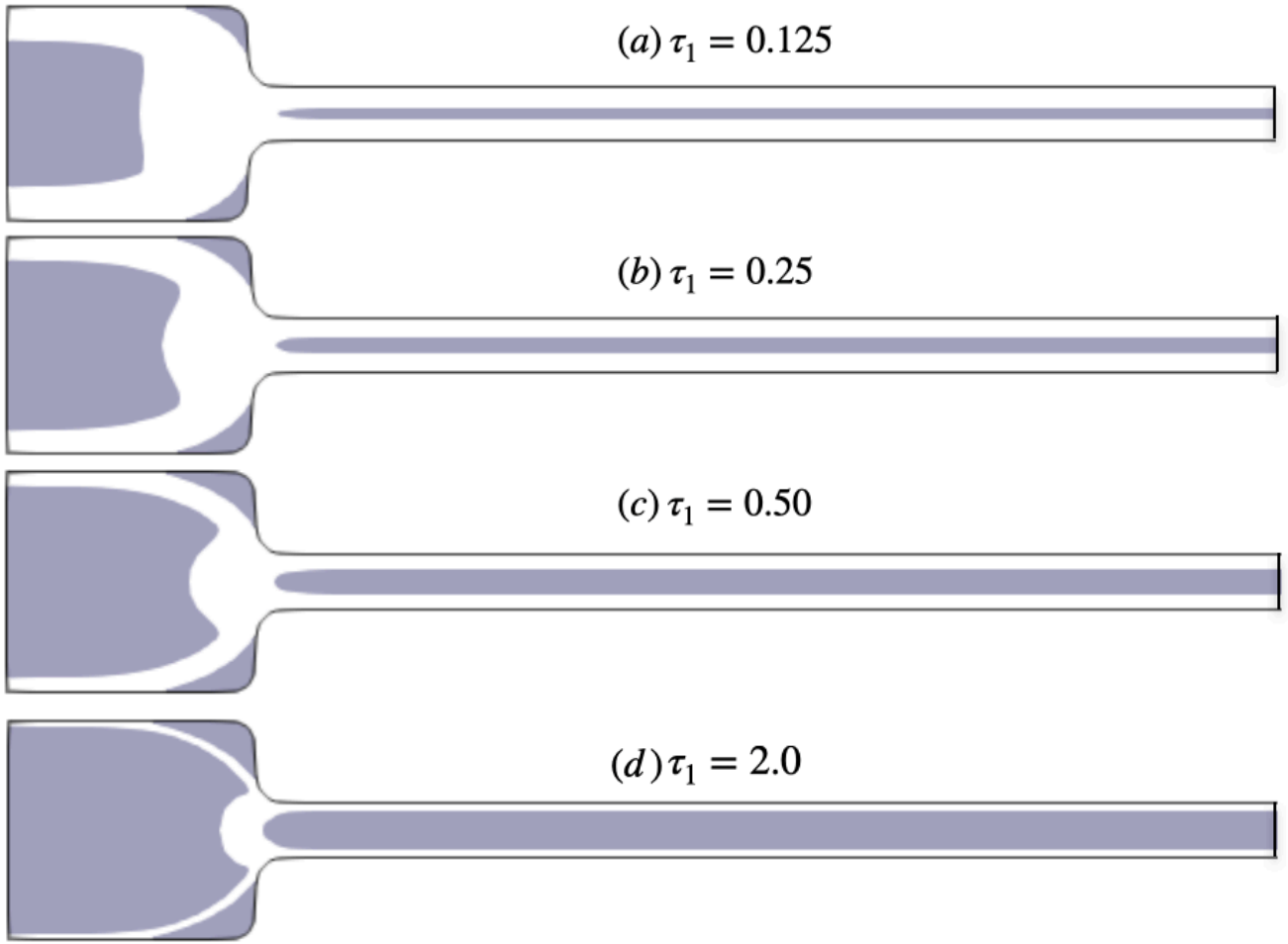
Contraction flow (smoothed)



- ✓ As predicted (u, λ, p) solutions
- ✓ Structuring level is predicting shape and extent of rigid zones

- **progressive growth of unyielded zones (shaded)**



Unyielded zones in upstream and downstream are not merging

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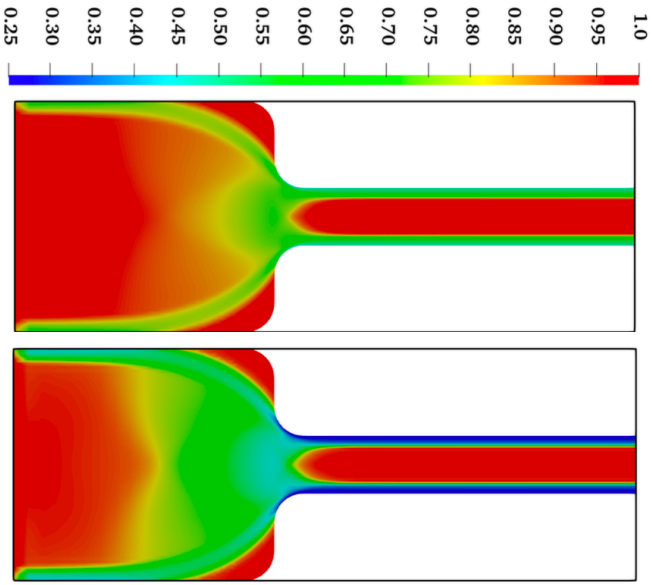
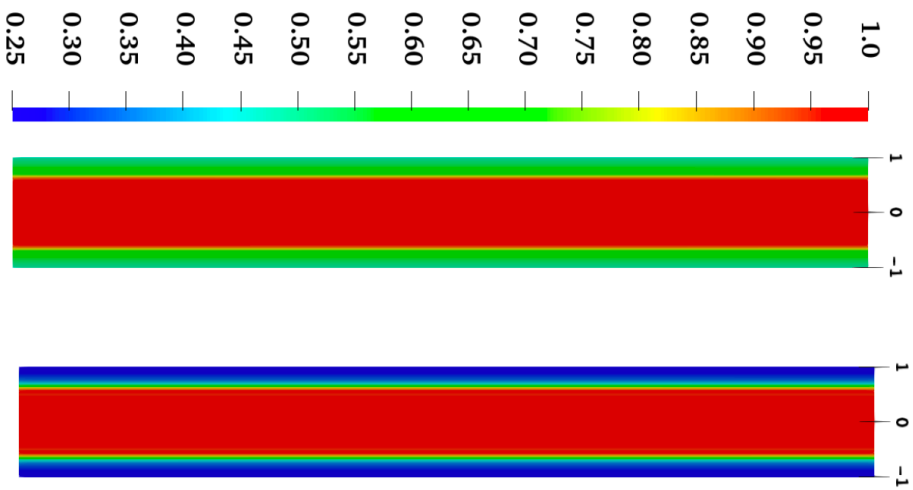
- **Material micro-structural level w.r.t. breakdown**

(i) Upstream channel & Entrance zone



(ii) Downstream channel



- **Inherent thixotropy speed-up the breakdown**
  - ✓ **Appearance of more breakdown layers**
  - ✓ **Applications: restart pressure in pipelines should not be over-estimated**

"Further investigation" regarding material structuring in thixo-elasovisocplastic

Summary

An accurate, robust, and efficient numerical solver for TVP flows is developed using

- ✓ **Higher order finite element method**
- ✓ **Monolitic Newton-multigrid**
  - **Adaptive discrete Newton's method with global convergent property**
  - **Geometric multigrid with local MPSC**

To analyze the quasi-Newtonian model for TVP materials for different flow simulations

- ✓ **Newtonian, VP, and TVP flow in Lid-driven cavity**
- ✓ **TVP flow in Couette devices**
- ✓ **TVP flow in curved contractions**

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