GPU Cluster Computing for FEM

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FEAST –

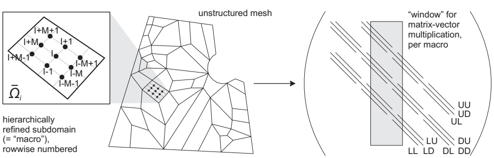
Hardware-oriented Numerics

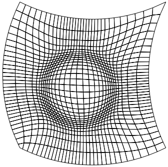
Structured vs. unstructured grids

Fully adaptive grids
Maximum flexibility
'Stochastic' numbering
Unstructured sparse matrices
Indirect addressing, very slow.

Locally structured grids
Logical tensor product
Fixed banded matrix structure
Direct addressing ($\Rightarrow$ fast)
r-adaptivity

Unstructured macro mesh of tensor product subdomains





Exploit local structure for tuned linear algebra and tailored multigrid smoothers

Solver approach

ScaRC – Scalable Recursive Clustering

- Hybrid multilevel domain decomposition method
- Minimal overlap by extended Dirichlet BCs
- Inspired by parallel MG ("best of both worlds")
  - Multiplicative between levels, global coarse grid problem (MG-like)
  - Additive horizontally: block-Jacobi / Schwarz smoother (DD-like)
- Hide local irregularities by MGs within the Schwarz smoother
- Embed in Krylov to alleviate Block-Jacobi character

global BiCGStab
preconditioned by
  global multilevel (V 1+1)
  additively smoothed by
    for all Ω_i: **local multigrid**
  coarse grid solver: UMFPACK

Multivariate problems

Block-structured systems

- Guiding idea: Tune scalar case once per architecture instead of over and over again per application
- Equation-wise ordering of the unknowns
- Block-wise treatment enables multivariate ScaRC solvers

Examples

- Linearised elasticity with compressible material (2x2 blocks)
- Saddle point problems: Stokes (3x3 with zero blocks), elasticity with (nearly) incompressible material, Navier-Stokes with stabilisation (3x3 blocks)

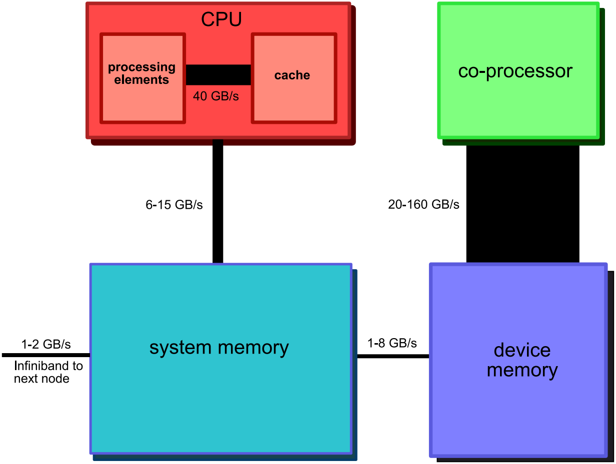
$$\begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{pmatrix} \begin{pmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \end{pmatrix} = \mathbf{f}, \quad \begin{pmatrix} \mathbf{A}_{11} & \mathbf{0} & \mathbf{B}_1 \\ \mathbf{0} & \mathbf{A}_{22} & \mathbf{B}_2 \\ \mathbf{B}_1^\mathsf{T} & \mathbf{B}_2^\mathsf{T} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \mathbf{p} \end{pmatrix} = \mathbf{f}, \quad \begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \mathbf{B}_1 \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \mathbf{B}_2 \\ \mathbf{B}_1^\mathsf{T} & \mathbf{B}_2^\mathsf{T} & \mathbf{C}_C \end{pmatrix} \begin{pmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \mathbf{p} \end{pmatrix} = \mathbf{f}$$

$\mathbf{A}_{11}$ and $\mathbf{A}_{22}$ correspond to scalar (elliptic) operators

$\Rightarrow$ Tuned linear algebra **and** tuned solvers

Co-processor integration into FEAST

Bandwidth in a CPU/GPU node



Mixed Precision Multigrid

	Core2Duo (double)		GTX 280 (mixed)		
Level	time(s)	MFLOP/s	time(s)	MFLOP/s	speedup
7	0.021	1405	0.009	2788	2.3x
8	0.094	1114	0.012	8086	7.8x
9	0.453	886	0.026	15179	17.4x
10	1.962	805	0.073	21406	26.9x

- Poisson on unitsquare, Dirichlet BCs, TP grid, *not a matrix stencil*
- *Converges to wrong solution in single precision*
- 1M DOF, multigrid, FE-accurate in less than 0.1 seconds!
- 27x faster than CPU, exactly same results as pure double
- 1.7x faster than pure double on GPU
- defect calculation alone: 46.5 GFLOP/s, 50x speedup (single vs. single)

Minimally invasive integration

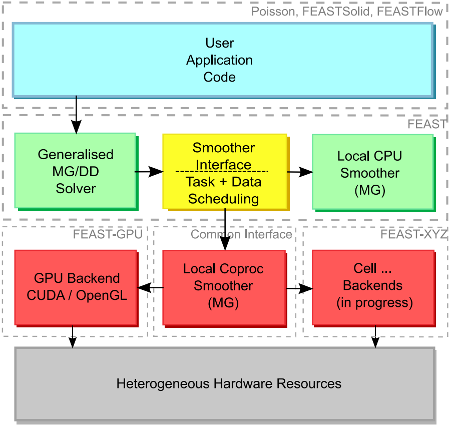
global BiCGStab
preconditioned by
  global multilevel (V 1+1)
  additively smoothed by
    for all Ω_i: **local multigrid**
  coarse grid solver: UMFPACK

All outer work: CPU, double

Local MGs: GPU, single

Same accuracy and functionality mandatory

Oblivious of the application



Some results

Linearised elasticity

$$\begin{pmatrix} \color{red}{\mathbf{A}_{11}} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \color{red}{\mathbf{A}_{22}} \end{pmatrix} \begin{pmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \end{pmatrix} = \mathbf{f}$$

$$\begin{pmatrix} (2\mu+\lambda)\partial_{xx} + \mu\partial_{yy} & (\mu+\lambda)\partial_{xy} \\ (\mu+\lambda)\partial_{yx} & \mu\partial_{xx} + (2\mu+\lambda)\partial_{yy} \end{pmatrix}$$

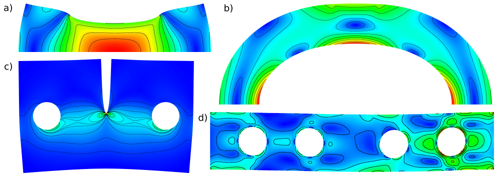
global multivariate BiCGStab
block-preconditioned by
Global multivariate multilevel (V 1+1)
additively smoothed (block GS) by

for all Ω_i: solve $\mathbf{A}_{11}\mathbf{c}_1 = \mathbf{d}_1$ by
local scalar multigrid

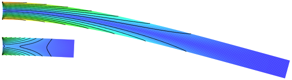
update RHS: $\mathbf{d}_2 = \mathbf{d}_2 - \mathbf{A}_{21}\mathbf{c}_1$

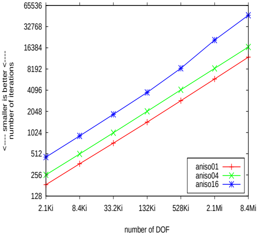
for all Ω_i: solve $\mathbf{A}_{22}\mathbf{c}_2 = \mathbf{d}_2$ by
local scalar multigrid

coarse grid solver: UMFPACK



Accuracy

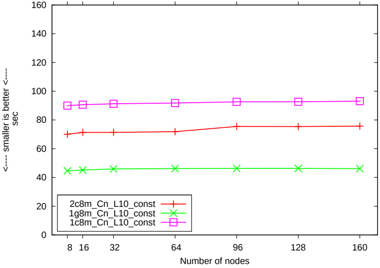


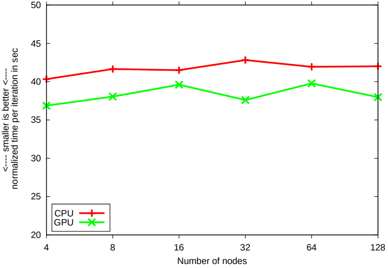


Cantilever beam, aniso 1:1, 1:4, 1:16
Hard, very ill-conditioned CSM test
CG solver: $>$ 2x iterations per refinement
GPU-ScaRC solver: same results as CPU

aniso04	Iterations		Volume		y-Displacement	
refinement L	CPU	GPU	CPU	GPU	CPU	GPU
8	4	4	1.6087641E-3	1.6087641E-3	-2.8083499E-3	-2.8083499E-3
9	4	4	1.6087641E-3	1.6087641E-3	-2.8083628E-3	-2.8083628E-3
10	4.5	4.5	1.6087641E-3	1.6087641E-3	-2.8083667E-3	-2.8083667E-3
aniso16						
8	6	6	6.7176398E-3	6.7176398E-3	-6.6216232E-2	-6.6216232E-2
9	**6**	**5.5**	6.7176427E-3	6.7176427E-3	-6.621655**1**E-2	-6.621655**2**E-2
10	5.5	5.5	6.7176516E-3	6.7176516E-3	-6.621750**1**E-2	-6.621750**2**E-2

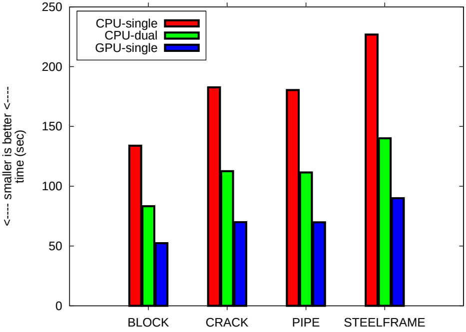
Weak scalability





- Outdated cluster, dual Xeon EM64T singlecore
- one NVIDIA Quadro FX 1400 per node (one generation behind the Xeons, 20 GB/s BW)
- Poisson problem (left): up to 1.3 B DOF, 160 nodes
- Elasticity (right): up to 1 B DOF, 128 nodes

Absolute speedup



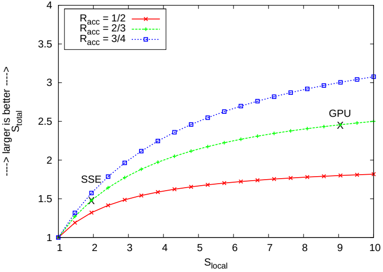
- 16 nodes, Opteron 2214 dualcore
- NVIDIA Quadro FX 5600 (76 GB/s BW), OpenGL
- Problem size 128 M DOF
- Dualcore 1.6x faster than singlecore
- GPU 2.6x faster than singlecore, 1.6x than dual

Acceleration analysis

Speedup analysis

- Addition of GPUs increases resources
- $\Rightarrow$ Correct model: strong scalability inside each node
- Accelerable fraction of the elasticity solver: $2/3$
- Remaining time spent in MPI and the outer solver

Accelerable fraction R_{acc}:	66%
Local speedup S_{local}:	9x
Total speedup S_{total}:	2.6x
Theoretical limit S_{max}:	3x



Stationary Navier-Stokes

$$\begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \mathbf{B}_1 \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \mathbf{B}_2 \\ \mathbf{B}_1^\mathsf{T} & \mathbf{B}_2^\mathsf{T} & \mathbf{C} \end{pmatrix} \begin{pmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \\ \mathbf{p} \end{pmatrix} = \begin{pmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \mathbf{g} \end{pmatrix}$$

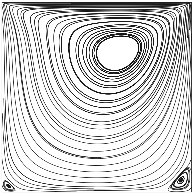
- 4-node cluster
- Opteron 2214 dualcore
- GeForce 8800 GTX (90 GB/s BW), CUDA
- Driven cavity and channel flow around a cylinder

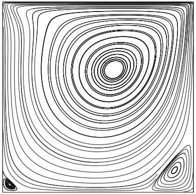
fixed point iteration
assemble linearised subproblems and solve with
global BiCGStab (reduce initial residual by 1 digit)
Block-Schurcomplement preconditioner
1) approx. solve for velocities with
global MG (V 1+0), additively smoothed by

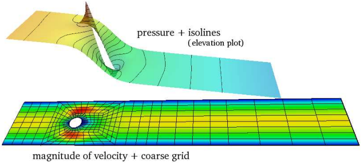
for all Ω_i: solve for $\mathbf{u}_1$ with **local MG**

for all Ω_i: solve for $\mathbf{u}_2$ with **local MG**

2) update RHS: $\mathbf{d}_3 = -\mathbf{d}_3 + \mathbf{B}^\mathsf{T}(\mathbf{c}_1, \mathbf{c}_2)^\mathsf{T}$
3) scale $\mathbf{c}_3 = (\mathbf{M}_p^\mathsf{L})^{-1}\mathbf{d}_3$







Navier-Stokes results

Speedup analysis

	R_{acc}		S_{local}		S_{total}	
	L9	L10	L9	L10	L9	L10
DC Re100	41%	46%	6x	12x	1.4x	1.8x
DC Re250	56%	58%	5.5x	11.5x	1.9x	2.1x
Channel flow	60%	–	6x	–	1.9x	–

Important consequence: Ratio between assembly and linear solve changes significantly

DC Re100		DC Re250		Channel flow	
plain	accel.	plain	accel.	plain	accel.
29:71	50:48	11:89	25:75	13:87	26:74

Conclusions

Conclusions

- Hardware-oriented numerics prevents existing codes being worthless in a few years
- Mixed precision schemes exploit the available bandwidth without sacrificing accuracy
- GPUs as local preconditioners in a large-scale parallel FEM package
- Not limited to GPUs, applicable to all kinds of hardware accelerators
- Minimally invasive approach, no changes to application code
- Excellent local acceleration, global acceleration limited by 'sequential' part
- Future work: Design solver schemes with higher acceleration potential without sacrificing numerical efficiency

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