FEM techniques for interfacial flows

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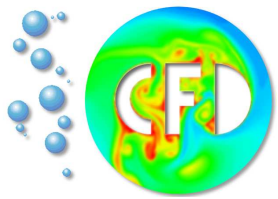


Benchmarking

Why benchmark

Why spend valuable time and effort to establish benchmark test cases?

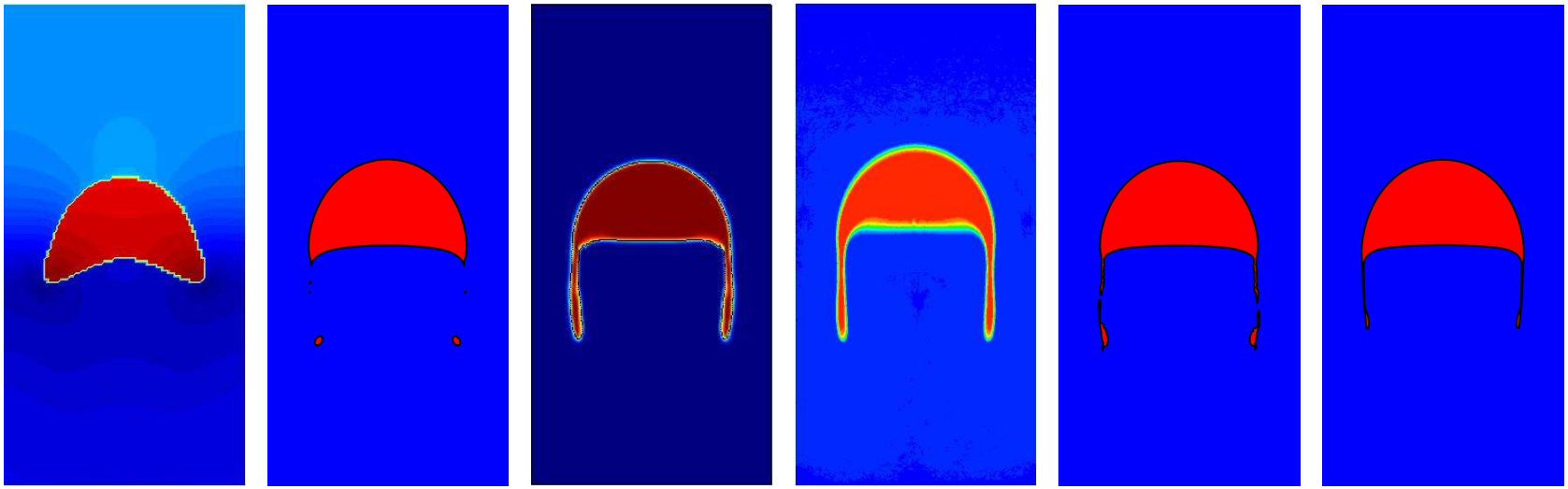
- Validation
- Comparison
- Evaluation

Why benchmark

Why spend valuable time and effort to establish benchmark test cases?

What is the CPU cost for achieving a certain accuracy?

Validation?



1) A. Smolianski; *Finite-element/level-set/operator-splitting (FELSOS) approach for computing two-fluid unsteady flows with free moving interfaces*, Int. J. Numer. Meth. Fluids 2005; 48:231-269.

2) S. Hysing, S. Turek, D. Kuzmin, N. Parolini, E. Burman, S. Ganesan, and L. Tobiska; *Proposal for quantitative benchmark computations of bubble dynamics*, Submitted to Int. J. Numer. Meth. Fluids.

Benchmarks

Development of quantitative two-phase flow benchmarks for critical evaluation of new and existing methods

Bubble benchmark quantities

- Center of mass
- Circularity
- Rise velocity

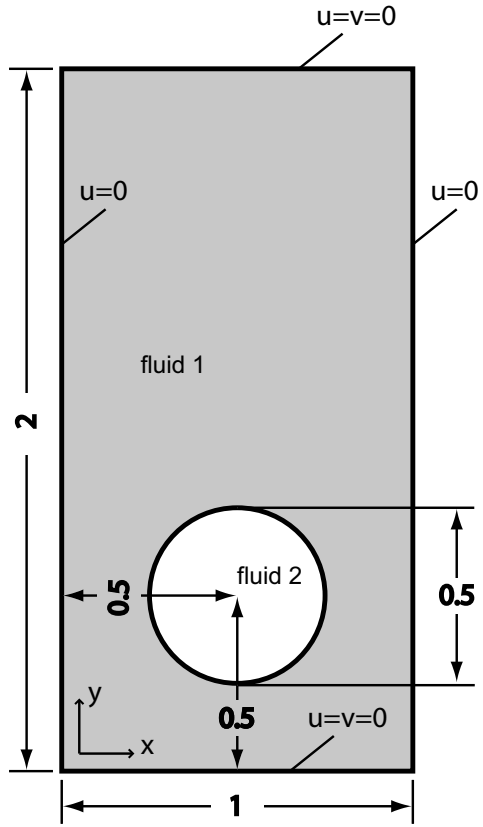


Figure: Initial configuration and boundary conditions for the test cases

Benchmark Quantities

- Center of mass:

$$\mathbf{x}_c = \int_{\Omega_2} \mathbf{x}\ d\mathbf{x} / \int_{\Omega_2} 1\ d\mathbf{x}$$

- Circularity:

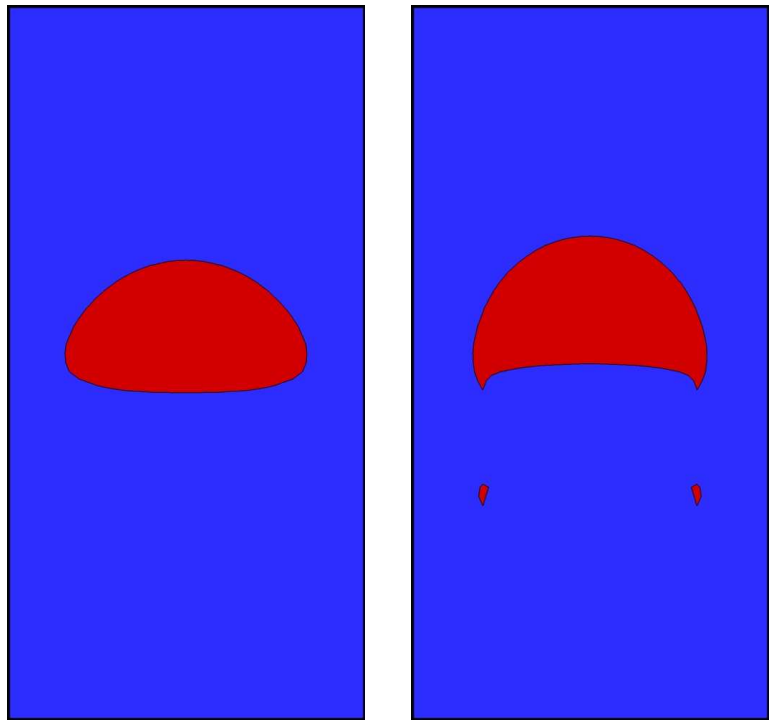
$$\phi = \frac{\text{perimeter of area-equivalent circle}}{\text{perimeter of } \Omega_2}$$

- Rise velocity:

$$\mathbf{U} = \int_{\Omega_2} \mathbf{u}\ d\mathbf{x} / \int_{\Omega_2} 1\ d\mathbf{x}$$

Benchmark Test Cases

Test Case	1	2
ρ_1 (liquid)	1000	1000
ρ_2 (gas)	1	100
μ_1 (liquid)	10	10
μ_2 (gas)	0.1	1
g_y	-0.98	-0.98
σ	1.96	24.5
Re	35	35
Eo	125	10
ρ_1/ρ_2	1000	10
μ_1/μ_2	100	10



Test Case 1

Test Case 2

Benchmark Test Cases

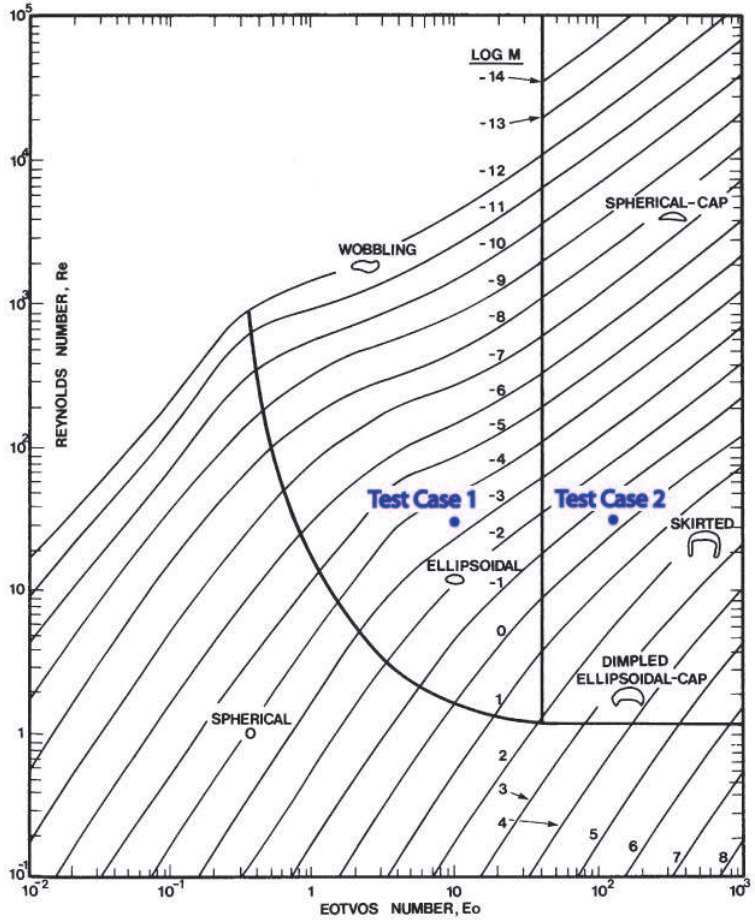


Figure: Shape regimes for bubbles and drops in unhindered gravitational motion through liquids [Clift *et al.*, Bubbles, Drops and Particles (1978)]

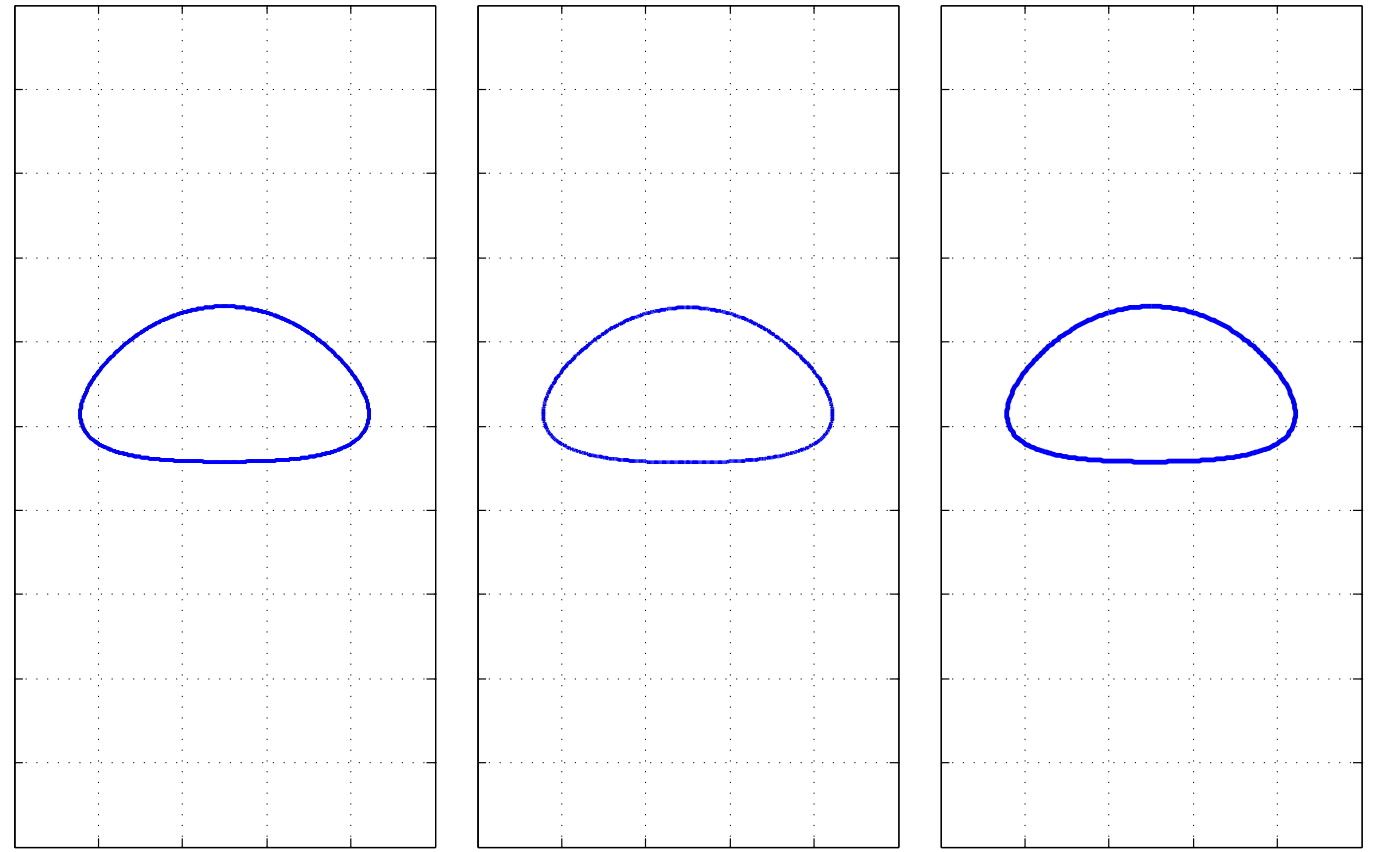
Preliminary computations

Participating Groups

	Group and Affiliation	Code/Method
1	Uni. Dortmund, Inst. of Applied Math. *S. Turek, D. Kuzmin, S. Hysing*	**TP2D** *FEM-Level Set*
2	EPFL Lausanne, Inst. of Analysis and Sci. Comp. *E. Burman, N. Parolini*	**FreeLIFE** *FEM-Level Set*
3	Uni. Magdeburg, Inst. of Analysis and Num. Math. *L. Tobiska, S. Ganesan*	**MooNMD** *FEM-ALE*

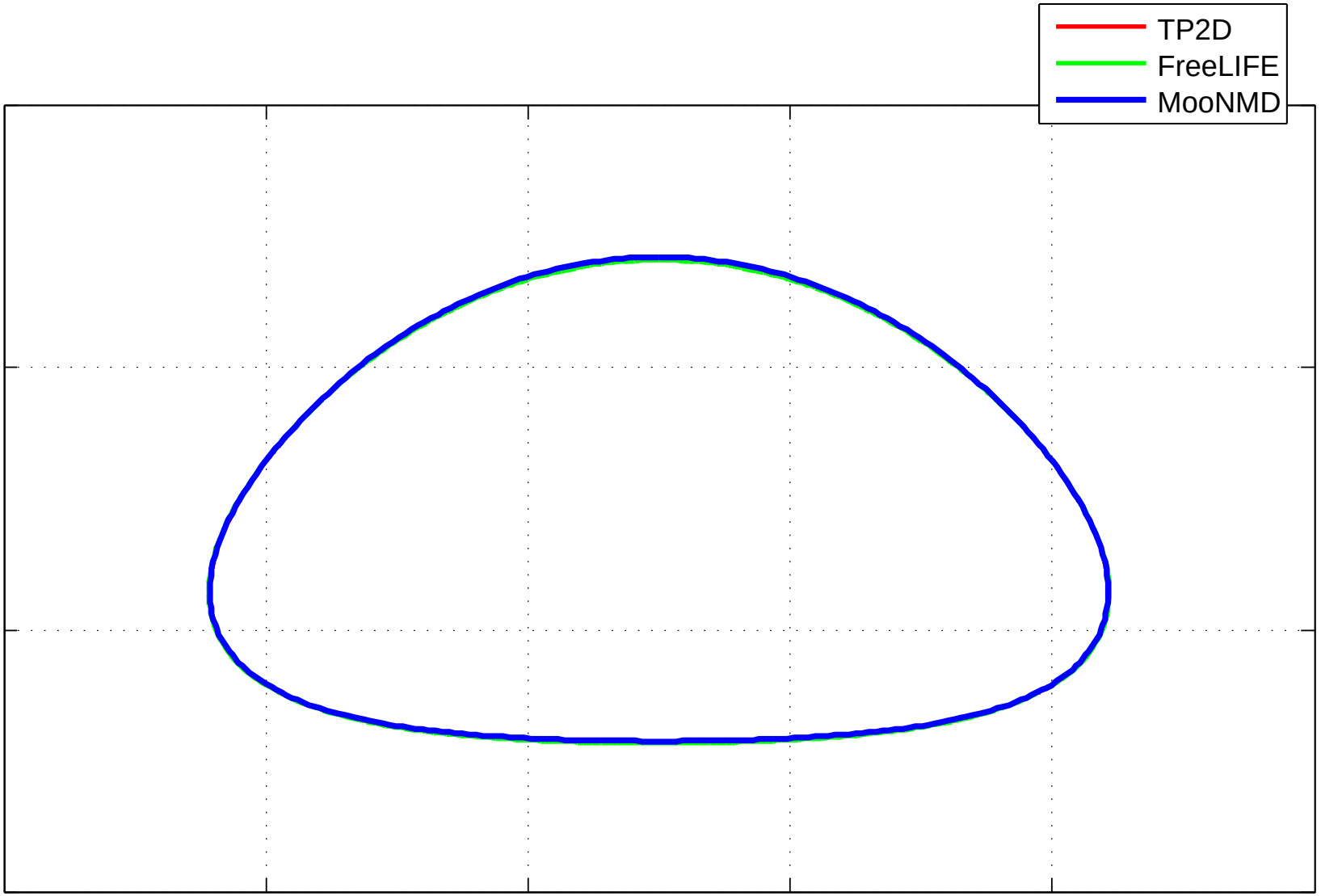
Test Case 1

Visual comparison

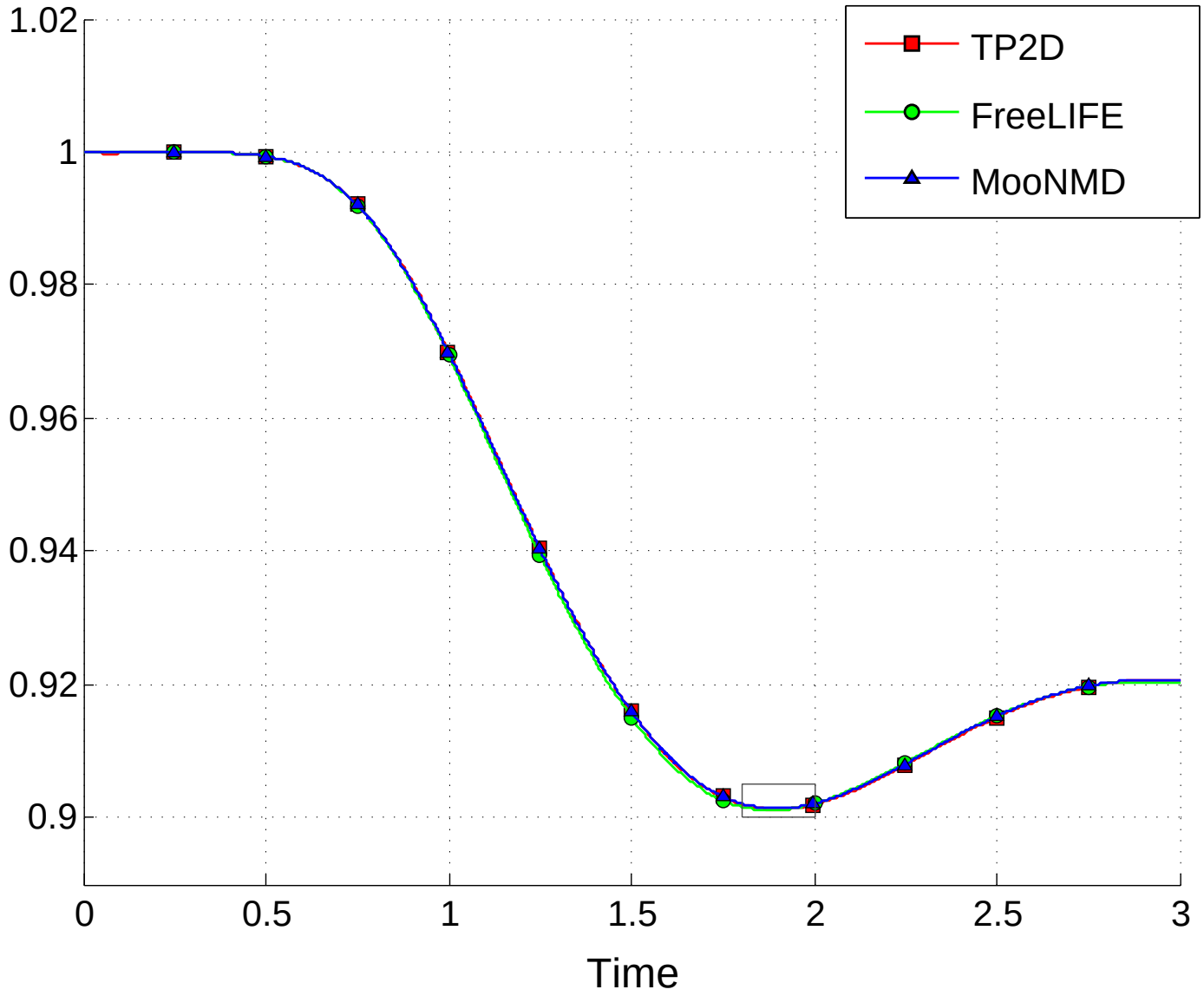


TP2D | *FreeLIFE* | *MooNMD*

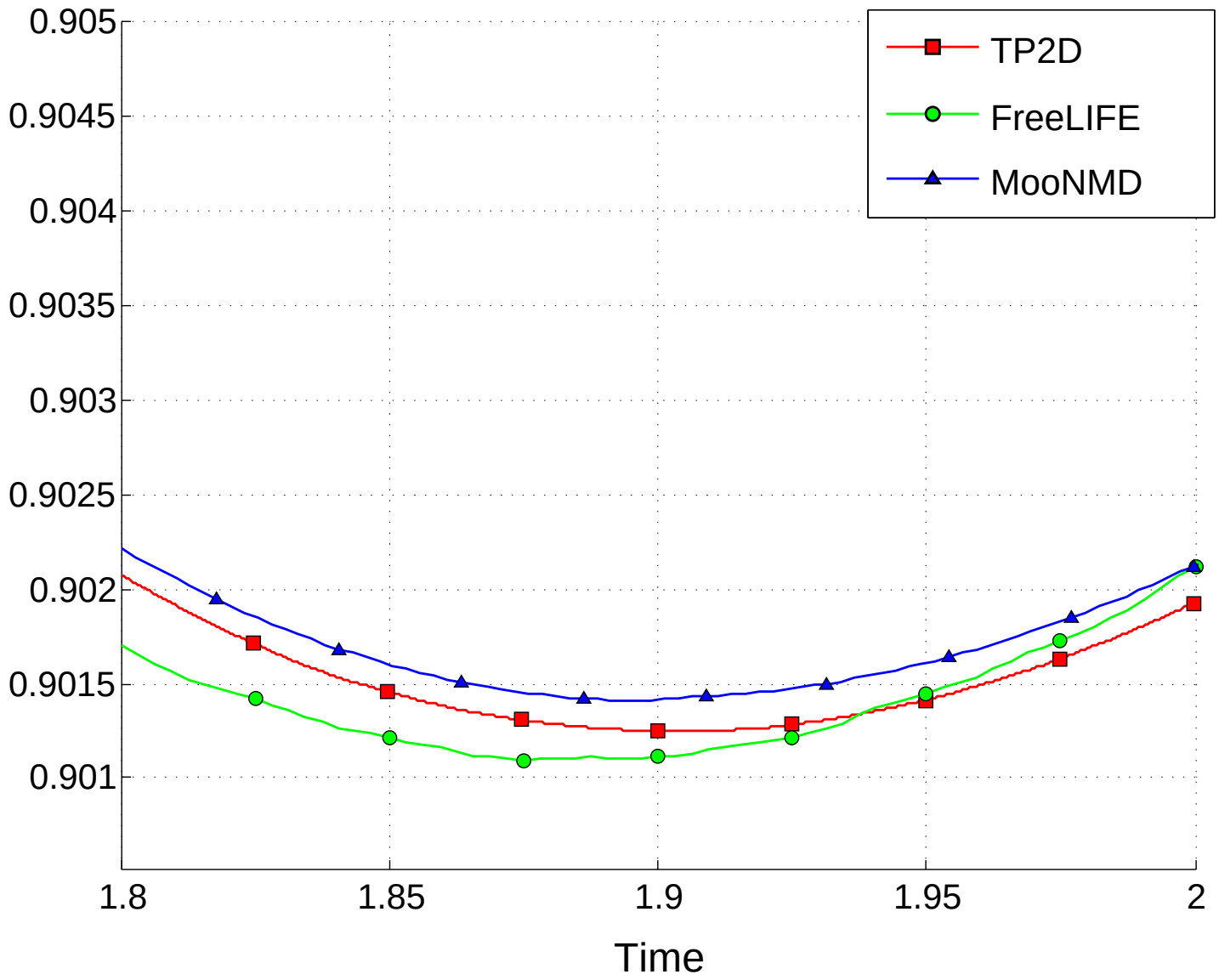
Visual comparison



Benchmark quantities - circularity



Benchmark quantities - circularity



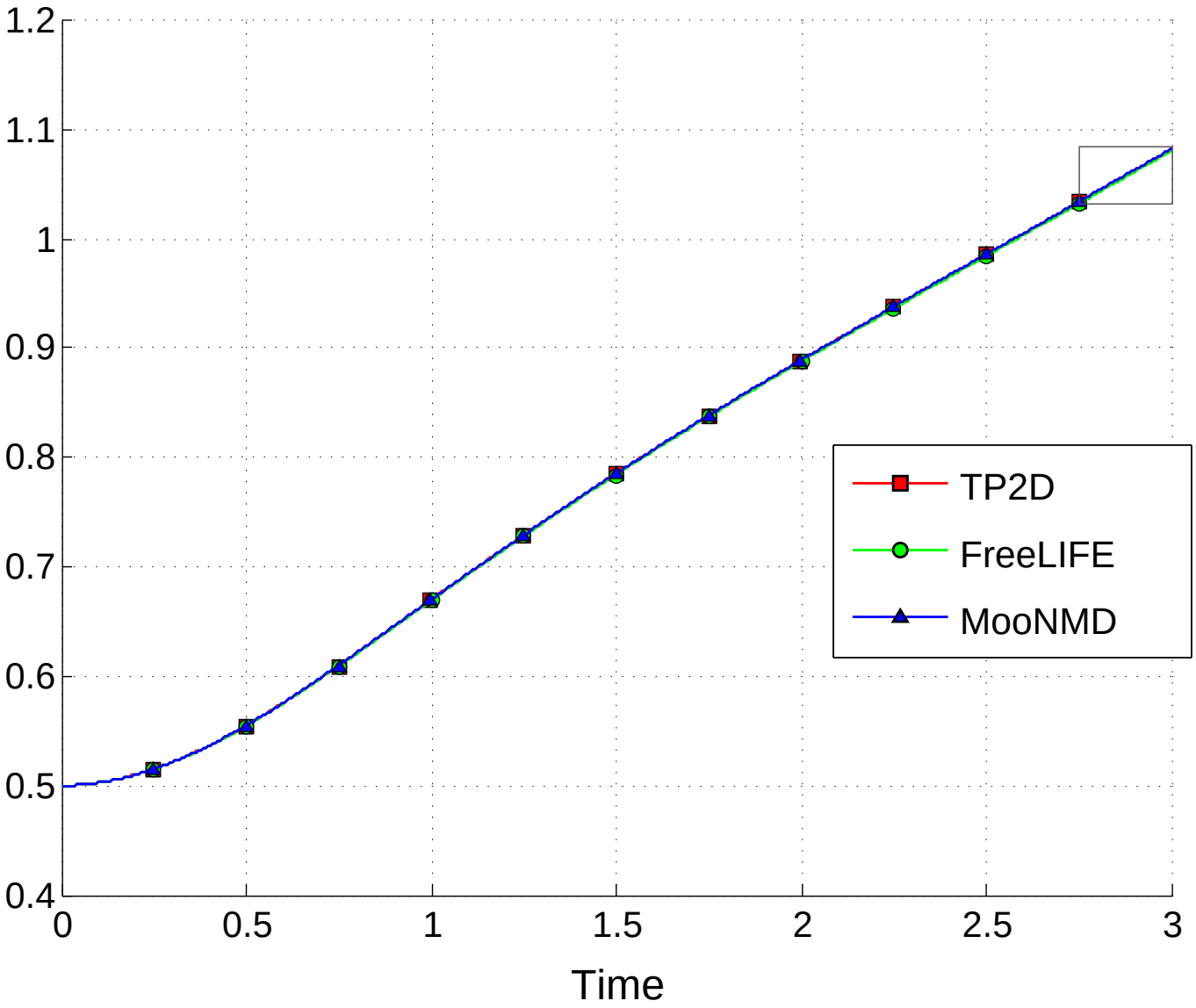
Benchmark quantities - circularity

GridLevel	1	2	3	4
Minimum circularity, $\not\subset_{min}$				
TP2D	0.9016	0.9014	0.9014	0.9013
FreeLIFE		0.9060	0.9021	0.9011
MooNMD	0.9022	0.9018	0.9013	0.9014
Incidence time, $t\|_{\not\subset=\not\subset_{min}}$				
TP2D	1.9234	1.8734	1.9070	1.9041
FreeLIFE		1.8375	1.9125	1.8750
MooNMD	1.8630	1.8883	1.9023	1.8987

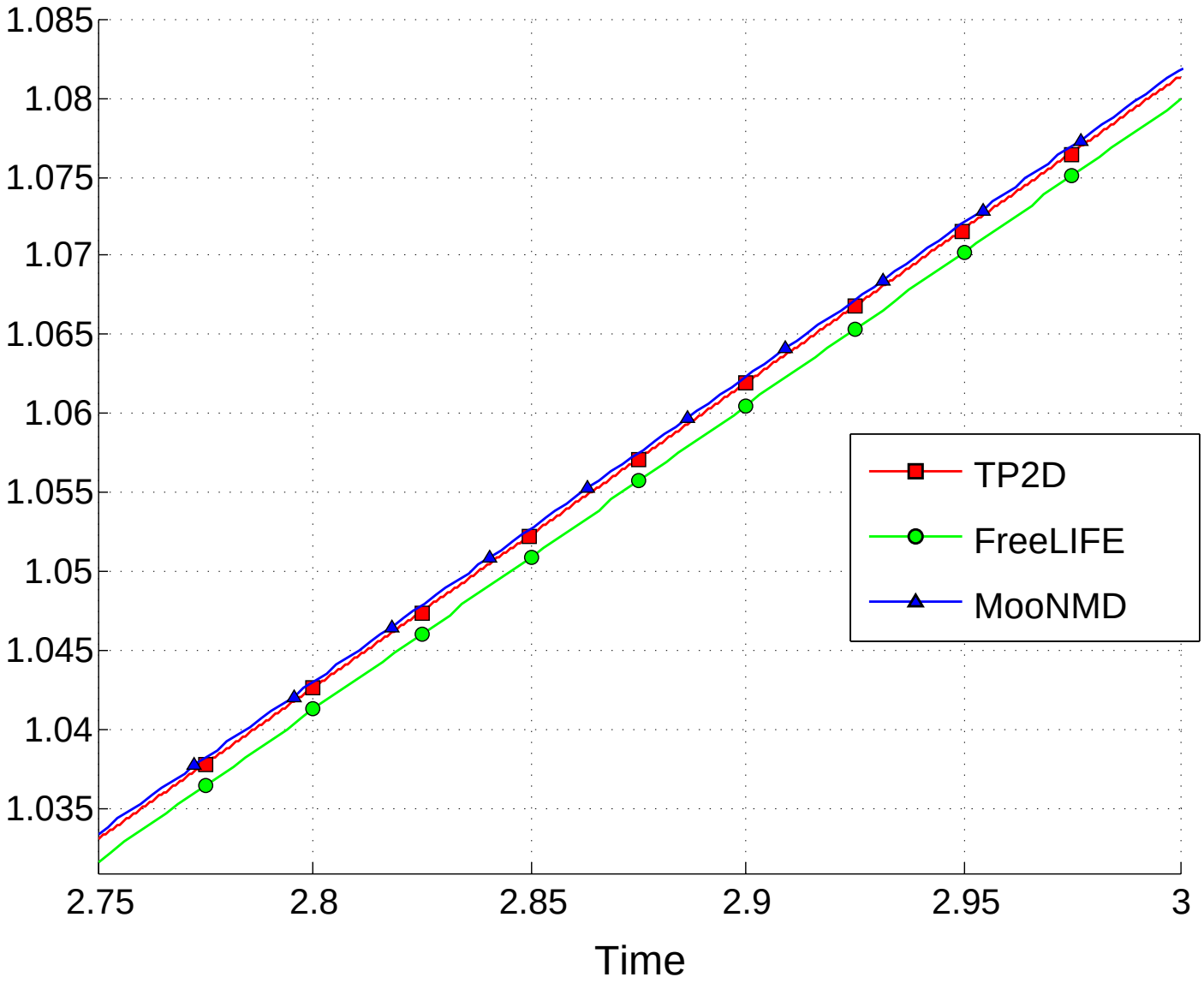
Reference target range

$$\not\subset_{min} = 0.9012 \pm 0.0002, \quad t|_{\not\subset=\not\subset_{min}} = 1.89 \pm 0.01$$

Benchmark quantities - center of mass



Benchmark quantities - center of mass



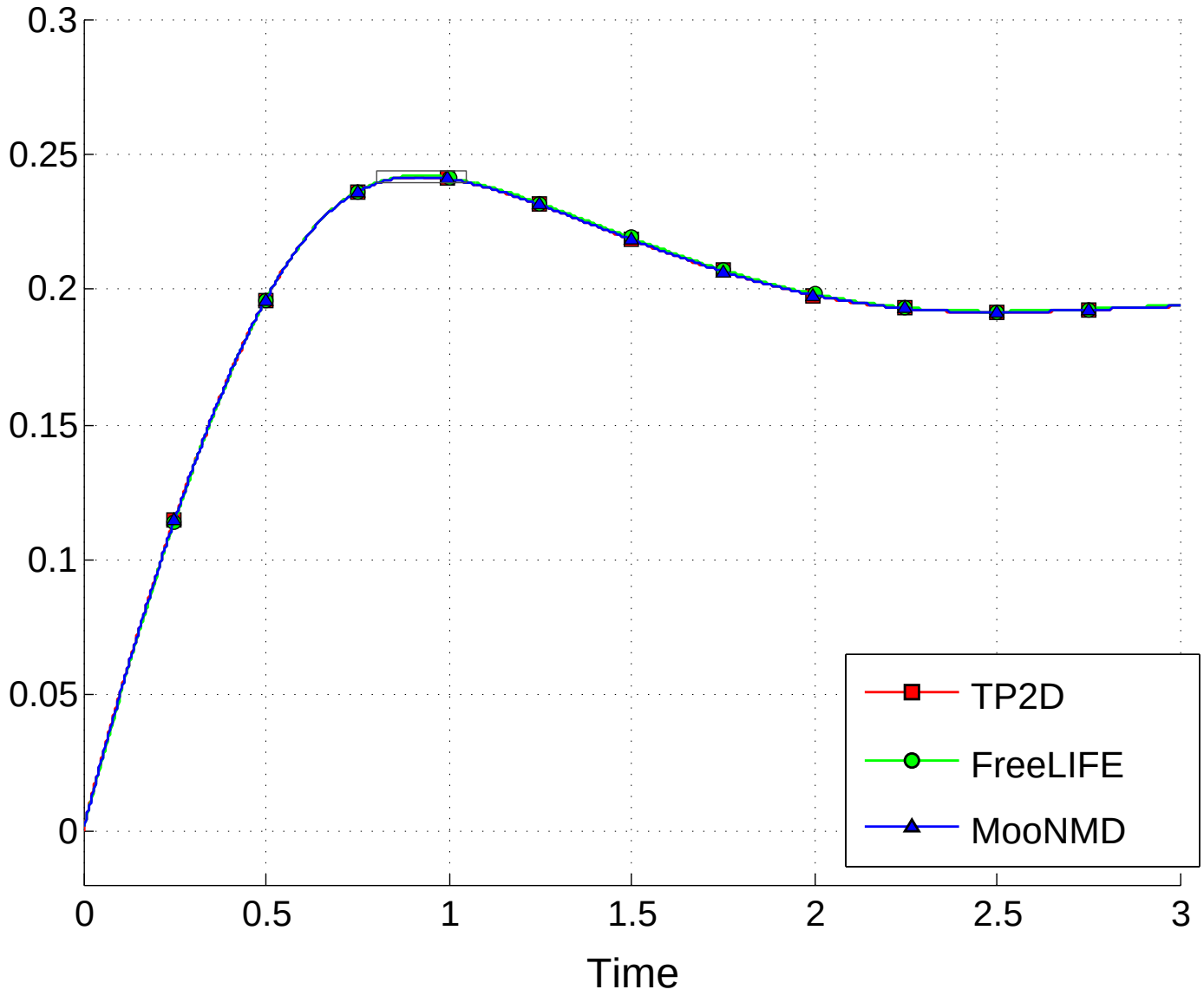
Benchmark quantities - center of mass

GridLevel	1	2	3	4
Center of mass, $y_c\|_{t=3}$				
TP2D	1.0818	1.0810	1.0812	1.0813
FreeLIFE		1.0715	1.0817	1.0799
MooNMD	1.0833	1.0823	1.0815	1.0817

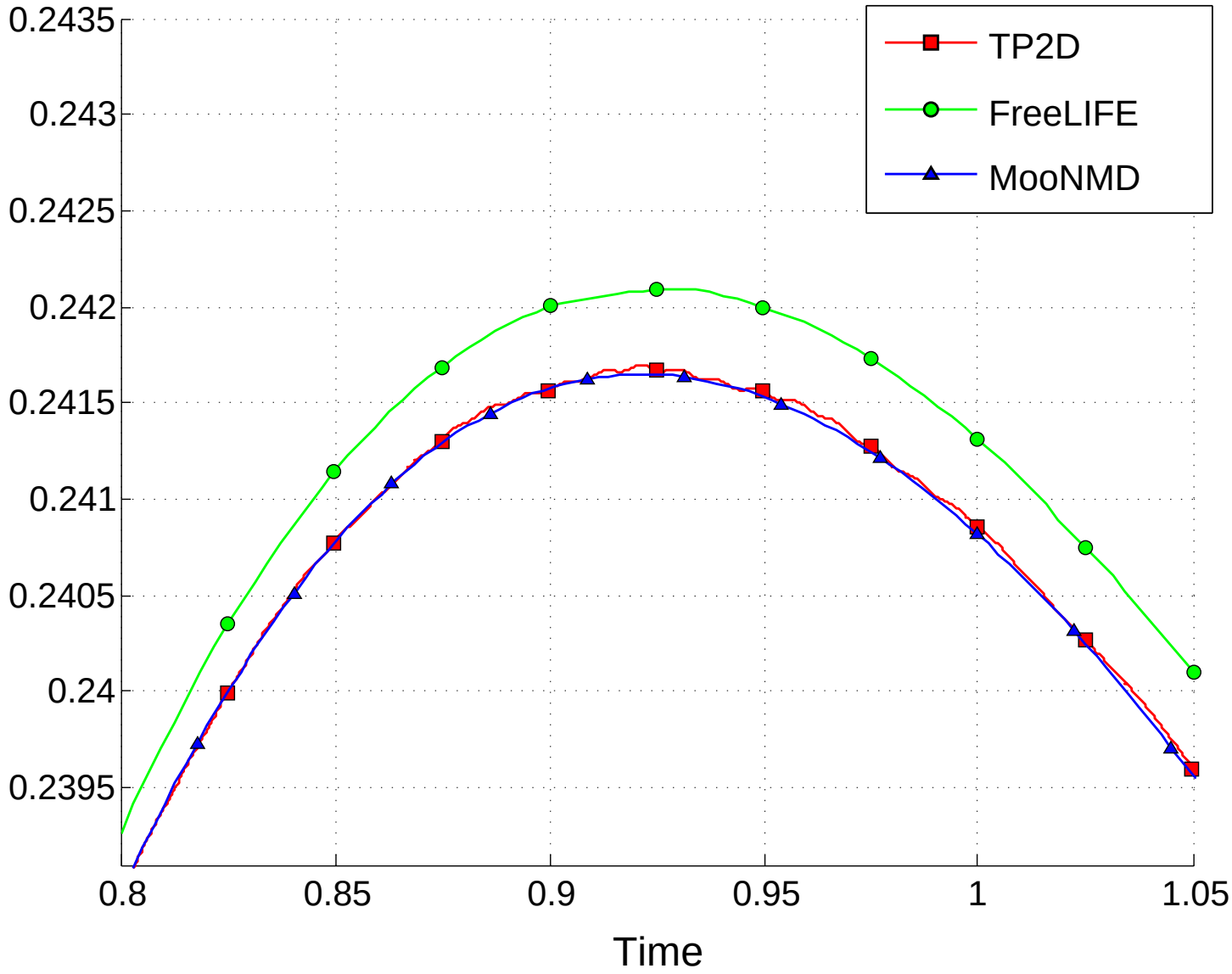
Reference target range

$$y_c|_{t=3} = 1.081 \pm 0.001$$

Benchmark quantities - rise velocity



Benchmark quantities - rise velocity



Benchmark quantities - rise velocity

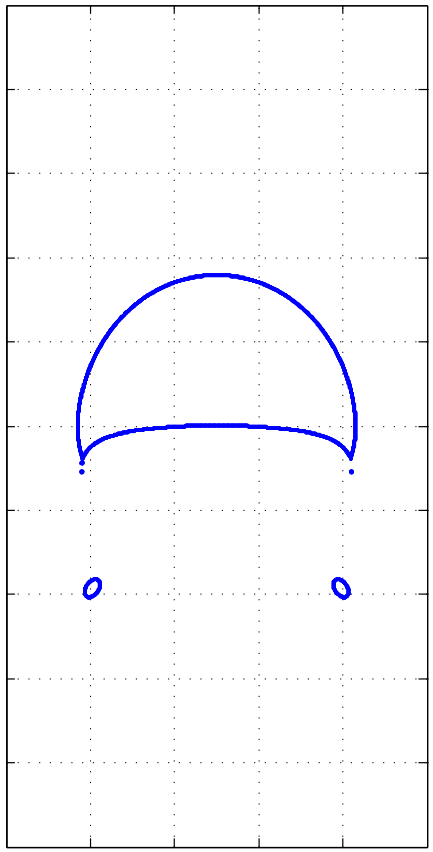
GridLevel	1	2	3	4
Maximum rise velocity, $V_{c,max}$				
TP2D	0.2418	0.2418	0.2419	0.2417
FreeLIFE		0.2427	0.2410	0.2421
MooNMD	0.2418	0.2417	0.2417	0.2417
Incidence time, $t\|_{V_c=V_{c,max}}$				
MooNMD	0.9236	0.9236	0.9249	0.9214
FreeLIFE		0.9000	0.9375	0.9313
TP2D	0.9141	0.9375	0.9281	0.9213

Reference target range

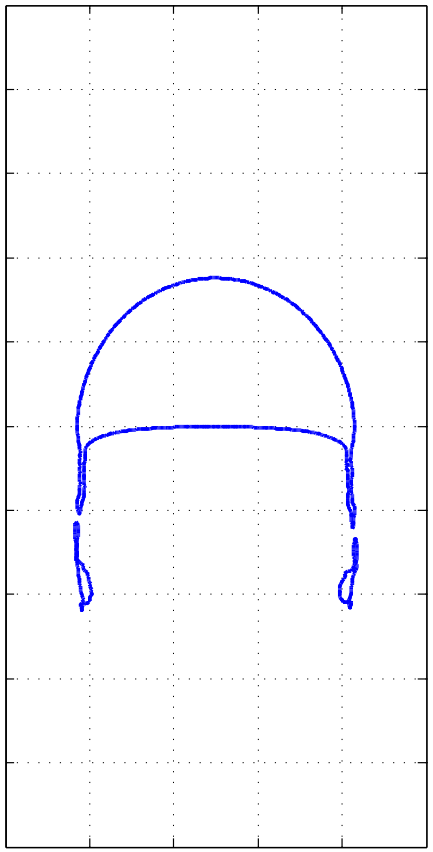
$$V_{c,max} = 0.2417, \quad t|_{V_c=V_{c,max}} = 0.9213\text{–}0.9214$$

Test Case 2

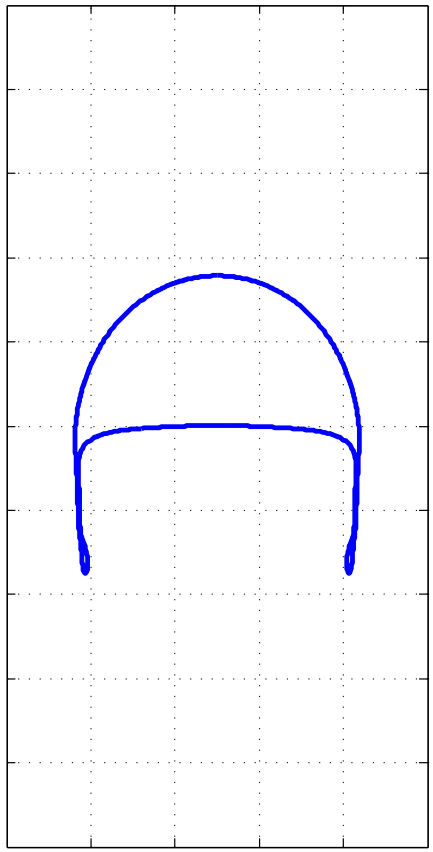
Visual comparison



TP2D

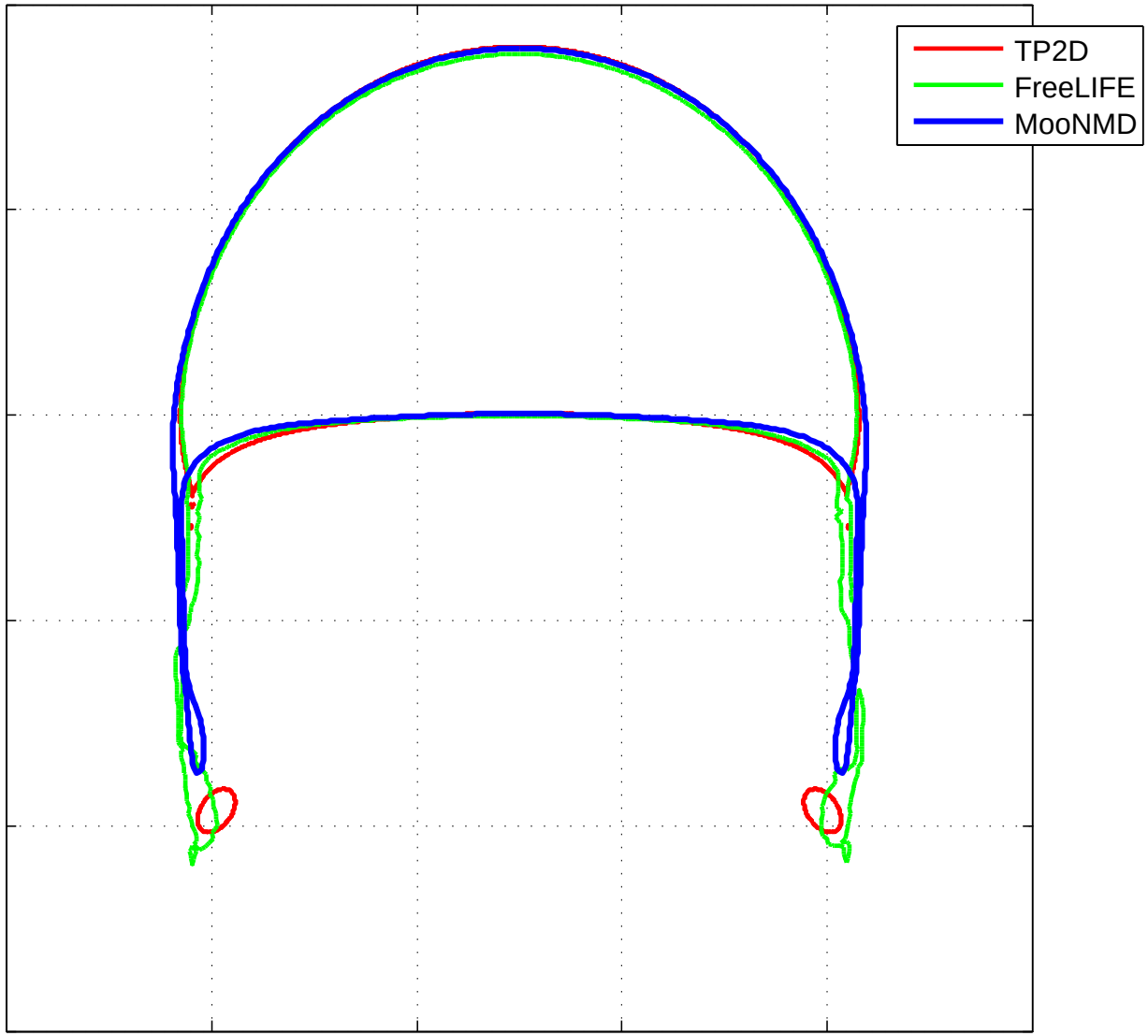


FreeLIFE

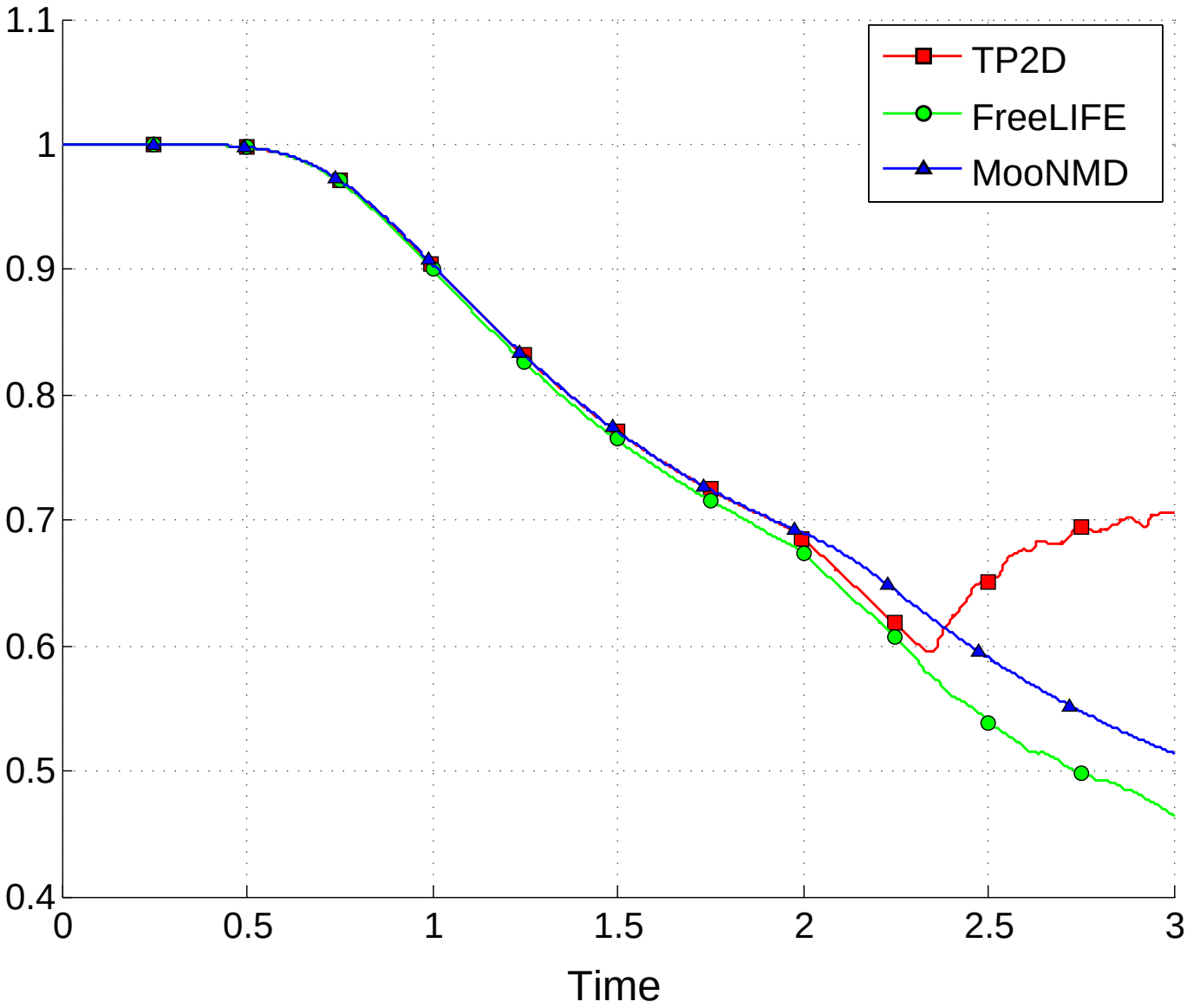


MooNMD

Visual comparison



Benchmark quantities - circularity



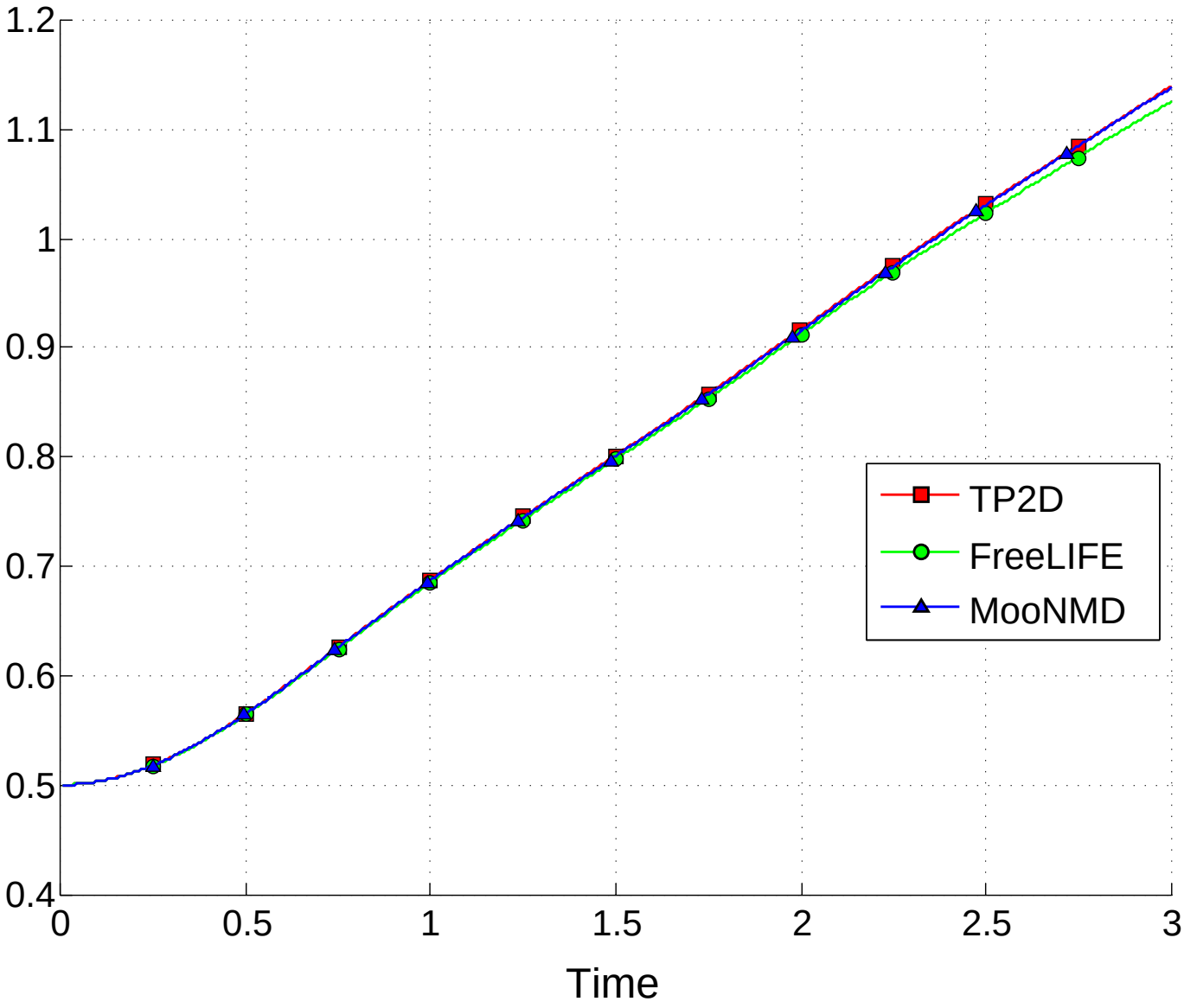
Benchmark quantities - circularity

GridLevel	1	2	3	4	5
Minimum circularity, $\not\subset_{min}$					
TP2D	0.5193	0.5717	0.5946	0.5943	0.5869
FreeLIFE			0.4868	0.5071	0.4647
MooNMD			-	0.5191	0.5144
Incidence time, $t\big\vert_{\not\subset=\not\subset_{min}}$					
TP2D	3.0000	2.4266	2.2988	2.3439	2.4004
FreeLIFE			2.7500	2.8438	3.0000
MooNMD			-	3.0000	3.0000

Reference target range

?

Benchmark quantities - center of mass



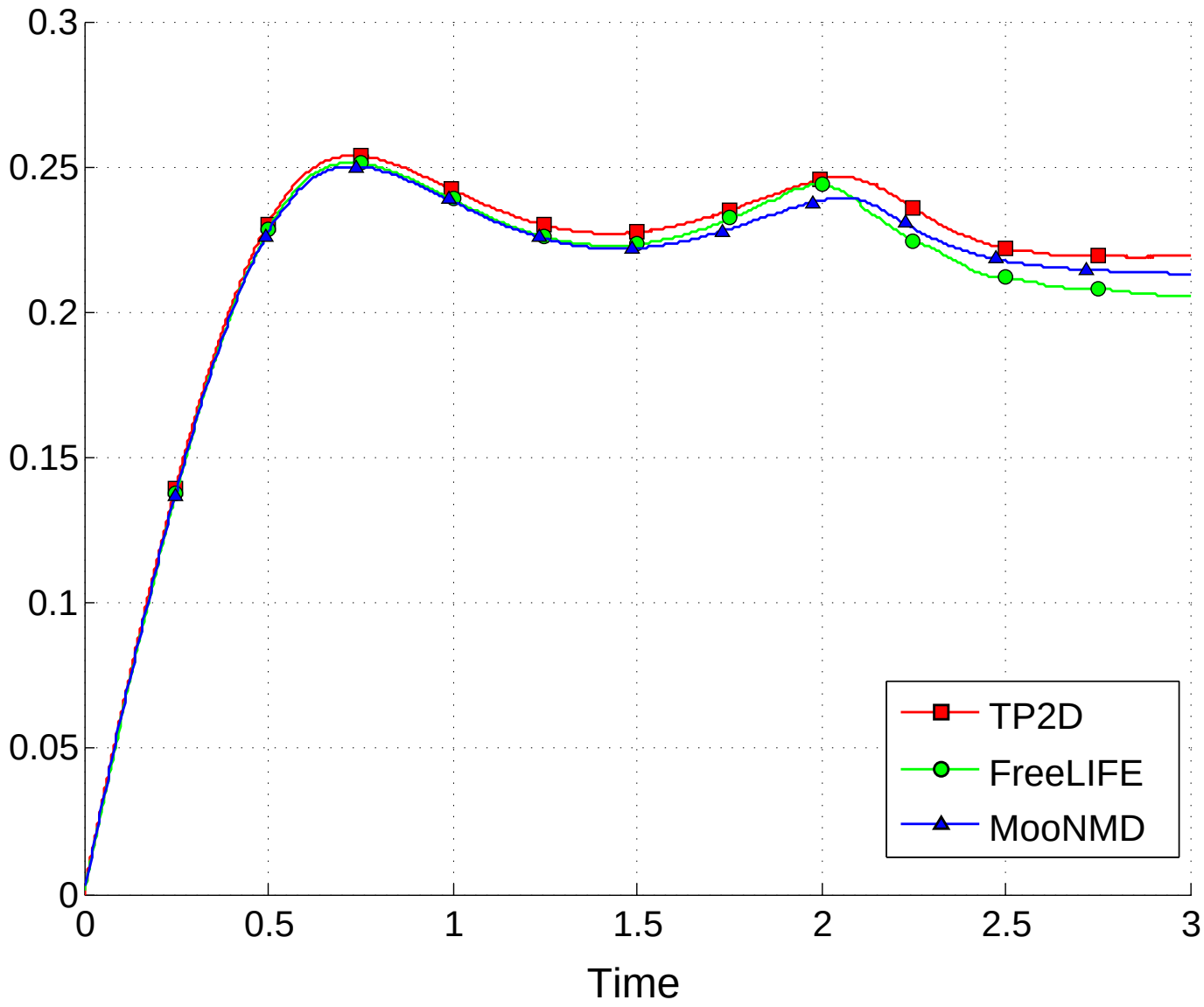
Benchmark quantities - center of mass

GridLevel	1	2	3	4	5
Center of mass, $y_c\|_{t=3}$					
TP2D	1.1303	1.1370	1.1377	1.1387	1.1380
FreeLIFE			1.0843	1.1099	1.1249
MooNMD			-	1.1380	1.1376

Reference target range

?

Benchmark quantities - rise velocity



Benchmark quantities - rise velocity

GridLevel	1	2	3	4	5
First rise velocity maximum, $V_{c,max\ 1}$					
TP2D	0.2790	0.2638	0.2570	0.2538	0.2524
FreeLIFE			0.2563	0.2518	0.2514
MooNMD			0.2503	0.2502	0.2502
First Incidence time, $t\|_{V_c=V_{c,max\ 1}}$					
TP2D	0.7641	0.7250	0.7430	0.7340	0.7332
FreeLIFE			0.7750	0.7188	0.7281
MooNMD			0.7317	0.7317	0.7317

Reference target range

$$V_{c,max\ 1} = 0.25 \pm 0.01, \quad t|_{V_c=V_{c,max\ 1}} = 0.73 \pm 0.02$$

Benchmark quantities - rise velocity

GridLevel	1	2	3	4	5
Second rise velocity maximum, $V_{c,max\ 2}$					
TP2D	0.2749	0.2597	0.2522	0.2467	0.2434
FreeLIFE			0.2397	0.2384	0.2440
MooNMD			0.2390	0.2393	0.2393
Second Incidence time, $t\|_{V_c=V_{c,max\ 2}}$					
TP2D	1.9375	1.9688	2.0234	2.0553	2.0705
FreeLIFE			1.9875	1.9062	1.9844
MooNMD			2.0650	2.0600	2.0600

Reference target range

?

Conclusions

Conclusions

- Proposed two benchmarks
- Established target reference values for the first benchmark
- Hinted at difficulties during break up in the second benchmark

Participate?

If you would like to participate in this or other numerical benchmarking projects please visit our benchmarking forum at:

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or send an email to:

ture@featflow.de