# Optimization Techniques for Incompressible Flow Problems

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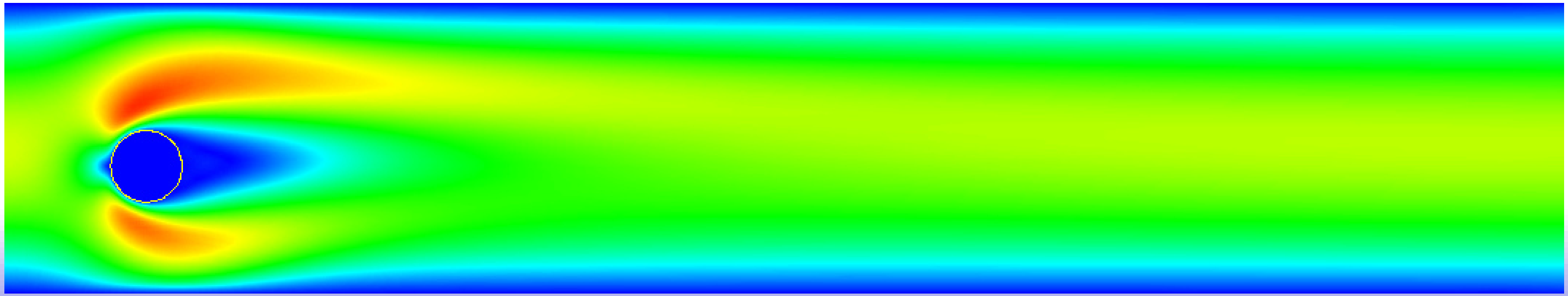


# Outline

- Introductory Summary:
  - What is the Topic?
- Current State:
  - Optimization in the Stationary Case
  - Nonstationary Case
- Outlook

# Optimization with PDE's

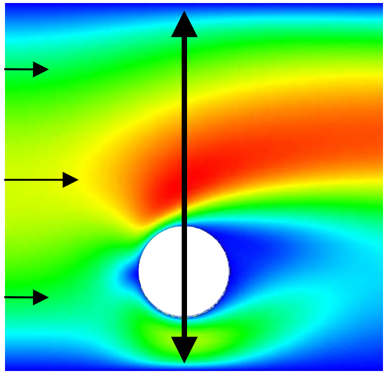
Formulate a minimization problem with side constrains:

$$\min_q J(u(q), p(q), q) \qquad = \text{drag, lift, ...}$$

$$\text{such that} \qquad \text{Navier Stokes}(u, p) \text{ is fulfilled}$$

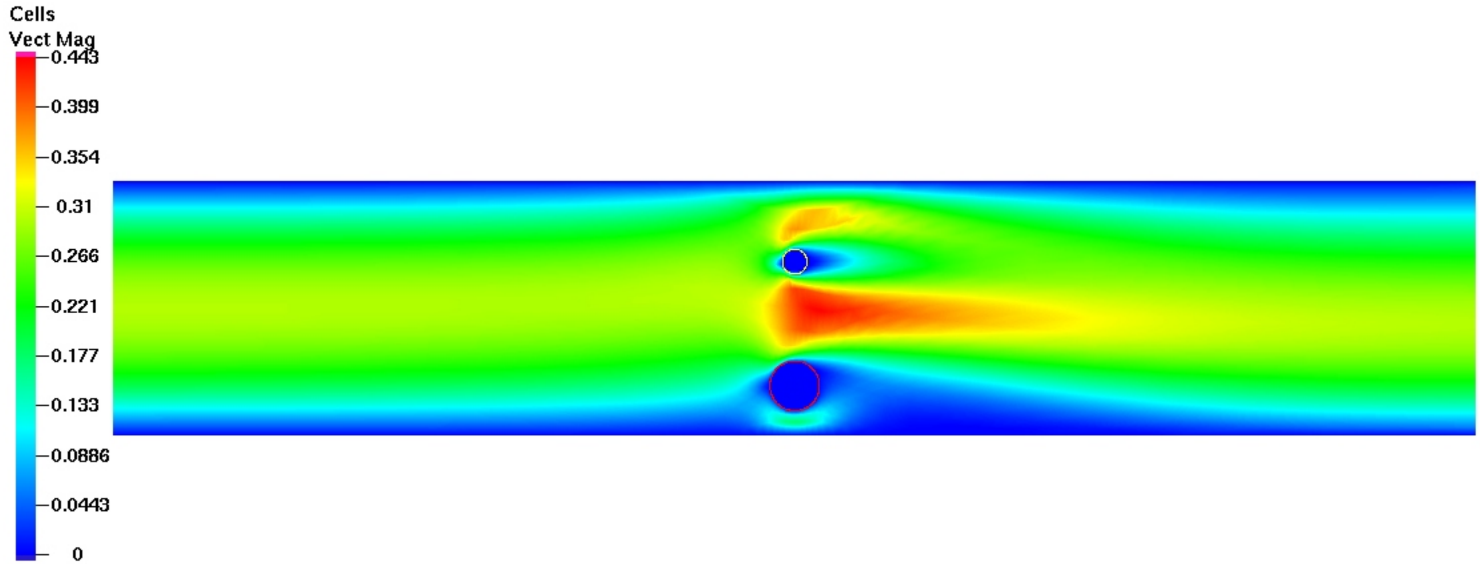
$\Rightarrow$ Natural extension to simulation:

Use the simulation
to automatically optimize something…



# Current state: Stationary case

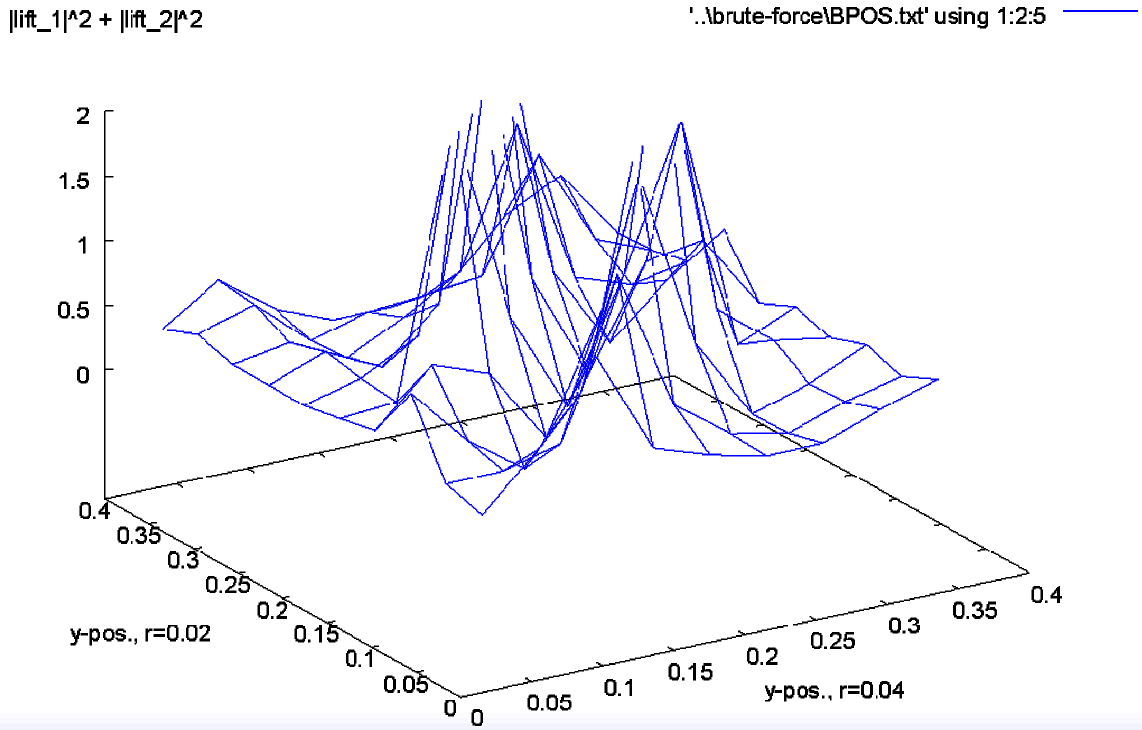
**2D-Optimization:** Find the heights where two balls hover




$$J(y_1, y_2) = |\mathrm{lift}_1|^2 + |\mathrm{lift}_2|^2$$

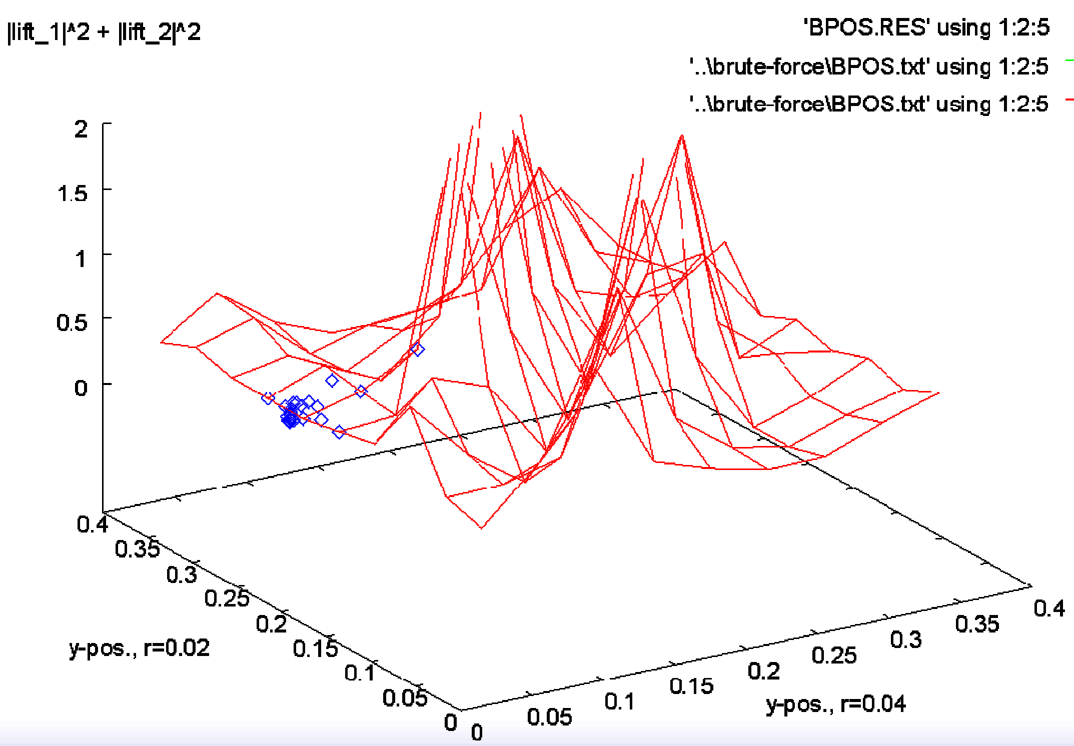
# Current state: Stationary case

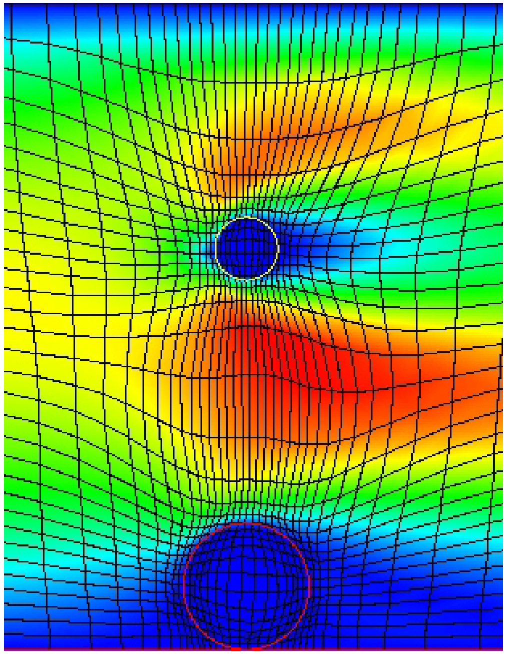
**Rough overview:** 90 evaluations



# Current state: Stationary case

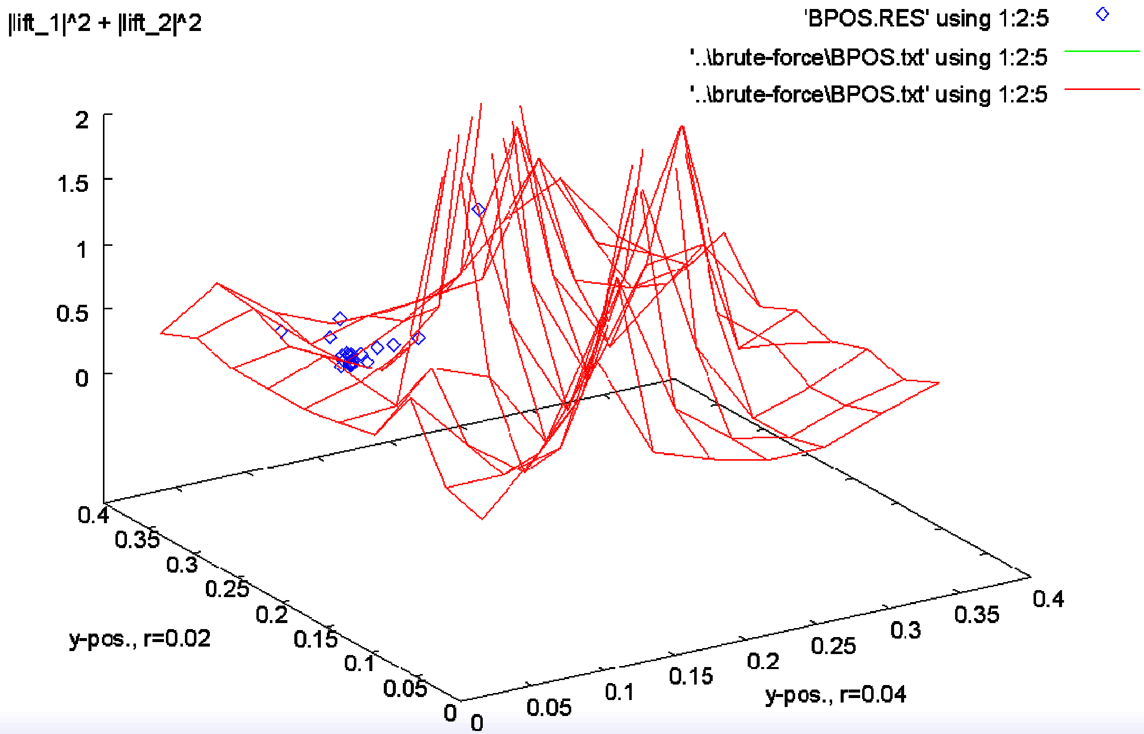
**Compass search:** 32 evaluations until J(y1,y2) < 0.16

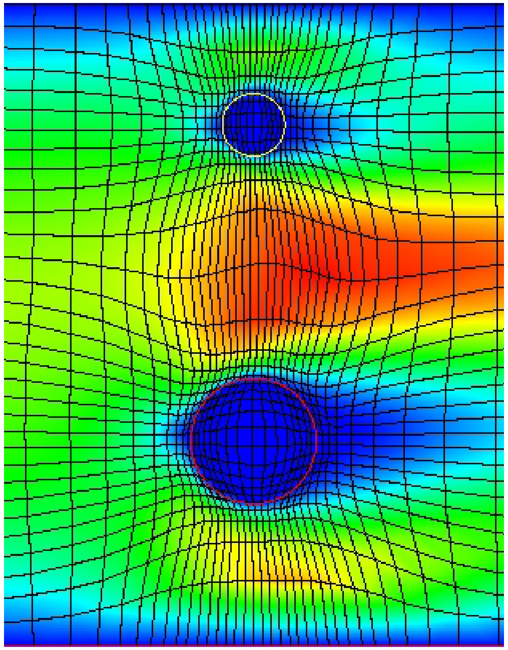


# Current state: Stationary case

**Nelder Mead:** 8 evaluations until J(y1,y2) < 0.07



# Current state: Stationary case

**Next steps:**

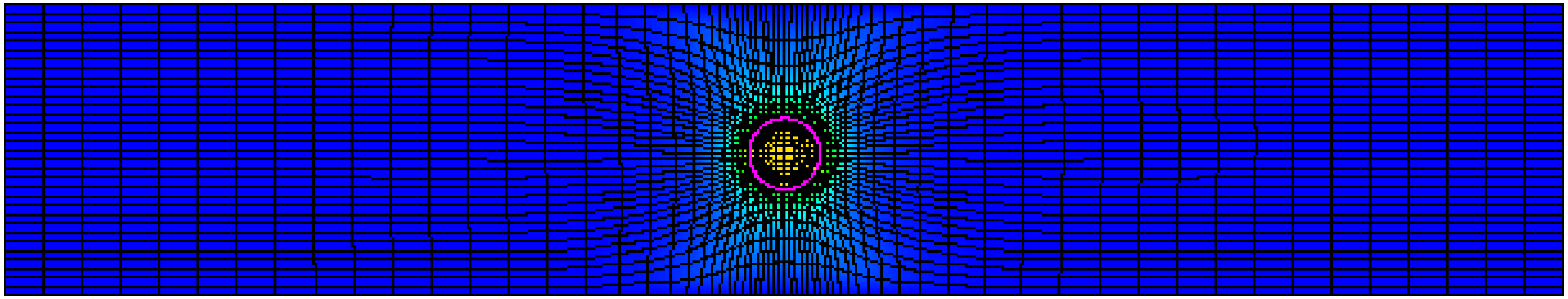
- More enhanced algorithms & tests
  - Line search with reconstructed gradients, Trust region,…
  - Gravity & more real life situations

- Tests with higher dimensions
  - 3rd dimension: inflow velocity

- Advanced grid deformation depending on the error

# Current state: Nonstationary case

Grid deformation in the nonstationary case



# Current state: Nonstationary case

Modification of Navier-Stokes equation was necessary

$\rightarrow$ to include movement of the mesh

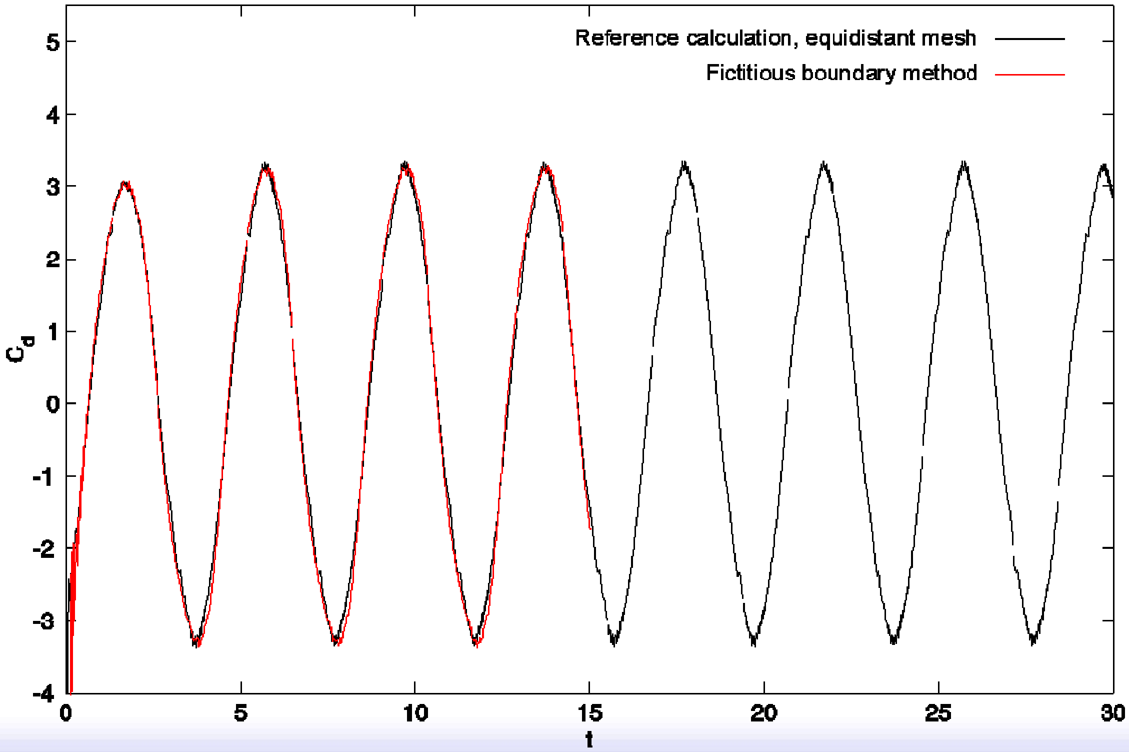
$$\begin{aligned} u_t + \nu\Delta u + u\nabla u + \nabla p &= f \\ \nabla \cdot u &= 0 \end{aligned}$$

$$\Rightarrow \quad \begin{aligned} u_t + \nu\Delta u + u\nabla(u - v) + \nabla p &= f \\ \nabla \cdot u &= 0 \end{aligned}$$

$$v = \text{grid velocity}$$

# Current state: Nonstationary case

Comparison of the drag to reference:



# Current state: Nonstationary case

**Next steps:**

- Muliple objects with different size
- Movement of elements depending on the fluid
- Collision models
- Apply optimization algorithms to different situations