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Level Set Extrapolation of Immersed Boundary Data in Unfitted Finite Element Methods for Deformable Interfaces

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Conservation laws in evolving domains

$$\begin{aligned} \frac{\partial u}{\partial t} + \nabla \cdot [\mathbf{f}(u) - \kappa \nabla u] &= 0 && \text{in } \Omega_+(t), \\ [\mathbf{f}(u) - \kappa \nabla u] \cdot \mathbf{n}_+ &= g_\Gamma(u, u_\Gamma) && \text{on } \Gamma(t), \\ u &= u_0 && \text{in } \Omega_+(0) \end{aligned}$$

Dirichlet-type outer condition

$$s_D(w, u) = \int_{\Omega \setminus \Omega_+(t)} \gamma_\Omega w (u - u_-) \mathrm{d}\mathbf{x}$$

Fictitious domain formulation

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_\Omega H(\phi)wu\mathrm{d}\mathbf{x} - \int_\Omega H(\phi)\nabla w\cdot[\mathbf{f}(u_\Gamma)-\kappa\nabla u]\mathrm{d}\mathbf{x} + s(w,u)$$
$$+\int_{\Gamma(t)} w[\mathbf{f}(u_\Gamma)\cdot\mathbf{n}_+ - v_n u_\Gamma - \kappa\partial_n u]\mathrm{d}s = 0, \quad \forall w\in V(\Omega)$$

$u_\Omega = H(\phi)u + (1-H(\phi))u_-$

Dirichlet-type ghost penalties

$$s_D(w,u) = \int_\Omega \gamma_\Omega w(u-u_\Omega)\mathrm{d}\mathbf{x}$$

Extrapolation using level sets

To be defined: continuous extensions $U, \partial_n U, V$ for calculating

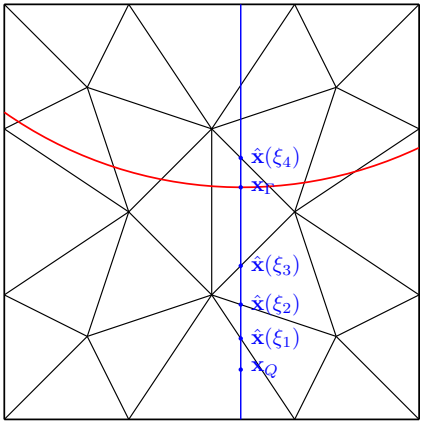
$$G(\phi_h, u_h, u_\Gamma) = F - \kappa \partial_n U - VU, \qquad \mathbf{v}_h = -V\mathbf{q}_h, \qquad u_{\Omega,h}(u_h, U)$$

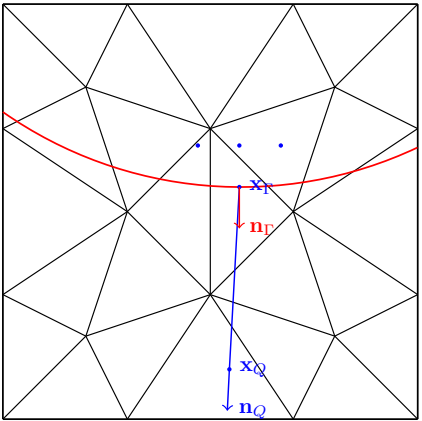
Main steps:

1. closest-point search
2. gradient reconstruction
3. extrapolation

Requirements: simplicity, efficiency, accuracy

Extrapolation using level sets





Summary

- approximation of surface integrals by volume integrals
- fast closest-point search algorithm
- new way to define and calculate compact-support extensions
- narrow-band integration of terms involving extended fluxes, ghost penalties and velocity fields

Outlook

- theoretical studies of proposed approach
- new approximate delta functions with compact support
- application to interface problems and PDE systems

Thank you for your attention!

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