Efficient Numerical Methods and Simulation Techniques for Granular flow

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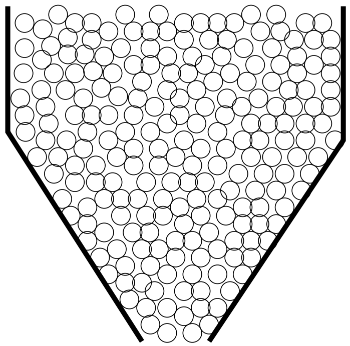
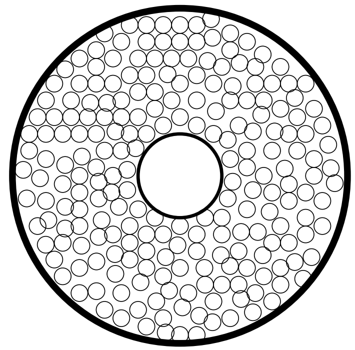
Motivation of this work

- **The flow of granular materials**
  - Example of application
    Pharmaceutical Industry, Food Processing, Soil Mechanics ...



Granular material storage

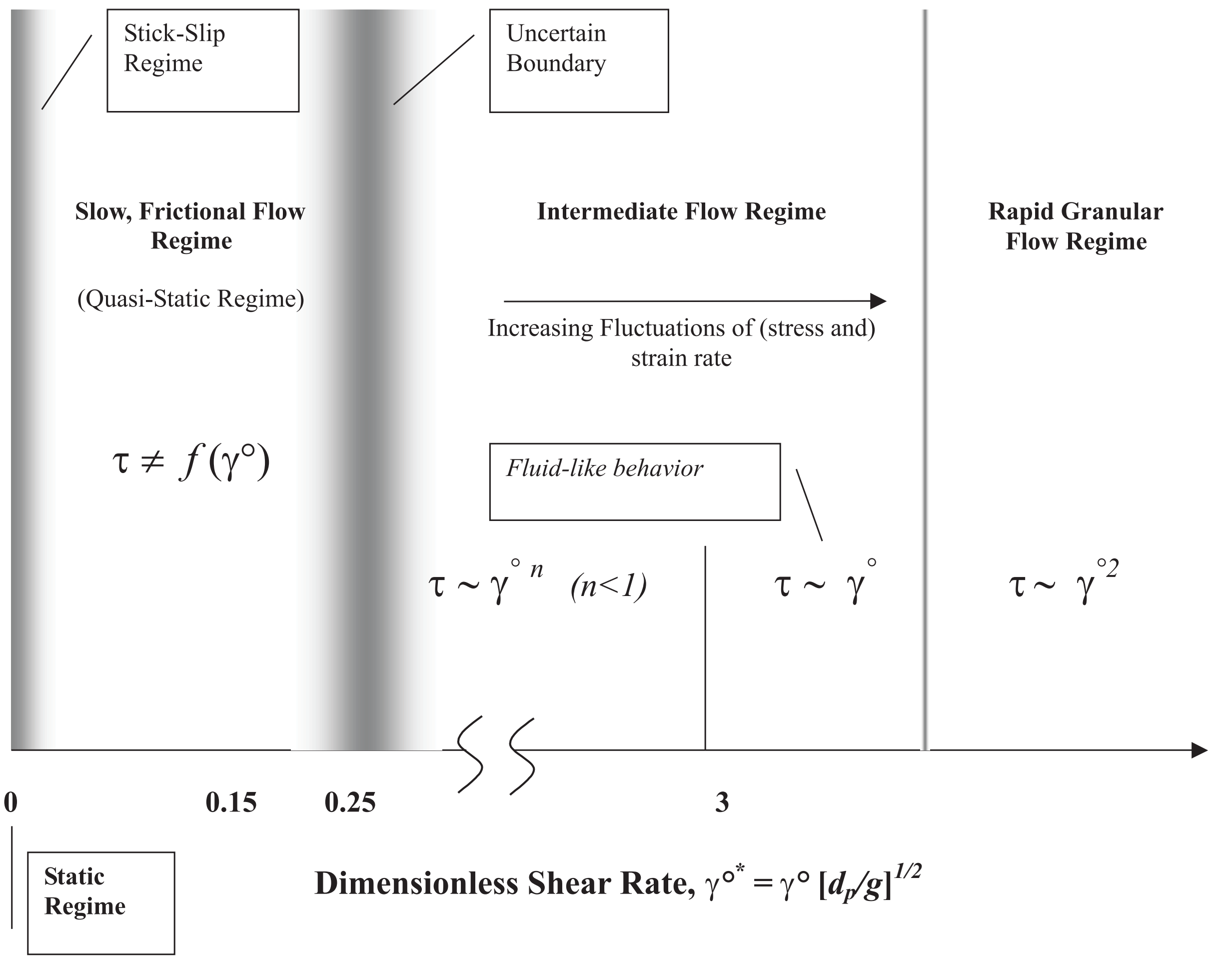


Couette flow

- **What about the viscosity !!?**
  - From engineering point of view this material does not exibit viscosity!!
  - From mathematical and numerical point of view we are able to set this type of problem into the same range of flow with generalized viscosity !?

Regimes of powder flow

Analogous to fluid flow, the powder regimes could be represented as a function of dimensionless shear rate $\gamma^{o*} = \gamma^{o}[d_p/g]^{1/2}$ which plays the similar role as the Reynolds number Re for fluids (Tardos et al).



Regimes of powder flow

- **Quasi-static regime**
  - Any movement between two static states can be neglected
  - The static equilibrium equation can be applied
  - No flow field can be predicted!

  This circumscribes the range of applications of this approach

- **Slow and frictional regime** (Schaeffer (1987))
  - The frictional forces between particles are predominant
  - Inertial effect is added to the static equations
  - Consideration of the continuity, yield condition and flow rule
  - All flow fields can be computed

- **Intermediate and rapid granular regimes**
  - Inter-particle friction energy
  - Collisional energy is important, too

Our contribution has the goal of supporting the slow and frictional regime

General equations for slow powder (Tardos)

- **General equation of motion for a powder**

$$\rho \frac{D\boldsymbol{u}}{Dt} = -\nabla p + \nabla \cdot \left[\frac{q(p,\rho)}{\left\| \mathbf{D} - \frac{1}{n} \nabla \cdot \boldsymbol{u} I \right\|} \left(\mathbf{D} - \frac{1}{n} \nabla \cdot \boldsymbol{u} I \right) \right] + \rho g, \text{ with}$$

- **Continuity equation**

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0, \text{ and}$$

- **Normality condition**

$$\nabla \cdot \boldsymbol{u} = \frac{\partial q(p,\rho)}{\partial p} \left\| \mathbf{D} - \frac{1}{n} \nabla \cdot \boldsymbol{u} I \right\|$$

- **The yield condition $q(p, \rho)$ is given by:**

Powder properties	Non-cohesive	Cohesive
Incompressible	$p \sin \phi$ (Schaeffer model)	$p \sin \phi + c \cos \phi$
Compressible	$p \sin \phi \left[2 - \frac{p}{\rho^{\frac{1}{\beta}}} \right]$	$p \sin \phi \rho^{\frac{1}{\beta}} - C \frac{(p - \rho^{\frac{1}{\beta}})^2}{\rho^{\frac{1}{\beta}}}$

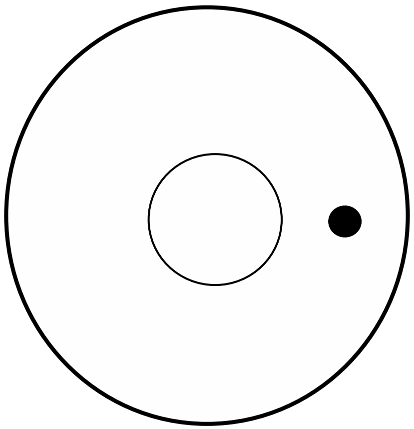
where $0.001 < \beta < 0.01$

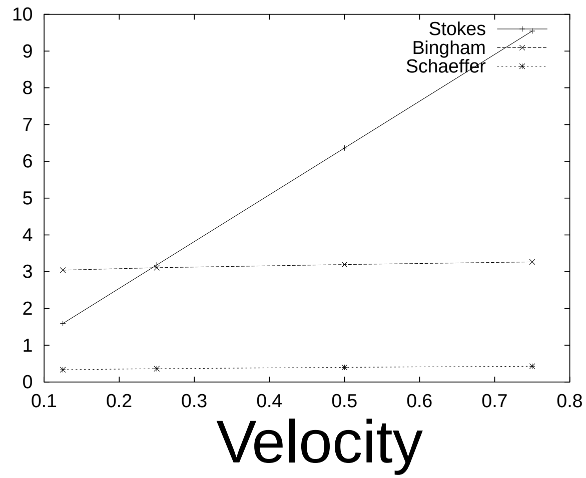
Aspect of the numerical simulations

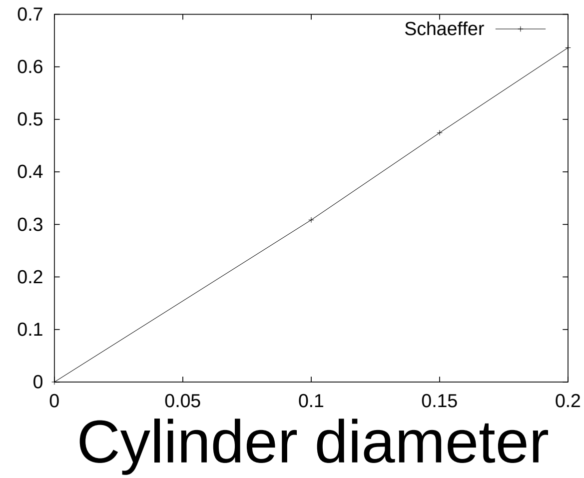
- **Spatial discretization**
  - nonconforming finite elements with edge oriented d.o.f's for velocity
  - piecewise constant finite elements for pressure
- **Nonlinear iteration:** Newton method
- **Linear solver:** multigrid for velocity, pressure and density
- **Coupled solver :** stationary and nonstationary problems
  - Defect correction method as outer iteration
  - The linear coupled subproblems are solved in one iteration step
- **Projection solver :** nonstationary problem
  - Decoupling step for the velocity $\boldsymbol{u}$, the pressure p, and the density ρ
  - perform only one iteration for pressure and the density each time step

Drag force in a granular medium

- The dependence of **drag force** with "grain **velocity**" in a couette flow around a cylinder for different material and for different cylinder diameter





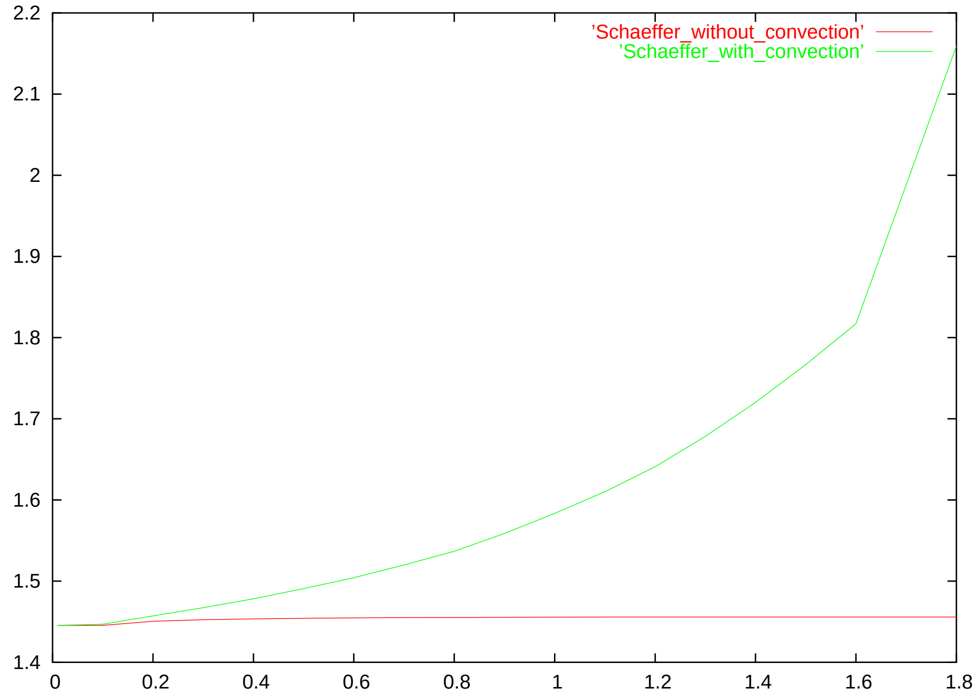


- The **drag force** for Schaeffer and Bingham flow acting on cylinder is **independent** of the grain **velocity**, contrary to the stokes flow

”When mechanical ploughs replaced draught animals, it was observed that ploughing at greater speeds does not require greater forces!”

Drag force and inertia effect

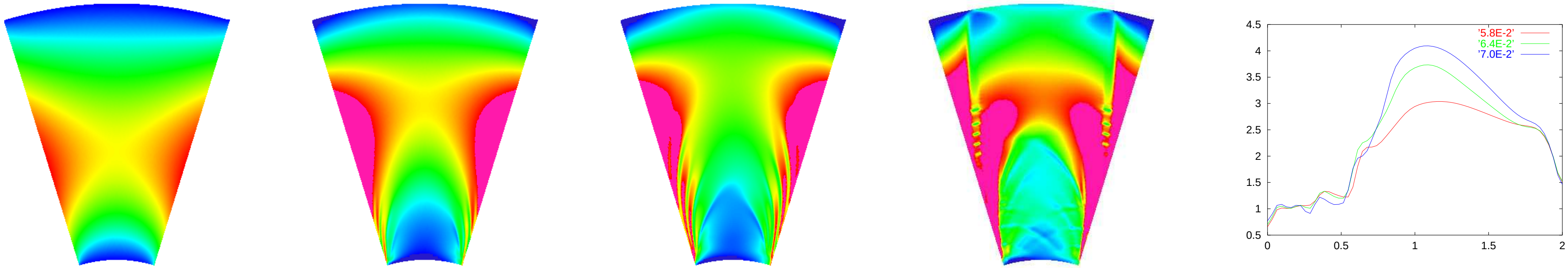
The dependence of the force with "velocity grain" for Schaeffer model with and without convection



- Inertia effect get relevant from certain speed
- The limit speed for which the assumption of slow flow remain valid !?

Incompressible Powder: Schaeffer model

- **Hoper configuration**
  - Developement of the pressure

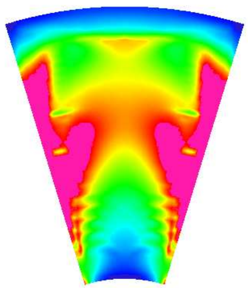


  - The inflow boundary condition not well preserved
  - The oscillation start from the outflow

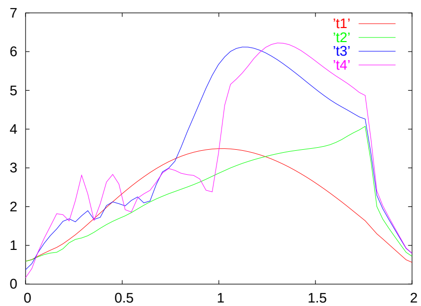
Is it possible that the apperance of the oscillations are only due to the artificial inflow and outflow boundary condition supplied to the hopper !??

Hopper configuration: boundary condition

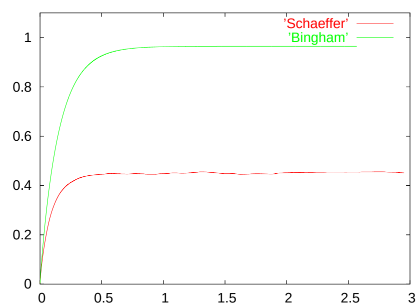
- **First remedy :** To preserve the boundary condition during the simulation we only apply newtonian law on the boundary



Pressure



Pressure wave

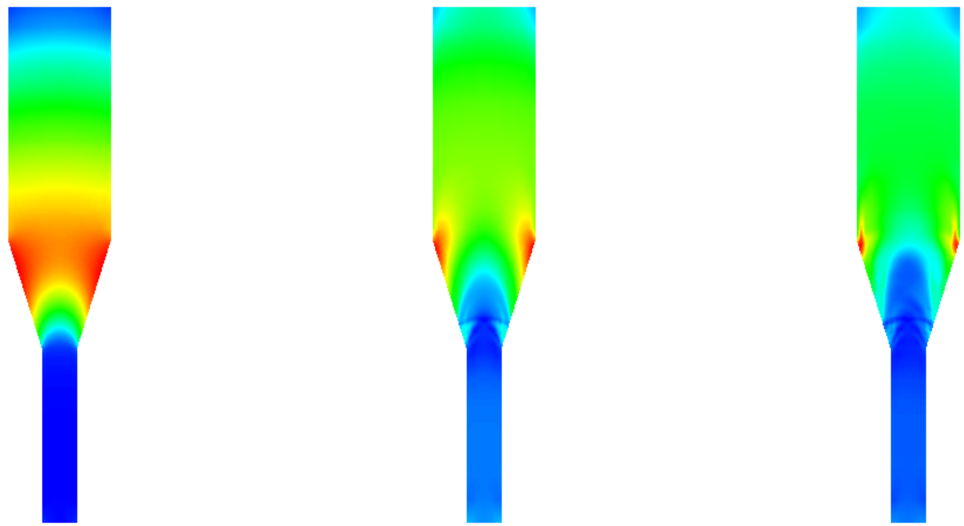


Flow rate

The oscillations appear again, but are not cleary seeing in terms of the flow rate!??

Silo configuration: boundary condition

- **Second remedy :** Silo configuration
  A long bin on the bottom and the top of the hoper in order to diminish the influence of the boundary condition onto the flow behaviour
  - Developement of the pressure and flow rate



- **Third remedy :** Integrate free surface boundary condition

Conclusion and Outlook

We conclude that using finite element methods together with the continuum mechanic approach of granular material is a useful tool

- The complete picture of the flow is involved, i.e. the velocity, the pressure as well as the stress
- The silo pressure is of complex nature, but we was able to reproduce the circumstance for which the pressure wave appears **(qualitatively)**
- The independence of the drag force with the velocity grain **(qualitatively)**
- Many questions are still to be answered
  - What is the appropriate boundary condition to supply for silo and and hopper configurations !?
  - How to assure the closure of the equations for couette device configuration, since the Navier Stokes equation posses no unique solution for the pressure with Dirichlet conditions !?
- What is next: make full compressible