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Benchmark computations for high density and viscosity ratio rising bubble problems using coupled Lattice Boltzmann-Level Set Methods

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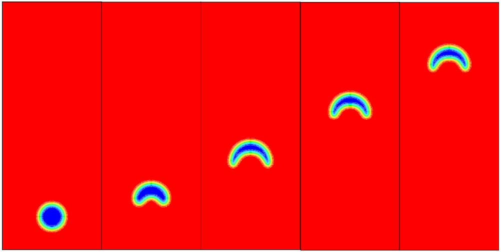
- Shan-Chen LB model (*Shan-Chen 1993*)
- Chemical potential model (*He et al. 1999 & Lee and Lin 2005*)
- Coupled LBM-Levelset schemes
  - One-Fluid Approach
  - Pressure Evolution Approach



Air bubble rising in water, $Eo = 100$, $Re = 16$

One-Fluid Coupled Approach

- Turn the N.E. into a desired form for LBM (Mehravaran, 2011)

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} + \frac{\nabla P}{\rho(\phi)} - \frac{\nabla \cdot \left(\mu(\phi)(\nabla \mathbf{u} + \nabla \mathbf{u}^T)\right)}{\rho(\phi)} = -\frac{\sigma \kappa(\phi) \mathbf{n}(\phi) \delta_\varepsilon(\phi)}{\rho(\phi)}$$

- Assume virtual variables $\bar{\rho} = 1$ and $\bar{\mu} = \mu/\rho$. Move $\frac{\nabla P}{\rho(\phi)}$ to the rhs

$$\bar{\rho} \partial_t \mathbf{u} + \bar{\rho} \mathbf{u} \cdot \nabla \mathbf{u} + \nabla P - \bar{\mu}(\phi) \nabla \cdot (\nabla \mathbf{u} + \nabla \mathbf{u}^T) = \bar{F}$$

$$\bar{F} = -\frac{\sigma \kappa(\phi) \mathbf{n}(\phi) \delta_\varepsilon(\phi)}{\rho(\phi)} + \frac{\nabla \mu(\phi) \cdot (\nabla \mathbf{u} + \nabla \mathbf{u}^T)}{\rho(\phi)} + \nabla P \left(1 - \frac{1}{\rho(\phi)}\right)$$

- Solve LBE for f_k with $\bar{F}$

$$\frac{\partial f_k}{\partial t} + e_k . \nabla f_k = \frac{1}{\tau}(f_k - f_k^{eq}) + \frac{(e_k - \mathbf{u}) \cdot \bar{F}}{\bar{\rho} c_s^2} f_k^{eq}$$

Pressure Evolution Coupled Appr.

- Intermolecular force *(He et al. 1999)*

$$F = \nabla\rho(\phi)c_s^2 - (\nabla p - F_s) \qquad \textit{where} \qquad F_s = \sigma\kappa(\phi)\mathbf{n}(\phi)\delta_\varepsilon(\phi)$$

- Define a pressure evolution distribution g_k

$$g_k = f_k c_s^2 + (p - \rho(\phi)c_s^2)w_k$$

- Rewrite LBE for evolution of pressure

$$\frac{\partial g_k}{\partial t} + e_k.\nabla g_k = \frac{1}{\tau}(g_k - g_k^{eq}) + (e_k - \mathbf{u})\cdot(\nabla\rho(\phi)c_s^2(\Gamma_k - w_k) - F_s)$$

$$\textit{where} \qquad \Gamma_k = f_k^{eq}/\rho(\phi)$$

Coupled Approaches: Summary

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▶ One-Fluid (Approach 1)

- One-fluid LBE for f_k
- Crank-Nicolson in time and mixed directional differencing in space
- Obtain velocity and pressure as usual

$$\bar{\rho}\mathbf{u} = \sum_k e_{k,i} f_k \quad , \quad P = \bar{\rho} c_s^2 = \sum_k f_k c_s^2$$

▶ Pressure Evolution (Approach 2)

- Write pressure evolution LBE for g_k
- Crank-Nicolson in time and mixed directional differencing in space
- Obtain velocity and pressure as momemts of g_k

$$\rho(\phi)\mathbf{u} = \frac{1}{c_s^2} \sum_k e_{k,i} \bar{g}_k - \frac{\delta t}{2} F_s \quad , \quad p = \sum_k \bar{g}_k + \frac{\delta t}{2} \mathbf{u} \cdot \nabla \rho c_s^2$$

▶ Solving the Level Set equation

$$\partial_t \phi + \mathbf{u} \cdot \nabla \phi = 0 \qquad \text{where} \qquad \phi(\mathbf{x}) = 0 \quad at \quad X = \Gamma$$

- 2nd order Runge-Kutta time integration
- 5th order WENO finite difference for the convective term
- Weighted reinitialization to the approximated signed distance function every $\triangle T$

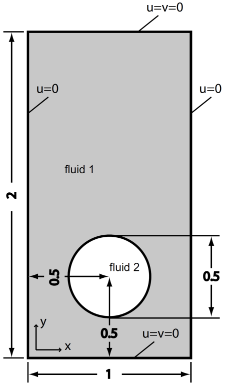
$$\phi_{new} = \alpha \phi_{dist} + (1 - \alpha)\phi_{old}$$

- Regularized *Heaviside function $H(\phi)$*, for density and viscosity profiles

$$\begin{cases} \rho(\phi) = \rho_l H(\phi) + \rho_g (1 - H(\phi)) \\ \mu(\phi) = \mu_l H(\phi) + \mu_g (1 - H(\phi)) \end{cases}$$

Rising bubble; Benchmarking Test Cases

"Proposal for quantitative benchmark computations of bubble dynamics" (2007)
Hysing, S.; Turek, S.; Kuzmin, D.; Parolini, N.; Burman, E.; Ganesan, S.; Tobiska, L.
see www.featflow.de/benchmarks/

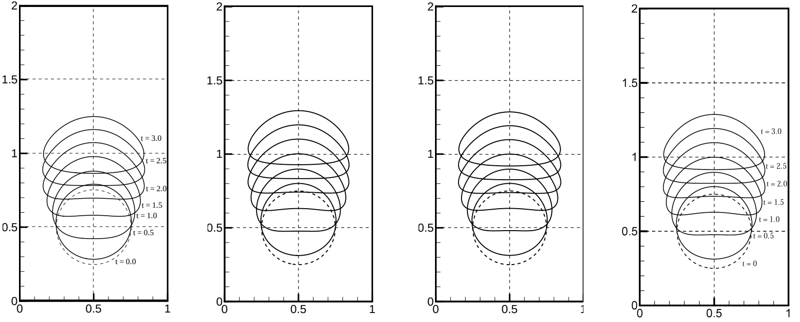


	ρ_l/ρ_g	μ_l/μ_g	Eo	Re
Test case 1	10	10	10	35
Test case 2	1000	100	125	35

$$Eo = \frac{4\rho_l g r_0^2}{\sigma} \quad , \quad Re = \frac{\rho_l \sqrt{g}(2r_0)^{3/2}}{\mu_l}$$

- **Rising bubble; Test Case 1**

Benchmarking time evolution, $0 < T < 3$



Shan-Chen | LBM-LS App. 1 | LBM-LS App. 2 | **FeatFLOW**

Numerical Results

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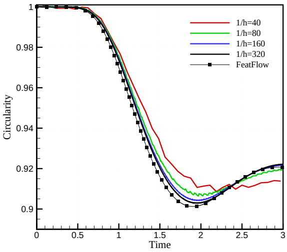
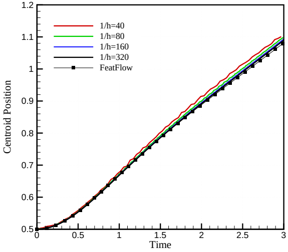
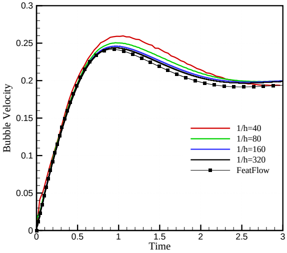
▶ **Rising bubble; Test Case 1** ⇒ LBM-LS Approach 1, grid effect, $0 < T < 3$

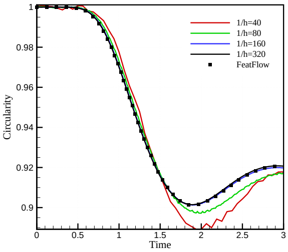


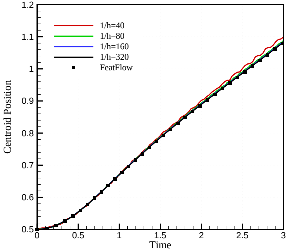


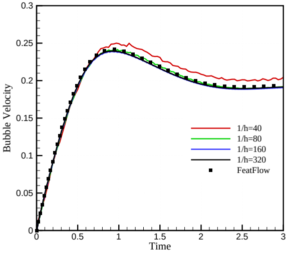


Numerical Results

▶ **Rising bubble; Test Case 1** $\Rightarrow$ LBM-LS Approach 2, grid effect, $0 < T < 3$



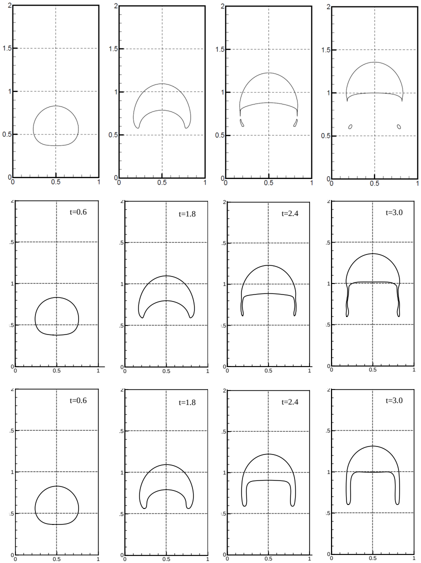




Numerical Result

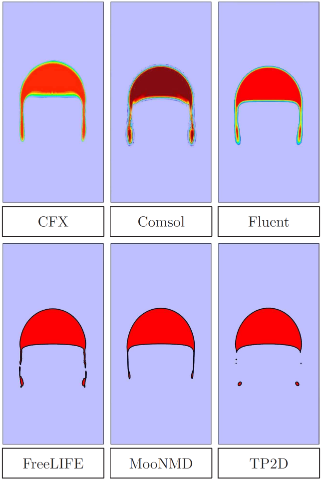
- **Rising bubble; Test Case 2**

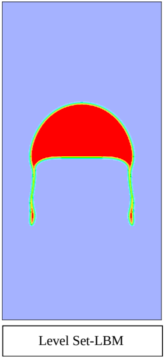
1. FeatFlow , 160 × 320
2. Approach 1, 160 × 320
3. Approach 2, 320 × 640



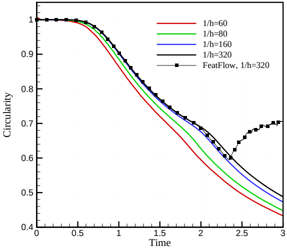
Numerical Results

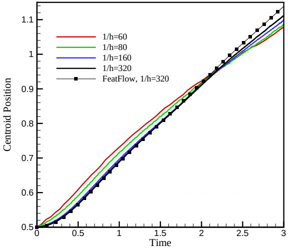
- **Rising bubble; Test Case 2** $\Rightarrow$ Benchmarking the picture norm!

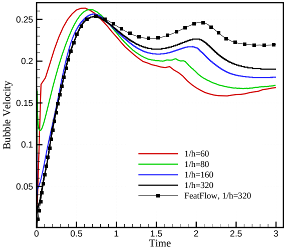




▶ **Rising bubble; Test Case 2** ⇒ LBM-LS Approach 1, grid effect, $0 < T < 3$







Numerical Results

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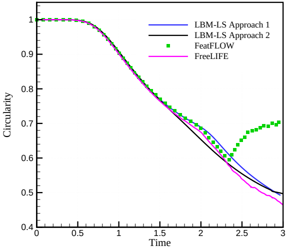
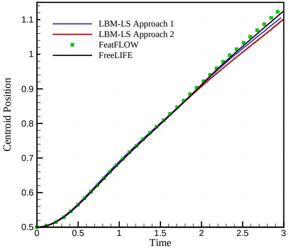
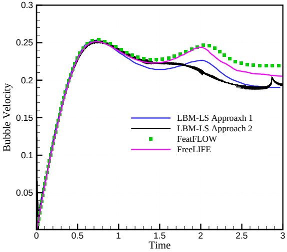
▶ **Rising bubble; Test Case 2** ⇒ Benchmark comparison, 320×640, $0 < T < 3$







Parallel Implementation

- **CPU Implementation**
  - Intel C compiler + OpenMP directives for shred-memory
  - LBM implemented using Array of Structures (AOS)

Compute platform	No. of Cores	Max. Bandwidth(GB/s)	Max. GFLOPS	Apprx. price (USD)
Intel Xeon X5680	24	32	20	1500

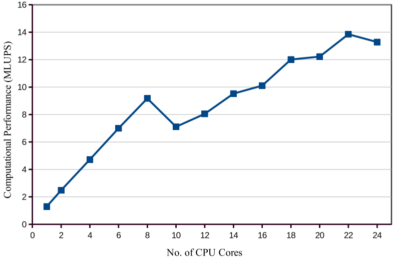
- **GPU Implementation**
  - nVIDIA's CUDA-C for many core GPUs
  - LBM implemented using Structure of Arrays (SOA)
  - Shared memory for LBM's streaming step
  - Constant cache for fast level set reinitialization

Compute platform	No. of Cores	Max. Bandwidth(GB/s)	Max. GFLOPS	Apprx. price (USD)
nVIDIA Tesla C2070	14(x32)	144	515	1900

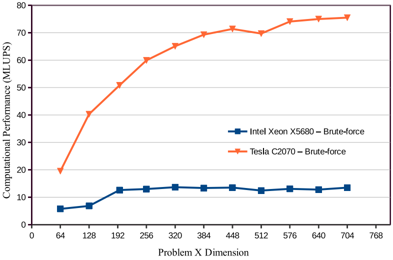
Parallel Implementation

▶ Computational Performance

- Favourable scaling in both weak and strong senses
- GPU vs. CPU performance
- GPU code is in average 5.5 times faster than CPU



Strong Scaling (160^2); Intel Xeon



Weak Scaling; Tesla vs. Intel Xeon

Summary

- Introduced two coupled 2nd order multiphase flow solvers
- Large density and viscosity ratios; High degree of stability and isotropy
- One-Fluid formulation coupled with level set provides sharp interfaces
- Pressure evolution scheme provides diffused interfaces
- Extensive benchmarking; accuracy of One-Fluid scheme comparable to FEM-based FeatFLOW solutions (www.featflow.de/benchmarks)
- Optimized CPU and GPU implementations; 5.5X speed-up on Tesla GPUs

Acknowledgement

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SPPEXA German Priority Programme 1648 Software for Exascale Computing

Thanks for your attention!