# Movement of a liquid in vegetative environments

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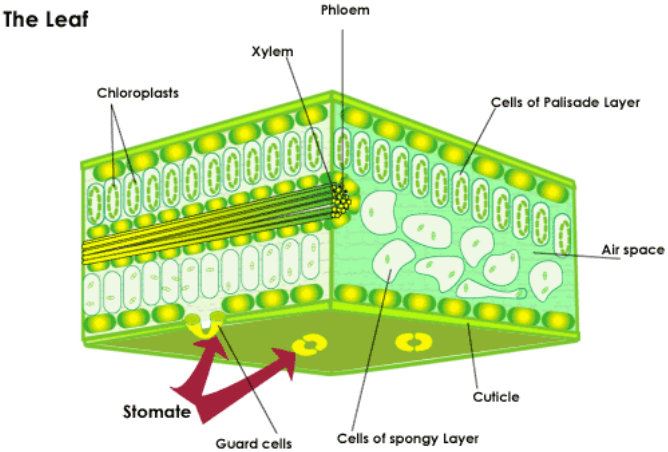
November 9, 2006

# Motivation

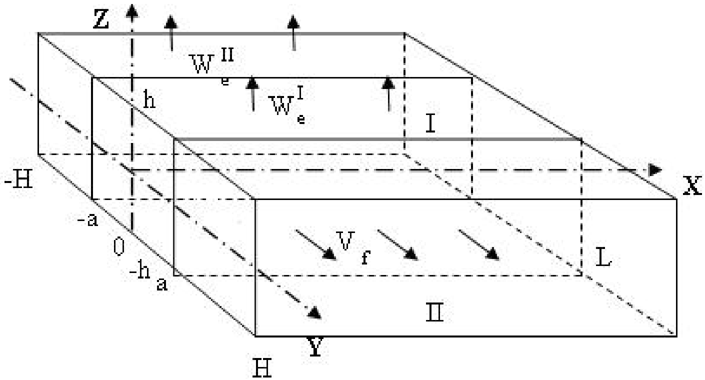
The solution of the given problem can be used:

- as the appendix in biological reseaches
- in connection with development of biotechnologies
- for reseach of evaporation from surface of leaves

# Cross-section through a leaf



# Schematic view



# Problem in 3D

- Equtions discribing movement of liquid in area I:

$$\frac{\partial V_x}{\partial x}+\frac{\partial V_y}{\partial y}+\frac{\partial V_z}{\partial z}=0$$

$$V_x=-\frac{K_x}{\mu}\cdot\frac{\partial P}{\partial x},\ V_y=-\frac{K_y}{\mu}\cdot\frac{\partial P}{\partial y},\ V_z=-\frac{K_z}{\mu}\cdot\frac{\partial P}{\partial z}$$

$$\frac{\partial C}{\partial t}+\frac{\partial(C\cdot V_x)}{\partial x}+\frac{\partial(C\cdot V_y)}{\partial y}+\frac{\partial(C\cdot V_z)}{\partial z}=D_c\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}\right)$$

- Equations discribing movement of liquid in area II:

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}=0$$

$$v_x=-\frac{k_x}{\mu}\left(\frac{\partial p}{\partial x}-\xi\frac{\partial \pi}{\partial x}\right),\ v_y=-\frac{k_y}{\mu}\left(\frac{\partial p}{\partial y}-\xi\frac{\partial \pi}{\partial y}\right),\ v_z=-\frac{k_z}{\mu}\left(\frac{\partial p}{\partial z}-\xi\frac{\partial \pi}{\partial z}\right)$$

$$\frac{\partial b}{\partial t}+\frac{\partial(b\cdot v_x)}{\partial x}+\frac{\partial(b\cdot v_y)}{\partial y}+\frac{\partial(b\cdot v_z)}{\partial z}=D_b\left(\frac{\partial^2 b}{\partial x^2}+\frac{\partial^2 b}{\partial y^2}+\frac{\partial^2 b}{\partial z^2}\right)-q_b$$

# Boundary and initial conditions in area I

| | | | |
|---|---|---|---|
| $x = 0:$ | $C = C^+$ | $P = P^+$ | |
| $x = L:$ | $C = C^-$ | $P = P^-$ | |
| $y = z = 0:$ | $V_y = 0$ | $V_z = 0$ | |
| $y = \pm a:$ | $V_x = 0$ | $V_z = 0$ | $V_y = V_f$ |
| $z = \pm h:$ | $V_z = \varepsilon^1(C)$ | | |
| $t = 0:$ | $C = C_0(x, y, z)$ | | |

*Where*

$$V_f = \xi_1(P - p - \zeta_s(\Pi - \pi))$$
$$\Pi = RTC, \pi = RTb$$

# Boundary and initial conditions in area II

$$x = 0: \quad \frac{\partial b}{\partial x} = 0 \quad \frac{\partial p}{\partial x} = 0$$

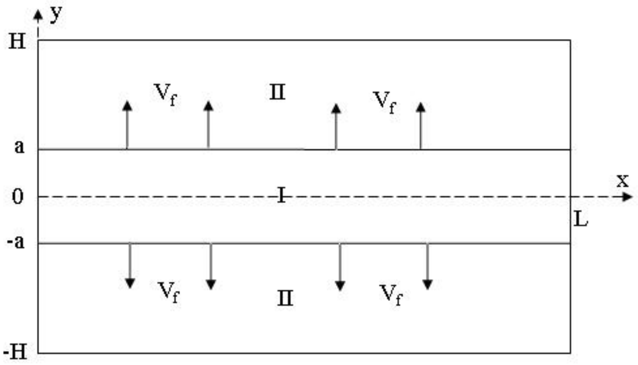
$$x = L: \quad \frac{\partial b}{\partial x} = 0 \quad \frac{\partial p}{\partial x} = 0$$

$$y = \pm H: \quad \frac{\partial b}{\partial y} = 0 \quad \frac{\partial p}{\partial y} = 0$$

$$z = \pm h: \quad v_z = \varepsilon^2(b) \quad \frac{\partial b}{\partial z} = 0$$

$$t = 0: \quad b = b_0(x, y, z)$$

# Reduction to 2D problem



# Reduction to 2D problem

- Equations discribing movement of liquid in area I:

$$\langle F(t,x,y)\rangle_z = \tfrac{1}{2h}\int_{-h}^{h} F(t,x,y,z)dz$$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = -\frac{1}{h}\varepsilon_I(C)$$

$$V_x = -\frac{K_x}{\mu}\cdot\frac{\partial P}{\partial x} \quad , \quad V_y = -\frac{K_y}{\mu}\cdot\frac{\partial P}{\partial y}$$

$$\chi_I\frac{\partial C}{\partial t} + \frac{\partial (C\cdot V_x)}{\partial x} + \frac{\partial (C\cdot V_y)}{\partial y} = \chi_I D_c\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right)$$

- Equations discribing movement of liquid in area II:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = -\frac{1}{h}\varepsilon_{II}(b)$$

$$v_x = -\frac{k_x}{\mu}\left(\frac{\partial p}{\partial x} - \xi\frac{\partial \pi}{\partial x}\right), v_y = -\frac{k_y}{\mu}\left(\frac{\partial p}{\partial y} - \xi\frac{\partial \pi}{\partial y}\right)$$

$$\chi_{II}\frac{\partial b}{\partial t} + \frac{\partial (b\cdot v_x)}{\partial x} + \frac{\partial (b\cdot v_y)}{\partial y} = \chi_{II} D_b\left(\frac{\partial^2 b}{\partial x^2} + \frac{\partial^2 b}{\partial y^2}\right) - q_b$$

# BIC's for 2D problem,area I

$$
\begin{array}{lll}
x = 0: & C = C^{+} & P = P^{+} \\
x = L: & C = C^{-} & P = P^{-} \\
y = 0: & V_y = 0 & \\
y = \pm a: & V_x = 0 & V_y = V_f \\
t = 0: & C = C_0(x, y) &
\end{array}
$$

$Where:$

$$
V_f = \xi_1(P - p - \zeta_s(\Pi - \pi))
$$

$$
\Pi = RTC, \pi = RTb
$$

# BIC's for 2D problem,area II

$$x = 0: \quad \frac{\partial b}{\partial x} = 0 \quad \frac{\partial p}{\partial x} = 0$$

$$x = L: \quad \frac{\partial b}{\partial x} = 0 \quad \frac{\partial p}{\partial x} = 0$$

$$y = \pm H: \quad \frac{\partial b}{\partial y} = 0 \quad \frac{\partial p}{\partial y} = 0$$

$$t = 0: \quad b = b_0(x, y)$$

# Nondimensionalize

Nondimensionalize all unknows $F = \{V_x, V_y, v_x, v_y, P, p, C, b\}$

$$
\begin{array}{ll}
x = x^\circ \cdot L, & y = y^\circ \cdot 2a \\
V_x = V_x^\circ \cdot V_x^\star, & V_y = V_y^\circ \cdot V_y^\star \\
v_x = v_x^\circ \cdot v_x^\star, & v_y = v_y^\circ \cdot v_y^\star \\
P = P^\circ \cdot P^\star, & p = p^\circ \cdot p^\star \\
C = C^\circ \cdot C^\star, & b = b^\circ \cdot b^\star
\end{array}
$$

Number of Strohal : $St = \frac{L}{t^\star V^\star}$

# Nondimensional equations

- Equations discribing movement of liquid in area I:

$$\frac{\partial V_x}{\partial x} + A_I \frac{\partial V_y}{\partial y} = -B_I \varepsilon_I(C)$$

$$V_x = -C_{Ix} \frac{\partial P}{\partial x} \quad , \quad V_y = -C_{Iy} \frac{\partial P}{\partial y}$$

$$\chi_I St \frac{\partial C}{\partial t} + \frac{\partial (C \cdot V_x)}{\partial x} + A_I \frac{\partial (C \cdot V_y)}{\partial y} = \chi_I \left( F_{Ix} \frac{\partial^2 C}{\partial x^2} + F_{Iy} \frac{\partial^2 C}{\partial y^2} \right)$$

*Where* $A_I = \frac{V_y^\star \cdot L}{V_x^\star \cdot 2a}$, $B_I = \frac{L}{h \cdot V_x^\star}$

$C_{Ix} = \frac{P^\star \cdot K_x}{V_x^\star \cdot L \cdot \mu}$, $C_{Iy} = \frac{P^\star \cdot K_y}{V_y^\star \cdot 2a \cdot \mu}$

$F_{Ix} = \frac{L \cdot D_C}{V_x^\star \cdot L^2}$, $F_{Iy} = \frac{L \cdot D_C}{V_x^\star \cdot (2a)^2}$

# Nondimensional equations

- Equations discribing movement of liquid in area II:

$$\frac{\partial v_x}{\partial x} + A_{II}\frac{\partial v_y}{\partial y} = -B_{II}\varepsilon_{II}(b)$$

$$v_x = -(C_{IIx}\frac{\partial p}{\partial x} - \varepsilon\frac{\partial b}{\partial x}) \quad , \quad v_y = -(C_{IIy}\frac{\partial p}{\partial y} - \varepsilon\frac{\partial b}{\partial y})$$

$$\chi_{II} St\frac{\partial b}{\partial t} + \frac{\partial(b\cdot v_x)}{\partial x} + A_{II}\frac{\partial(b\cdot v_y)}{\partial y} = \chi_{II}\left(F_{IIx}\frac{\partial^2 b}{\partial x^2} + F_{IIy}\frac{\partial^2 b}{\partial y^2}\right) - q_b^\star$$

*Where* $A_{II} = \frac{v_y^\star \cdot L}{v_x^\star \cdot 2(H-a)}$, $\quad B_{II} = \frac{L}{h\cdot v_x^\star}$

$$C_{IIx} = \frac{p^\star \cdot k_x}{v_x^\star \cdot L\cdot \mu}, \quad C_{IIy} = \frac{p^\star \cdot k_y}{V_y^\star \cdot 2(H-a)\cdot \mu}$$

$$F_{IIx} = \frac{L\cdot D_b}{v_x^\star \cdot L^2}, \quad F_{IIy} = \frac{L\cdot D_b}{V_x^\star \cdot (2(H-a))^2}$$

$$\varepsilon = \frac{\zeta\cdot R\cdot T\cdot b^\star \cdot k_x}{L\cdot v_x^\star \cdot \mu} \quad or \quad \varepsilon = \frac{\zeta\cdot R\cdot T\cdot b^\star \cdot k_y}{2(H-a)\cdot v_x^\star \cdot \mu}$$

# Expansion in $\varepsilon$, in zeroth order

$$F = F^{(0)} + \varepsilon F^{(1)} + \varepsilon^2 F^{(2)} + \ldots$$

- Equations discribing movement of liquid in area I:

$$\frac{\partial V_x^{(0)}}{\partial x} + A_I \frac{\partial V_y^{(0)}}{\partial y} = -B_I \varepsilon_I$$

$$V_x^{(0)} = -C_{Ix} \frac{\partial P^{(0)}}{\partial x} \quad , \quad V_y^{(0)} = -C_{Iy} \frac{\partial P^{(0)}}{\partial y}$$

$$\chi_I St \frac{\partial C^{(0)}}{\partial t} + \frac{\partial (C^{(0)} \cdot V_x^{(0)})}{\partial x} + A_I \frac{\partial (C^{(0)} \cdot V_y^{(0)})}{\partial y} = \chi_I (F_{Ix} \frac{\partial^2 C^{(0)}}{\partial x^2} + F_{Iy} \frac{\partial^2 C^{(0)}}{\partial y^2})$$

- Equations discribing movement of liquid in area II:

$$\frac{\partial v_x^{(0)}}{\partial x} + A_{II} \frac{\partial v_y^{(0)}}{\partial y} = -B_{II} \varepsilon_{II}$$

$$v_x^{(0)} = -C_{IIx} \frac{\partial p^{(0)}}{\partial x} \quad , \quad v_y^{(0)} = -C_{IIy} \frac{\partial p^{(0)}}{\partial y}$$

$$\chi_{II} St \frac{\partial b^{(0)}}{\partial t} + \frac{\partial (b^{(0)} \cdot v_x^{(0)})}{\partial x} + A_{II} \frac{\partial (b^{(0)} \cdot v_y^{(0)})}{\partial y} = \chi_{II} (F_{IIx} \frac{\partial^2 b^{(0)}}{\partial x^2} + F_{IIy} \frac{\partial^2 b^{(0)}}{\partial y^2}) - q_b^\star$$

# BIC's of zeroth order area I

$$
\begin{array}{lll}
x = 0: & C^{(0)} = C^{+} & P^{(0)} = P^{+} \\
x = 1: & C^{(0)} = C^{-} & P^{(0)} = P^{-} \\
y = 0: & V_y^{(0)} = 0 & \\
y = \pm 1: & V_x^{(0)} = 0 & V_y^{(0)} = \zeta_1 (P^{(0)} - p^{(0)}) \\
t = 0: & C^{(0)} = C_0^{(0)}(x, y) &
\end{array}
$$

# BIC's of zeroth order area II

$$x = 0: \qquad \frac{\partial b^{(0)}}{\partial x} = 0 \qquad \frac{\partial p^{(0)}}{\partial x} = 0$$

$$x = 1: \qquad \frac{\partial b^{(0)}}{\partial x} = 0 \qquad \frac{\partial p^{(0)}}{\partial x} = 0$$

$$y = \pm\frac{H}{(H-a)}: \qquad \frac{\partial b^{(0)}}{\partial y} = 0 \qquad \frac{\partial p^{(0)}}{\partial y} = 0$$

$$t = 0: \qquad b^{(0)} = b_0^{(0)}(x, y)$$

# Expansion in $\varepsilon$,in first order

- Equations discribing movement of liquid in area I:

$$\frac{\partial V_x^{(1)}}{\partial x} + A_I \frac{\partial V_y^{(1)}}{\partial y} = 0$$

$$V_x^{(1)} = -C_{Ix} \frac{\partial P^{(1)}}{\partial x} \quad , \quad V_y^{(1)} = -C_{Iy} \frac{\partial P^{(1)}}{\partial y}$$

$$\chi_I St \frac{\partial C^{(1)}}{\partial t} + \frac{\partial}{\partial x}(C^{(1)} \cdot V_x^{(0)} + C^{(0)} \cdot V_x^{(1)}) + A_I \frac{\partial}{\partial y}(C^{(1)} \cdot V_y^{(0)} + C^{(0)} \cdot V_y^{(1)}) = \\ = \chi_I \left( F_{Ix} \frac{\partial^2 C^{(1)}}{\partial x^2} + F_{Iy} \frac{\partial^2 C^{(1)}}{\partial y^2} \right)$$

- Equations discribing movement of liquid in area II:

$$\frac{\partial v_x^{(1)}}{\partial x} + A_{II} \frac{\partial v_y^{(1)}}{\partial y} = 0$$

$$v_x^{(1)} = -C_{IIx}\left(\frac{\partial p^{(1)}}{\partial x} - \frac{\partial b^{(0)}}{\partial x}\right) \quad , \quad v_y^{(1)} = -C_{IIy}\left(\frac{\partial p^{(1)}}{\partial y} - \frac{\partial b^{(0)}}{\partial y}\right)$$

$$\chi_{II} St \frac{\partial b^{(1)}}{\partial t} + \frac{\partial}{\partial x}(b^{(1)} \cdot v_x^{(0)} + b^{(0)} \cdot v_x^{(1)}) + A_{II} \frac{\partial}{\partial y}(b^{(1)} \cdot v_y^{(0)} + b^{(0)} \cdot v_y^{(1)}) = \\ = \chi_{II} \left( F_{IIx} \frac{\partial^2 b^{(1)}}{\partial x^2} + F_{IIy} \frac{\partial^2 b^{(1)}}{\partial y^2} \right)$$

# BIC's of first order in area I

$$
\begin{array}{lll}
x = 0: & C^{(1)} = 0 & P^{(1)} = 0 \\
x = 1: & C^{(1)} = 0 & P^{(1)} = 0 \\
y = 0: & V_y^{(1)} = 0 & \\
y = \pm 1: & V_x^{(1)} = 0 & V_y^{(1)} = \zeta_1 (P^{(1)} - p^{(1)}) \\
t = 0: & C^{(1)} = C_0^{(1)}(x, y) &
\end{array}
$$

# BIC's of first order,in area II

$$x = 0: \qquad \frac{\partial b^{(1)}}{\partial x} = 0 \qquad \frac{\partial p^{(1)}}{\partial x} = 0$$

$$x = 1: \qquad \frac{\partial b^{(1)}}{\partial x} = 0 \qquad \frac{\partial p^{(1)}}{\partial x} = 0$$

$$y = \pm\frac{H}{(H-a)}: \qquad \frac{\partial b^{(1)}}{\partial y} = 0 \qquad \frac{\partial p^{(1)}}{\partial y} = 0$$

$$t = 0: \qquad b^{(1)} = b_0^{(1)}(x, y)$$

# Expansion in $\varepsilon$,in second order

- Equations discribing movement of liquid in area I:

$$\frac{\partial V_x^{(2)}}{\partial x} + A_I \frac{\partial V_y^{(2)}}{\partial y} = 0$$

$$V_x^{(2)} = -C_{Ix} \frac{\partial P^{(2)}}{\partial x} \quad , \quad V_y^{(2)} = -C_{Iy} \frac{\partial P^{(2)}}{\partial y}$$

$$\chi_I St \frac{\partial C^{(2)}}{\partial t} + \frac{\partial}{\partial x}(C^{(2)} \cdot V_x^{(1)} + C^{(1)} \cdot V_x^{(2)}) + A_I \frac{\partial}{\partial y}(C^{(2)} \cdot V_y^{(1)} + C^{(1)} \cdot V_y^{(2)}) = = \chi_I \left( F_{Ix} \frac{\partial^2 C^{(2)}}{\partial x^2} + F_{Iy} \frac{\partial^2 C^{(2)}}{\partial y^2} \right)$$

- Equations discribing movement of liquid in area II:

$$\frac{\partial v_x^{(2)}}{\partial x} + A_{II} \frac{\partial v_y^{(2)}}{\partial y} = 0$$

$$v_x^{(2)} = -C_{IIx}\left(\frac{\partial p^{(2)}}{\partial x} - \frac{\partial b^{(1)}}{\partial x}\right) \quad , \quad v_y^{(1)} = -C_{IIy}\left(\frac{\partial p^{(2)}}{\partial y} - \frac{\partial b^{(1)}}{\partial y}\right)$$

$$\chi_{II} St \frac{\partial b^{(2)}}{\partial t} + \frac{\partial}{\partial x}(b^{(2)} \cdot v_x^{(1)} + b^{(1)} \cdot v_x^{(2)}) + A_{II} \frac{\partial}{\partial y}(b^{(2)} \cdot v_y^{(1)} + b^{(1)} \cdot v_y^{(2)}) = = \chi_{II} \left( F_{IIx} \frac{\partial^2 b^{(2)}}{\partial x^2} + F_{IIy} \frac{\partial^2 b^{(2)}}{\partial y^2} \right)$$

# BIC's of second order in area I

$$
\begin{array}{lll}
x = 0: & C^{(2)} = 0 & P^{(2)} = 0 \\
x = 1: & C^{(2)} = 0 & P^{(2)} = 0 \\
y = 0: & V_y^{(2)} = 0 & \\
y = \pm 1: & V_x^{(2)} = 0 & V_y^{(2)} = \zeta_1 (P^{(2)} - p^{(2)}) \\
t = 0: & C^{(2)} = C_0^{(2)}(x, y) &
\end{array}
$$

# BIC's of second order in area II

$$x = 0: \quad \frac{\partial b^{(2)}}{\partial x} = 0 \quad \frac{\partial p^{(2)}}{\partial x} = 0$$

$$x = 1: \quad \frac{\partial b^{(2)}}{\partial x} = 0 \quad \frac{\partial p^{(2)}}{\partial x} = 0$$

$$y = \pm\frac{H}{(H-a)}: \quad \frac{\partial b^{(2)}}{\partial y} = 0 \quad \frac{\partial p^{(2)}}{\partial y} = 0$$

$$t = 0: \quad b^{(2)} = b_0^{(2)}(x, y)$$

# amplitude of pressure in area I

$$F^{(0)} = \sum_{n=1}^{\infty} F_n(y)\cos(\pi n x)$$

$$P_n(y) = exp\left(-\pi n\sqrt{\frac{C_{Ix}}{C_{Iy}A_I}}y\right)\int\frac{\left(exp\left(\pi n\sqrt{\frac{C_{Ix}}{C_{Iy}A_I}}y\right)\right)B_I\epsilon^I(y)}{-2\pi n\sqrt{\frac{C_{Ix}}{C_{Iy}A_I}}}dy +$$
$$+exp\left(\pi n\sqrt{\frac{C_{Ix}}{C_{Iy}A_I}}y\right)\int\frac{\left(exp\left(-\pi n\sqrt{\frac{C_{Ix}}{C_{Iy}A_I}}y\right)\right)B_I\epsilon^I(y)}{-2\pi n\sqrt{\frac{C_{Ix}}{C_{Iy}A_I}}}dy +$$
$$+D1\cdot exp\left(\pi n\sqrt{\frac{C_{Ix}}{C_{Iy}A_I}}y\right) + D2\cdot exp\left(-\pi n\sqrt{\frac{C_{Ix}}{C_{Iy}A_I}}y\right)$$

# amplitude of pressure in area II

$$p_n(y) = exp\left(-\pi n\sqrt{\frac{C_{IIx}}{C_{IIy}A_{II}}}y\right)\int\frac{\left(exp\left(\pi n\sqrt{\frac{C_{IIx}}{C_{IIy}A_{II}}}y\right)\right)B_{II}\epsilon^{II}(y)}{-2\pi n\sqrt{\frac{C_{IIx}}{C_{Iy}A_{II}}}}dy +$$

$$+exp\left(\pi n\sqrt{\frac{C_{IIx}}{C_{IIy}A_{II}}}y\right)\int\frac{\left(exp\left(-\pi n\sqrt{\frac{C_{IIx}}{C_{IIy}A_{II}}}y\right)\right)B_{II}\epsilon^{II}(y)}{-2\pi n\sqrt{\frac{C_{IIx}}{C_{IIy}A_{II}}}}dy +$$

$$+K1\cdot exp\left(\pi n\sqrt{\frac{C_{IIx}}{C_{IIy}A_{II}}}y\right) + K2\cdot exp\left(-\pi n\sqrt{\frac{C_{IIx}}{C_{IIy}A_{II}}}y\right)$$

# velocity in area I

$$V_x = \sum_{n=1}^{\infty} C_{Ix}\pi n \sin(\pi n x)(exp\left(-\pi n\sqrt{\frac{C_{Ix}}{C_{Iy}A_I}}y\right)\int \frac{\left(exp\left(\pi n\sqrt{\frac{C_{Ix}}{C_{Iy}A_I}}y\right)\right)B_I\epsilon^I(y)}{-2\pi n\sqrt{\frac{C_{Ix}}{C_{Iy}A_I}}}dy+$$

$$+exp\left(\pi n\sqrt{\frac{C_{Ix}}{C_{Iy}A_I}}y\right)\int \frac{\left(exp\left(-\pi n\sqrt{\frac{C_{Ix}}{C_{Iy}A_I}}y\right)\right)B_I\epsilon^I(y)}{-2\pi n\sqrt{\frac{C_{Ix}}{C_{Iy}A_I}}}dy + D1\cdot exp\left(\pi n\sqrt{\frac{C_{Ix}}{C_{Iy}A_I}}y\right)+$$

$$+D2\cdot exp\left(-\pi n\sqrt{\frac{C_{Ix}}{C_{Iy}A_I}}y\right))$$

$$V_y = \sum_{n=1}^{\infty} C_{Ix}\cos(\pi n x)(\frac{B_I\varepsilon^I(y)}{\pi n\sqrt{\frac{C_{Ix}}{C_{Iy}A_I}}}+$$

$$+\frac{1}{2}exp\left(-\pi n\sqrt{\frac{C_{Ix}}{C_{Iy}A_I}}y\right)\int\left(exp\left(\pi n\sqrt{\frac{C_{Ix}}{C_{Iy}A_I}}y\right)\right)B_I\epsilon^I(y)dy+$$

$$+\frac{1}{2}exp\left(\pi n\sqrt{\frac{C_{Ix}}{C_{Iy}A_I}}y\right)\int\left(exp\left(-\pi n\sqrt{\frac{C_{Ix}}{C_{Iy}A_I}}y\right)\right)B_I\epsilon^I(y)dy-$$

$$-(\pi n\sqrt{\frac{C_{Ix}}{C_{Iy}A_I}})(D1\cdot exp\left(\pi n\sqrt{\frac{C_{Ix}}{C_{Iy}A_I}}y\right)+D2\cdot exp\left(-\pi n\sqrt{\frac{C_{Ix}}{C_{Iy}A_I}}y\right))$$

# velocity in area II

$$v_x = \sum_{n=1}^{\infty} C_{IIx} \pi n \sin(\pi n x)(exp\left(-\pi n \sqrt{\frac{C_{IIx}}{C_{IIy} A_{II}}} y\right) \int \frac{\left(exp\left(\pi n \sqrt{\frac{C_{IIx}}{C_{IIy} A_{II}}} y\right)\right) B_I I \epsilon^{II}(y)}{-2\pi n \sqrt{\frac{C_{IIx}}{C_{IIy} A_{II}}}} dy +$$

$$+ exp\left(\pi n \sqrt{\frac{C_{IIx}}{C_{IIy} A_{II}}} y\right) \int \frac{\left(exp\left(-\pi n \sqrt{\frac{C_{IIx}}{C_{IIy} A_{II}}} y\right)\right) B_{II} \epsilon^{II}(y)}{-2\pi n \sqrt{\frac{C_{IIx}}{C_{IIy} A_{II}}}} dy + K1 \cdot exp\left(\pi n \sqrt{\frac{C_{IIx}}{C_{IIy} A_{II}}} y\right) +$$

$$+ K2 \cdot exp\left(-\pi n \sqrt{\frac{C_{IIx}}{C_{Iy} A_{II}}} y\right))$$

$$v_y = \sum_{n=1}^{\infty} C_{IIx} \cos(\pi n x)\left(\frac{B_I I \varepsilon^{II}(y)}{\pi n \sqrt{\frac{C_{IIx}}{C_{IIy} A_{II}}}} + \right.$$

$$+ \frac{1}{2} exp\left(-\pi n \sqrt{\frac{C_{IIx}}{C_{IIy} A_{II}}} y\right) \int \left(exp\left(\pi n \sqrt{\frac{C_{IIx}}{C_{IIy} A_{II}}} y\right)\right) B_{II} \epsilon^{II}(y) dy +$$

$$+ \frac{1}{2} exp\left(\pi n \sqrt{\frac{C_{IIx}}{C_{IIy} A_{II}}} y\right) \int \left(exp\left(-\pi n \sqrt{\frac{C_{IIx}}{C_{IIy} A_{II}}} y\right)\right) B_{II} \epsilon^{II}(y) dy -$$

$$\left. -(\pi n \sqrt{\frac{C_{IIx}}{C_{IIy} A_{II}}})(K1 \cdot exp\left(\pi n \sqrt{\frac{C_{IIx}}{C_{IIy} A_{II}}} y\right) + K2 \cdot exp\left(-\pi n \sqrt{\frac{C_{IIx}}{C_{IIy} A_{II}}} y\right)) \right)$$

# Conclusion

- We modeled the movement of a liquid in vegetative environments
- We reduced the problem to two dimensions
- We used an expansion of the solution in $\varepsilon$
- We got the solution in zeroth order by decomposition of variables

We can use the solution in zeroeth order to get the first and second order solutions and for the reseach biological experiments.