Analysis and postprocessing of Space-Time Compressed Flow Computations

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FEAST - Finite Element Analysis & Solution Tools

Nonlinear (anisotropic) diffusion models:

$$\partial_t \rho - \mathrm{div}\,(\mathsf{A}(\nabla \rho_\sigma)\nabla \rho) = f(\rho)$$

← Usable for:

- Noise elimination combined with edge enhancement
- Simplification of images/data sets by segmentation

→ Perona-Malik, Catte-Lions-Morel-Coll, Weikert, Mikula, Rumpf

→ $\mathsf{A}(\nabla \rho_\sigma) := B(x)^T \begin{pmatrix} \alpha & 0 \\ 0 & G(\|\nabla \rho_\sigma\|)\,\mathrm{Id}_{d-1} \end{pmatrix} B(x)$,
$G(s) := \frac{\epsilon}{1+s^2/\lambda^2}$

→ Possible extensions:

- Structure tensor for better differentiation between corners and edges in image features
- Coherence enhancing

Nonlinear anisotrophic transport-diffusion models:

$$\partial_t \rho + \mathbf{v} \cdot \nabla \rho - \mathrm{div}\,(\mathsf{A}(\mathbf{v}, \nabla \rho_\sigma)\nabla \rho) = f(\rho)$$

← Usable for instationary flow visualizations

→ $\mathsf{A}(\mathbf{v}, \nabla \rho_\sigma) := B(x)^T \begin{pmatrix} \alpha(\|\mathbf{v}\|) & 0 \\ 0 & G(\|\nabla \rho_\sigma\|)\,\mathrm{Id}_{d-1} \end{pmatrix} B(x),$
$\alpha(\|v\|)(x) = \frac{\beta^2 \max(\|v(x)\|, \|v\|_{min})^2 \tau}{2}$, $G(s) := \frac{\epsilon}{1+s^2/\lambda^2}$

→ Visualizes flows in the whole domain

→ Creates scale-space representations of flow features

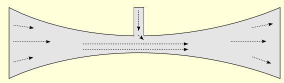
→ Flow field clustering

⇒ Possible replacement for particle tracing

Numerical and Algorithmic Aspects:

1. Discretization techniques
   - The operator $A(\cdot)$ and function $f(\cdot)$ were linearized (reducing the problem to a linear one)
   - A stable semi-implicit Euler scheme was used for time discretization (Crank-Nicholson also possible)
   - We were using a conforming linear quadrilateral FEM for space discretization
   - The transport operator was stabilized by streamline diffusion ⇒ Implementation by modifying $\alpha(\cdot)$
2. Efficient (parallel) numerics (Feast)
   - Simplified parallelization of computations by domain decomposition (ScaRC)
   - Fast adaption/combination of various solvers
   - Better use of modern computers by exploiting local structures even for meshes generated by refining unstructured coarse grids (see [5])
3. Parameter Choices/Blending strategies
   - Diffusion dominated ⇒ $\beta = 10.0$
   - Solutions start with pure noise
   - Solutions get coarser with time ⇒ Blending of several solutions
   - Different blending strategies possible:
     – Sine based ansatz → Only one solution dominates
     – Bézier-spline based ansatz → all solutions contribute

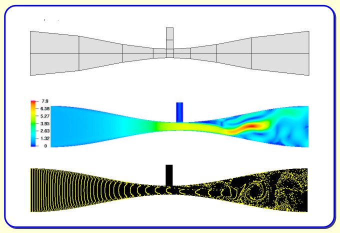
Complex CFD configuration:



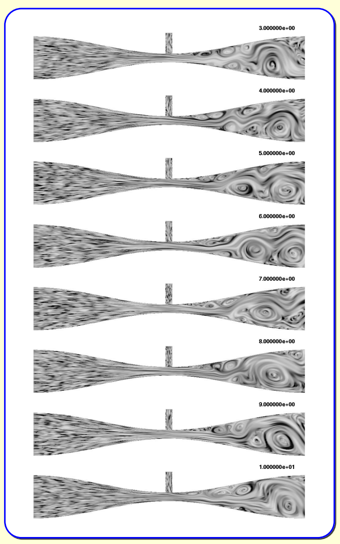
The 'Venturi pipe' (or 'Venturi tube'), named after 18th-19th-century Italian physicist Giovanni Battista Venturi, is a short pipe with a narrow throat in the middle. It is used to measure fluid flows and as a pump. Fluid passing the throat speeds up and causes a pressure drop in the middle of the pipe. If an additional pipe is attached to that throat, passing fluid creates a powerful suction effect. For example sailing boats can be drained simply by their movement. We were using a simplified configuration without any valves or additional features.

Application to CFD data:

We precalculated instationary flow data for this problem (81920 cells, 84500 nodes) using our numerical package FeatFlow . Here an example of the flow and a particle tracing of it:

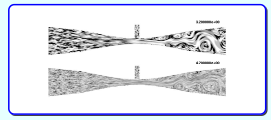


The precalculated data was later read in and linearly interpolated in time to get a flow field $\mathbf{v}(\mathbf{t_n})$ for the nonlinear anisotropic transport-diffusion problem. A time step of $\tau = 0.0025$ and contrast enhancement was used. Four solutions were blended together to generate the following picture sequence (only the blended solution is shown):



Blending:

The solutions progress with time in the scale-space toward coarser features. Several solutions have to be blended together to approximate a desired scale level. The resulting scale can get very uneven, if not enough solutions are used for blending. Two solutions are not adequate for our purposes as can be seen in the following example:



Convergence rates:

Convergence rates are computed for the Venturi pipe configuration with sine waves as initial solution.

level	elements	nodes
1	20	34
2	80	107
3	320	373
4	1280	1385
5	5120	5329
6	20480	20897
7	81920	82753

Preconditioned (ADITRIGS) BICG-stab (see [6]):

level	$\tau = 0.001$	#it	$\tau = 0.002$	#it	$\tau = 0.003$	#it
5	0.08753	6	0.08345	6	0.12957	7
6	0.08973	6	0.11818	7	0.19461	9
7	0.20600	9	0.37173	15	0.50550	21

Multigrid with ADITRIGS-smoother:

level	$\tau = 0.001$	#it	$\tau = 0.002$	#it	$\tau = 0.003$	#it
5	0.00001	1	0.00008	2	0.00030	2
6	0.00003	2	0.00043	2	0.00985	3
7	0.00027	2	0.06115	5	0.60475	28

BICG-stab preconditioned with one multigrid iteration:

level	$\tau = 0.001$	#it	$\tau = 0.002$	#it	$\tau = 0.003$	#it
5	0.00001	1	0.00001	2	0.00008	2
6	0.00001	1	0.00059	2	0.00275	3
7	0.00001	1	0.00641	3	0.05587	5

The combination of BICG and MG handles even the problem case for $\tau = 0.003$ and refinement level 7 far better than expected. Both methods seem to complement each other for large unsymmetric problems generated by the anisotropic transport diffusion problem.

Results:

- By using specialized solvers even large unsymmetric problems can be efficiently solved
- Flow features (especially vortices) can be easily detected/extracted
- "Streamlines" for instationary flows are generated (theoretically even in the 3D case!)

Work Programme:

- Treating the nonlinearities by using a fixed point iteration
- Reducing still occuring numerical oscillations by adding a cross-term to $G(\cdot)$ (see [1])
- Trying to use higher order schemes (Crank-Nicholson)

Further informations about the use of PDE in image processing can be found in [8, 7, 2]. Some pointers for detection of flow features by using anisotropic diffusion can be found in [3, 4]. A more in deep presentation of our latest work can be seen in our new preprint (available here).

Literatur

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