Universität Dortmund
technische universität
dortmund
Fakultät für Mathematik
IAM

Numerical Benchmarking of Fluid-Structure Interaction between elastic object and laminar incompressible flow

S. Turek, J. Hron, M. Razzaq, H. Wobker
TU Dortmund

with support by

M.Schäfer, M. Heck, M. Krafczyk, J. Tölke, S. Geller, H.-J. Bungartz, M. Brenk, R. Rannacher,
T. Dunne, W. Wall, A. Gerstenberger, E. Rank, A. Düster, S. Kollmannsberger
K.-U. Bletzinger, R. Wüchner, A.Kupzok, T. Gallinger, U. Israel

Key question

- ***Accurate and robust description of the interaction mechanisms w.r.t. highly dynamical and nonlinear behaviour and significant geometry changes?***
- That includes:
  - Quality of different **discretization** techniques (FEM, FV, FD, LBM, resp., beam, shell, volume elements) for FSI?
  - Robustness and numerical efficiency of the integrated **solver** components?
  - **Coupling** mechnisms?

1st step: *Identification of appropriate **FSI** setting for numerical benchmarking*

2nd step: ***FSI** benchmark setting due to experimental studies*

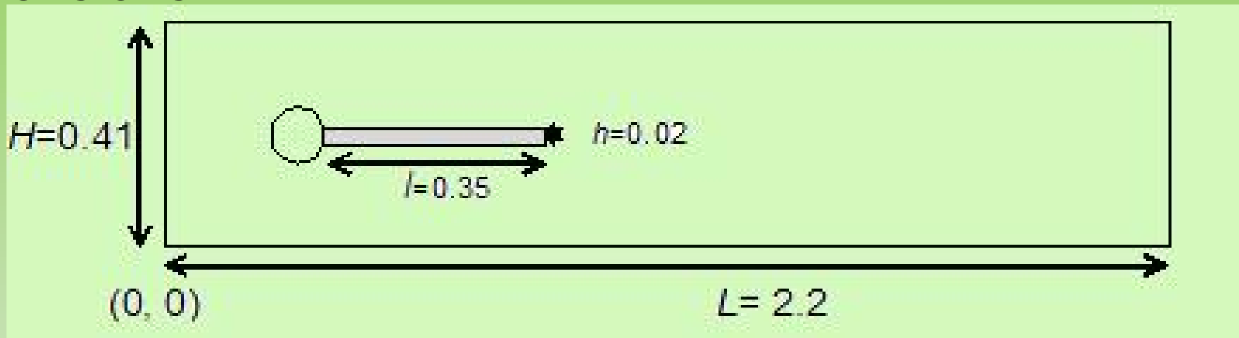
3rd step: *Extension to **FSI**-Optimisation benchmark*

Requirements for numerical FSI benchmarking

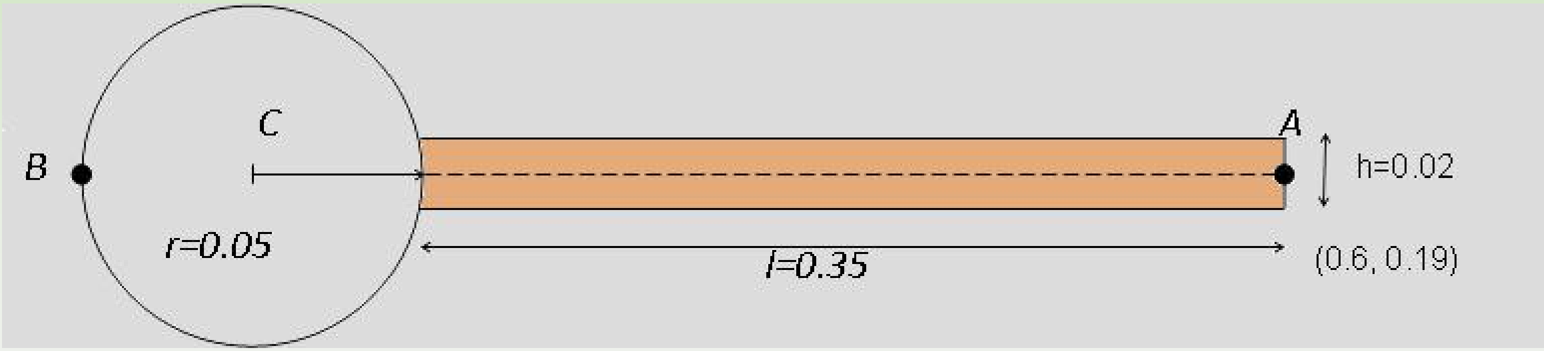
- Mainly based on the successful DFG *flow around cylinder* benchmark
- *Realistic materials*
  - **Incompressible Newtonian fluid,** laminar flow regime
  - Elastic solid, large deformations
- *Comparative evaluation*
  - Setup with periodical oscillations
  - Non-graphically based quantities
- *Computable configurations*
  - Laminar flow
  - Reasonable aspect ratios
  - Simple geometry (2D)

Computational domain

- Domain dimensions

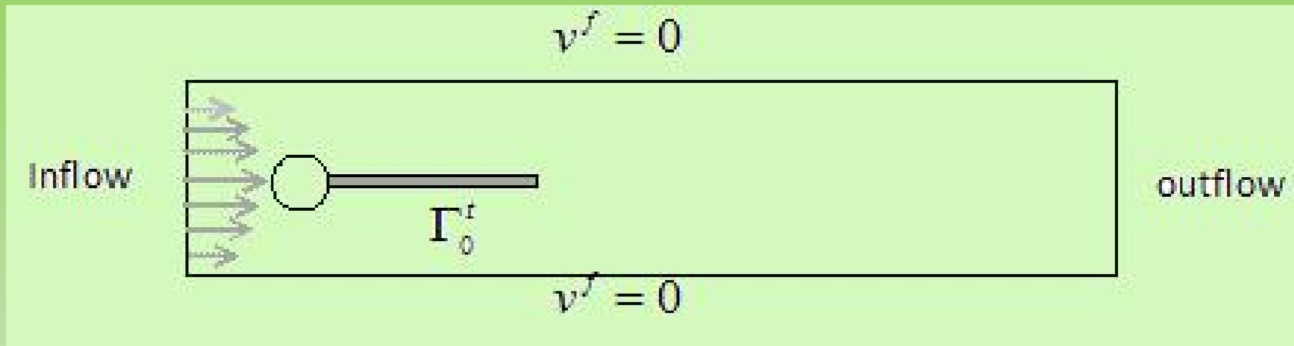


- Detail of the submerged structure



$$A(t=0)=(0.6,0.2),\quad B=(0.15,0.2), C=(0.2,0.2)$$

Boundary and initial conditions



Inflow Parabolic velocity profile is prescribed at the left end of the channel

Outflow Condition can be chosen by the user, assuming zero reference pressure (*stress free* or *do nothing*)

otherwise The *no-slip* condition is prescribed for the fluid on the other boundary parts. i.e. top and bottom wall and cylinder

Initial Zero velocity in the fluid and no deformation of the structure + smooth increase of the inflow profile

Fluid and structure properties

- **Incompressible** fluid with density ρ^f

$$\rho^f \frac{\partial \mathrm{v}^f}{\partial t} + \rho^f \left(\nabla \mathrm{v}^f\right)\mathrm{v}^f = \operatorname{div} \sigma^f \qquad \text{in } \Omega_t^f$$

$$\operatorname{div} \mathrm{v}^f = 0$$

$$\sigma^f = -p^f \mathrm{I} + \rho^f \nu^f \left(\nabla \mathrm{v}^f + \nabla \mathrm{v}^{f\,\mathrm{T}}\right)$$

- Elastic material with density $\rho^s, \mathrm{F} = \mathrm{I} + \nabla \mathrm{u}^s, J = \det \mathrm{F}$: St. Venant – Kirchhoff material

$$\rho^s \frac{\partial^2 \mathrm{u}^s}{\partial t^2} = \operatorname{div}\left(\sigma^s \mathrm{F}^{-\mathrm{T}}\right) \qquad \text{in } \Omega^s$$

$$\sigma^s = \frac{1}{J} \mathrm{F}\left(\lambda^s (\mathrm{tr}\mathrm{E})\mathrm{I} + 2\mu^s \mathrm{E}\right)\mathrm{F}^{\mathrm{T}}$$

$$\mathrm{E} = \frac{1}{2}\left(\mathrm{F}^{\mathrm{T}}\mathrm{F} - \mathrm{I}\right)$$

Suggested material parameters

solid

ρ^s density

ν^s Poisson ratio

μ^s shear modulus

fluid

ρ^f density

ν^f kinematic viscosity

Parameter	polybutadiene & glycerine	polypropylene & glycerine
$\rho^s [10^3 \mathrm{kg/m^3}]$	0.91	1.1
ν^s	0.50	0.42
$\mu^s [10^6 \mathrm{kg/ms^2}]$	0.53	317
$\rho^f [10^3 \mathrm{kg/m^3}]$	1.26	1.26
$\nu^f [10^{-3} \mathrm{m^2}/s]$	1.13	1.13

Parameter	FSI1	FSI2	FSI3
$\rho^s [10^3 \mathrm{kg/m^3}]$	1	1	1
ν^s	0.4	0.4	0.4
$\mu^s [10^6 \mathrm{kg/ms^2}]$	0.5	0.5	2.0
$\rho^f [10^3 \mathrm{kg/m^3}]$	1	1	1
$\nu^f [10^{-3} \mathrm{m^2}/s]$	1	1	1
$\bar{U} [\mathrm{m}/s]$	0.2	1	2

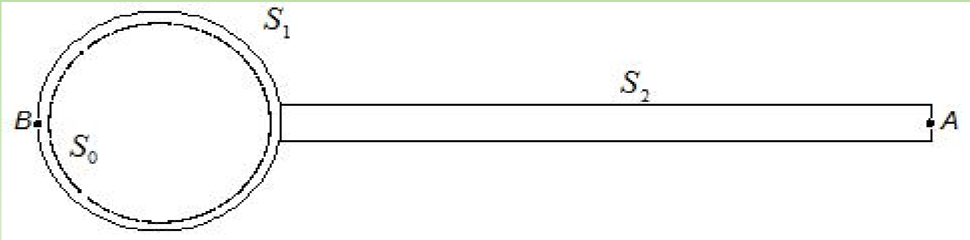
Parameter	FSI1	FSI2	FSI3
$\beta = \frac{\rho^s}{\rho^f}$	1	1	1
ν^s	0.4	0.4	0.4
$\mathrm{Ae} = \frac{E^s}{\rho^f \bar{U}^2}$	3.5×10^4	1.4×10^3	1.4×10^3
$\mathrm{Re} = \frac{\bar{U}d}{\nu^f}$	20	100	200
$\bar{U} [\mathrm{m/s}]$	0.2	1	2

Quantities of interest

- The position $A(t) = (x(t), y(t))$ of the end of the structure\item pressure difference between the points *A(t)* and *B*

$$\Delta p^{AB}(t) = p^{B}(t) - p^{A(t)}(t)$$

- Forces exerted by the fluid on the *whole body*, i.e. lift and drag forces acting on the cylinder and the structure together



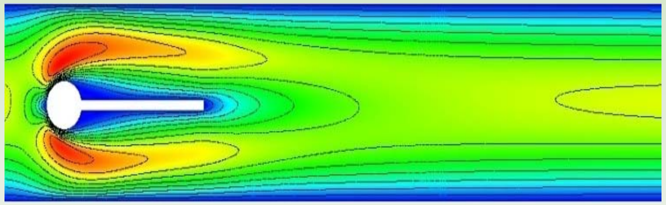
$$(F_D, F_L) = \int_S \sigma\, ndS = \int_{S_1} \sigma^f\, ndS + \int_{S_2} \sigma^{f|S}\, ndS = \int_{S_0} \sigma\, ndS$$

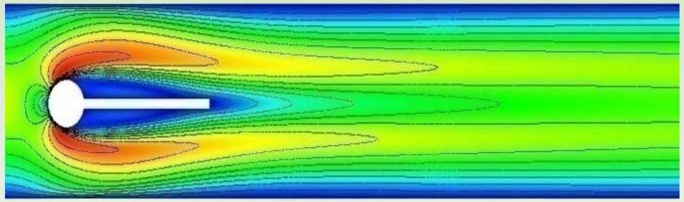
- Frequency and maximum amplitude
- Compare results for *one* full period and 3 different levels of spatial discretization *h* and 3 time step sizes Δt

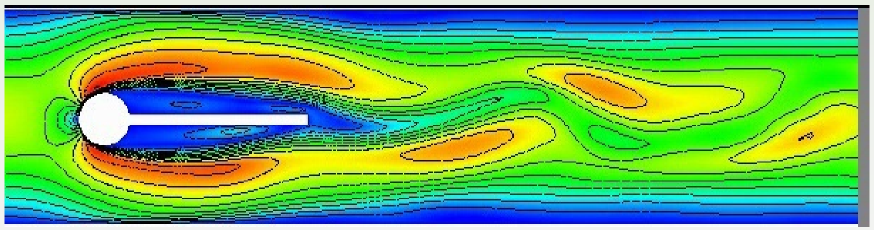
1.Step: CFD tests for validation

	CFD1	CFD2	CFD3
$\rho^f [10^3 \mathrm{kg/m^3}]$	1	1	1
$\nu^f [10^{-3} \mathrm{m^2/s}]$	1	1	1
$\bar{U} [\mathrm{m/s}]$	0.2	1	2
$\mathrm{Re} = \frac{\bar{U} d}{\nu^f}$	20	100	200
$\bar{U} [\mathrm{m/s}]$	0.2	1	2

Test	Drag	Lift
CFD1	14.29	1.119
CFD2	136.7	10.53
CFD3	439.4 ± 5.618 [4.395]	-11.89 ± 437.8 [4.395]



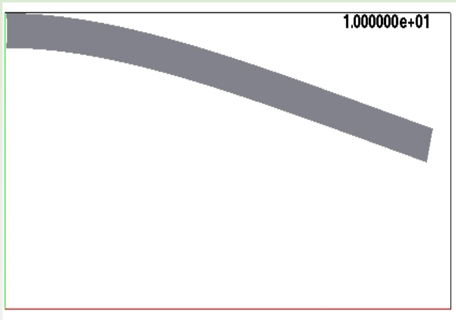


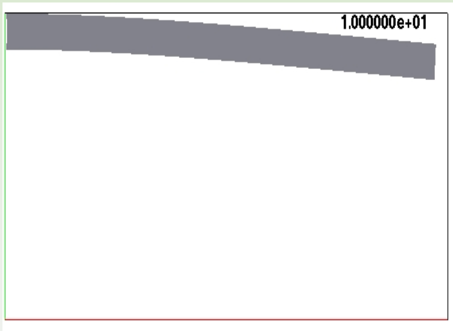


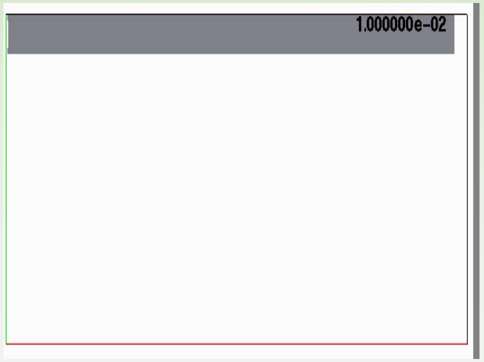
2.Step: CSM tests for validation

	CSM1	CSM2	CSM3
$\rho^s[10^3 \mathrm{kg/m}^3]$ ν^s $\mu^s[10^6 \mathrm{kg/ms}^2]$	**1** **0.4** **0.5**	**1** **0.4** **2.0**	**1** **0.4** **0.5**
$g[\mathrm{m/s}^2]$	**2**	**2**	**2**
$\beta = \frac{\rho^s}{\rho^f}$ ν^s $\mathrm{E}^s[10^6 \mathrm{kg/ms}^2]$	**1** **0.4** **1.4**	**1** **0.4** **5.6**	**1** **0.4** **1.4**
$g[\mathrm{m/s}^2]$	**2**	**2**	**2**

test	ux of A [$\times 10^{-3}$ m]	uy of A [$\times 10^{-3}$ m]
CSM1	-7.187	-66.10
CSM2	-0.4690	-16.97
CSM3	-14.305 ± 14.305 [1.0995]	-63.607 ± 65.160 [1.0995]



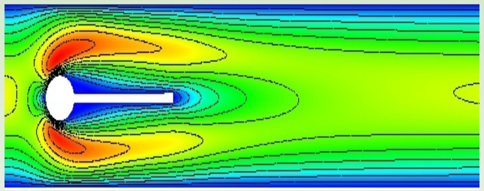


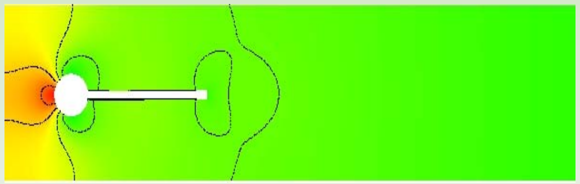


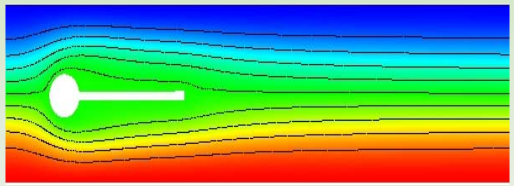
FSI1: steady, small deformations

parameter	FSI1	FSI2	FSI3
$\rho^s[10^3\mathrm{kg/m^3}]$ ν^s $\mu^s[10^6\mathrm{kg/ms^2}]$	1 0.4 0.5	1 0.4 0.5	1 0.4 2.0
$\rho^s[10^3\mathrm{kg/m^3}]$ $\nu^s[10^{-3}\mathrm{m^2/s}]$	1 1	1 1	1 1
$\bar{U}[\mathrm{m/s}]$	0.2	1	2

parameter	FSI1	FSI2	FSI3
$\beta = \frac{\rho^s}{\rho^f}$ ν^s $\mathrm{Ae} = \frac{E^s}{\rho^f \bar{U}^2}$	1 0.4 3.5×10^4	1 0.4 1.4×10^3	1 0.4 1.4×10^3
$\mathrm{Re} = \frac{\bar{U}d}{\nu^f}$ $\bar{U}[\mathrm{m/s}]$	20 0.2	100 1	200 2

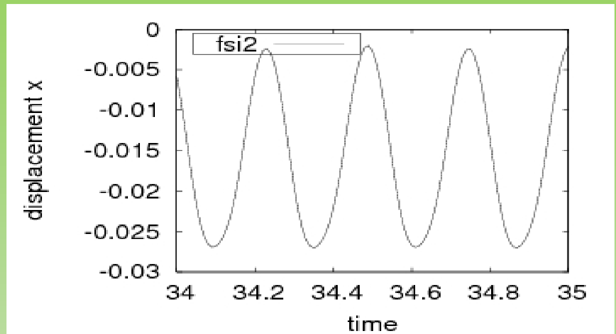


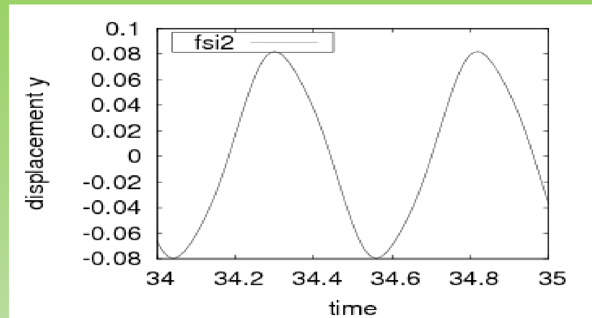


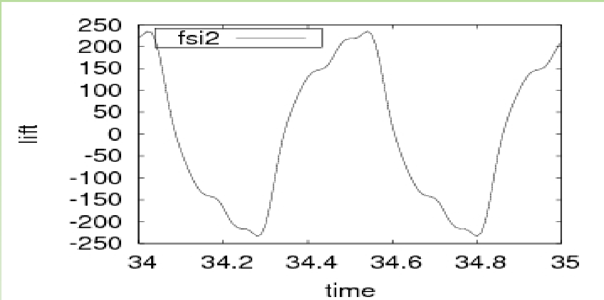


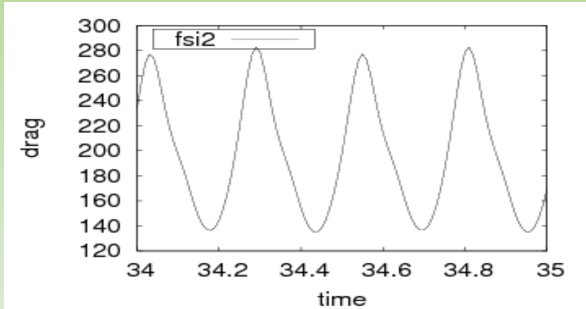
	ux of A [$\times10^{-3}$m]	uy of A [$\times10^{-3}$m]	drag	lift
FSI1	0.02270493	0.8208773	14.29426	0.763746

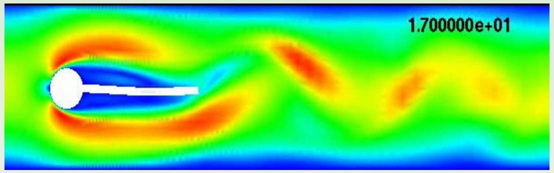
FSI2: large deformations, periodical oscillations

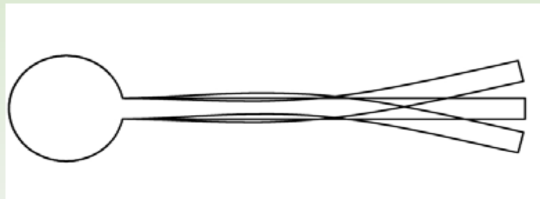






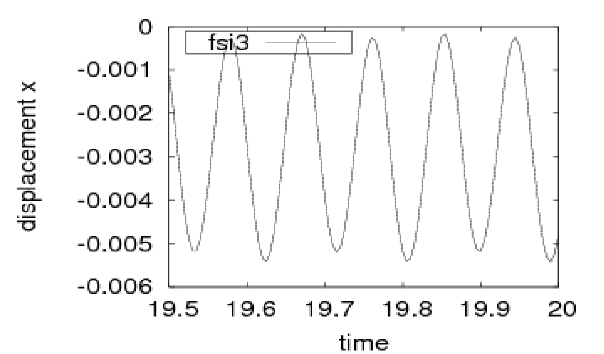


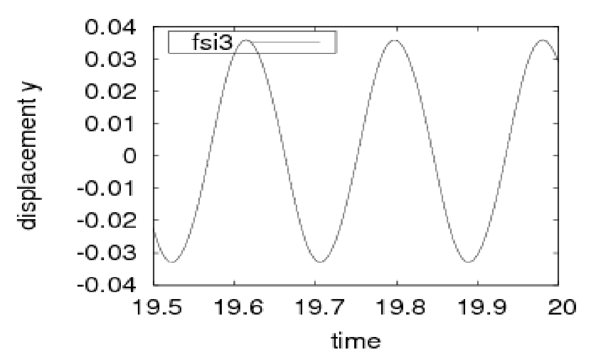


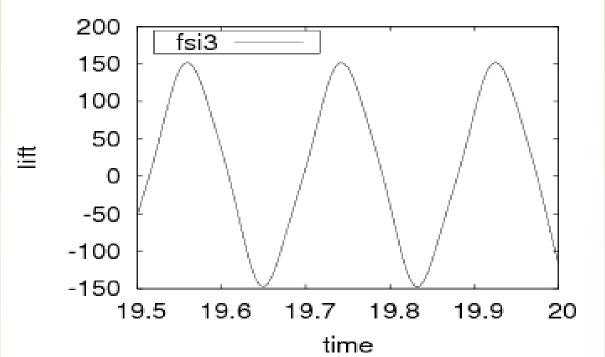


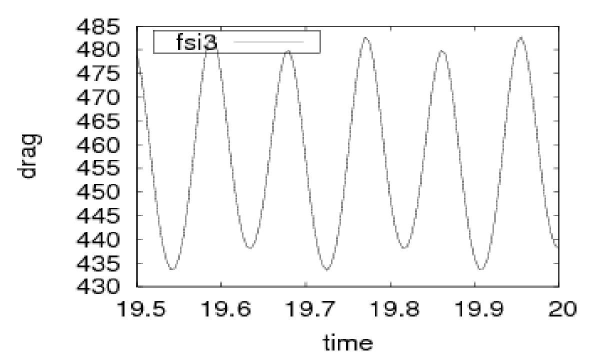
Test	ux of A [$\times 10^{-3}$ m]	uy of A [$\times 10^{-3}$ m]	drag	lift
FSI2	$-14.85 \pm 12.70 [3.86]$	$1.30 \pm 81.7 [1.93]$	$215.06 \pm 77.65 [3.86]$	$0.61 \pm 237.8 [1.93]$

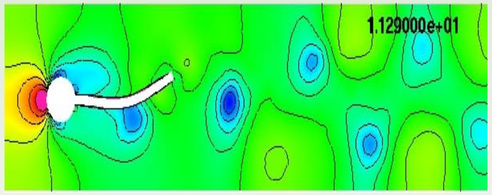
FSI3: large deformations, complex oscillations

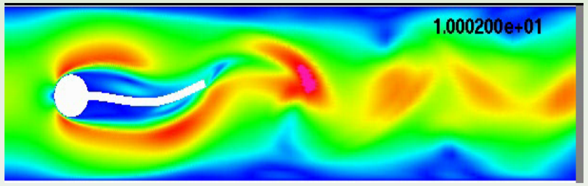


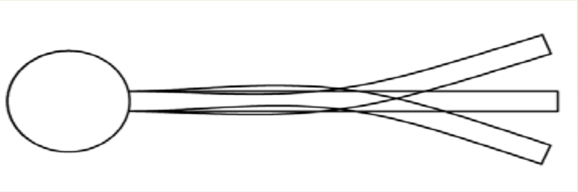












Test	ux of A [$\times 10^{-3}$m]	ux of A [$\times 10^{-3}$m]	drag	lift
FSI3	$-2.88 \pm 2.72[10.93]$	$1.47 \pm 34.99[5.46]$	$460.5 \pm 27.74[10.93]$	$2.50 \pm 153.91[5.46]$

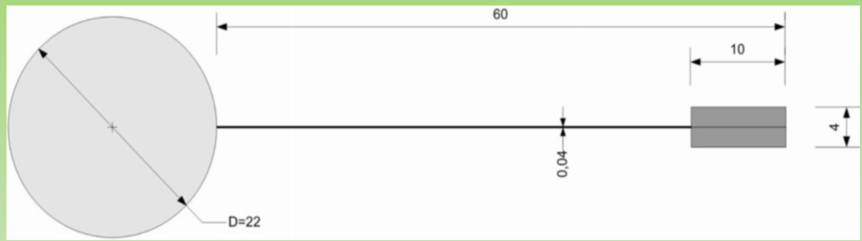
Status of numerical benchmarking

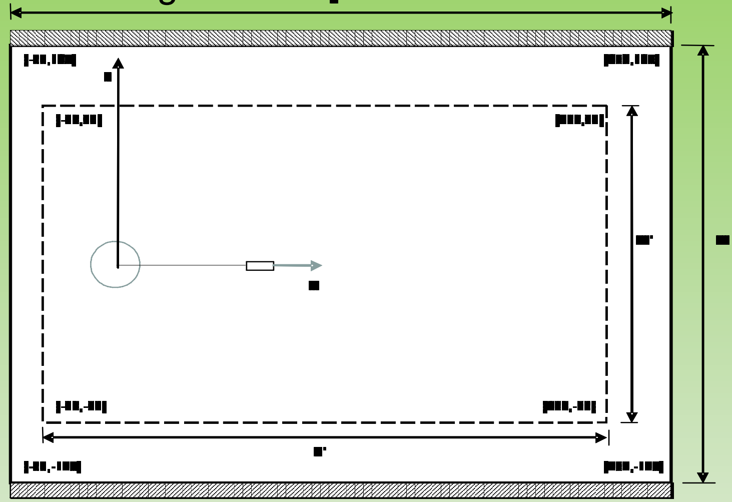
- Subtests for validating CFD and CSM components are available:
  - CSM1-3: "OK"
  - CFD1: "easy" → *Re=20*
  - CFD2: (also) "easy" → *Re=100*
  - CFD3: "non-trivial" → *Re=200*
- FSI settings with desired properties:
  - FSI1: "simple" → for validation only
  - FSI3: "hard" → due to CFD3
  - FSI2: fully oscillating while CFD2 (≈same *Re* number!) is steady

    ⇒ Excellent check for interaction mechanisms
- **Evaluation and comparison of mathematical and algorithmic components - *everybody is invited to participate.***

FSI4: Benchmarking of experimental data

- **Flustruc experiment**, Erlangen, http://www.lstm.uni-erlangen.de/flustruc/





fluid parameters	
density of the fluid kinematic viscosity	1.05e-6 [kg/mm^3] 164.0

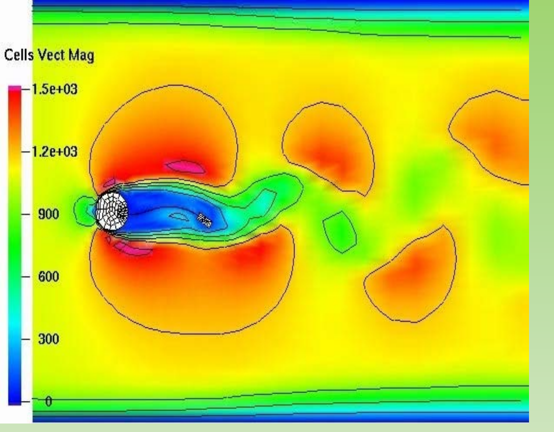
solid parameters	
density of the beam (steel) density of the rear mass shear modulus poisson ratio	7.85e-6[kg/mm^3] 7.8e-6 [kg/mm^3] 7.58e13 0.3

geometry parameter		value [mm]
channel length	*L*	338.0
channel width	*W*	240.0
cylinder center position	*C*	(0.0, 0.0)
cylinder radius	*R*	11.0
elastic structure length	*l*	50
elastic structure thickness	*w*	0.04
rear mass length	*w'*	10.0
rear mass thickness	*h'*	4.0
reference point (at t=0)	*A*	(71.0, 0.0)
reference point	*B*	(11.0, 0.0)

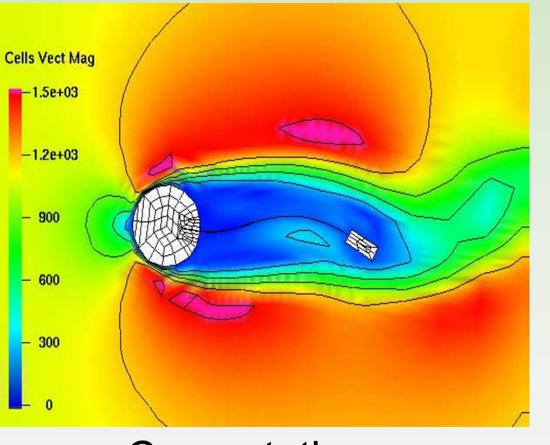
FSI4: New configuration

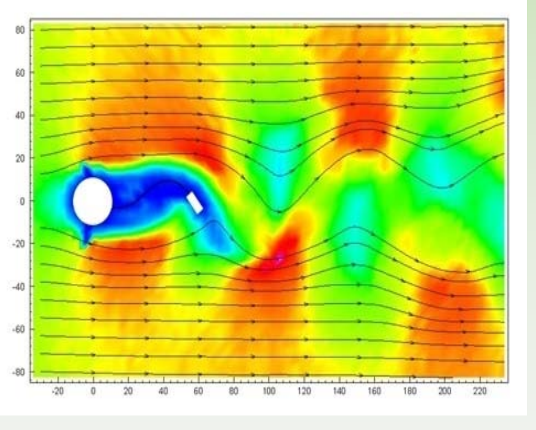
+ Laminar Flow (glycerine)
+ “2D“ flow and deformation
- Rotational degree of freedom
- Large aspect ratio (thin structure),
- Corners

Laminar: Velocity=1.07 m/s, Re=140



Zoomed

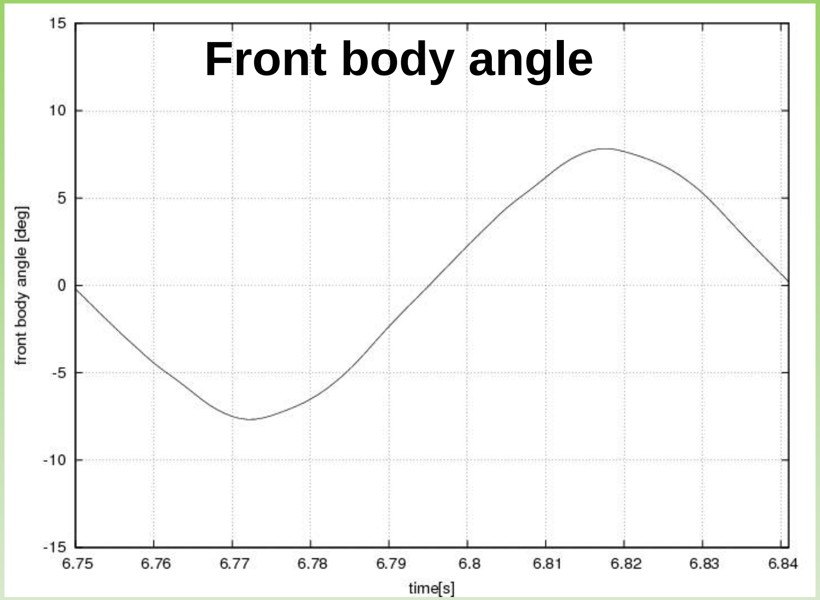




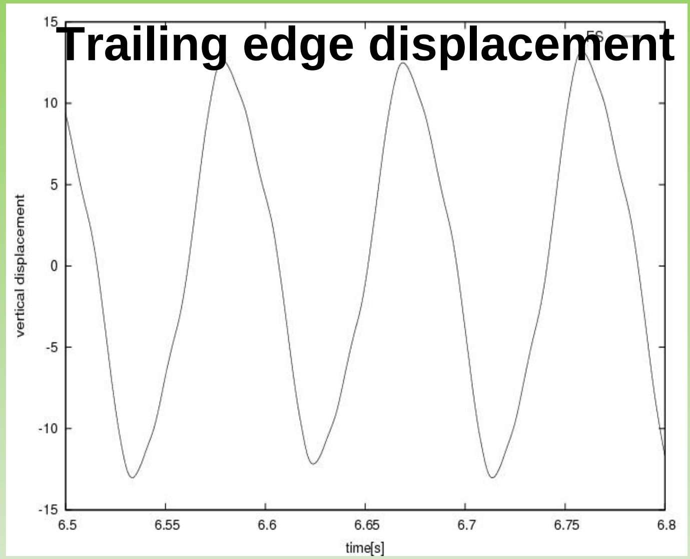
Flustruc experiment, Erlangen

Computation

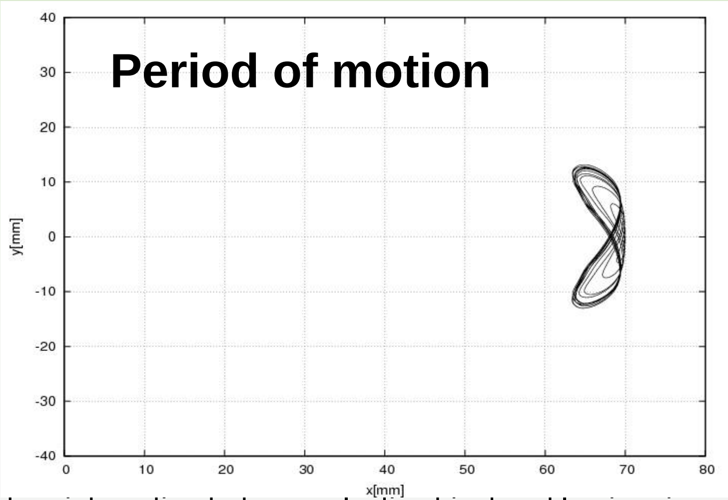
FSI4: Plots



Front body angle
front body angle [deg]
time[s]
15
10
5
0
-5
-10
-15
6.75
6.76
6.77
6.78
6.79
6.8
6.81
6.82
6.83
6.84



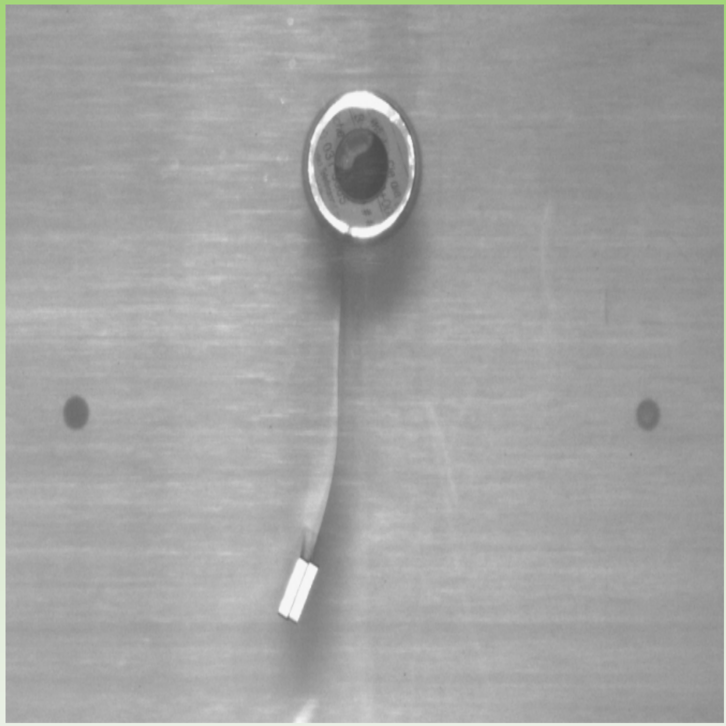
Trailing edge displacement
vertical displacement
time[s]
15
10
5
0
-5
-10
-15
6.5
6.55
6.6
6.65
6.7
6.75
6.8



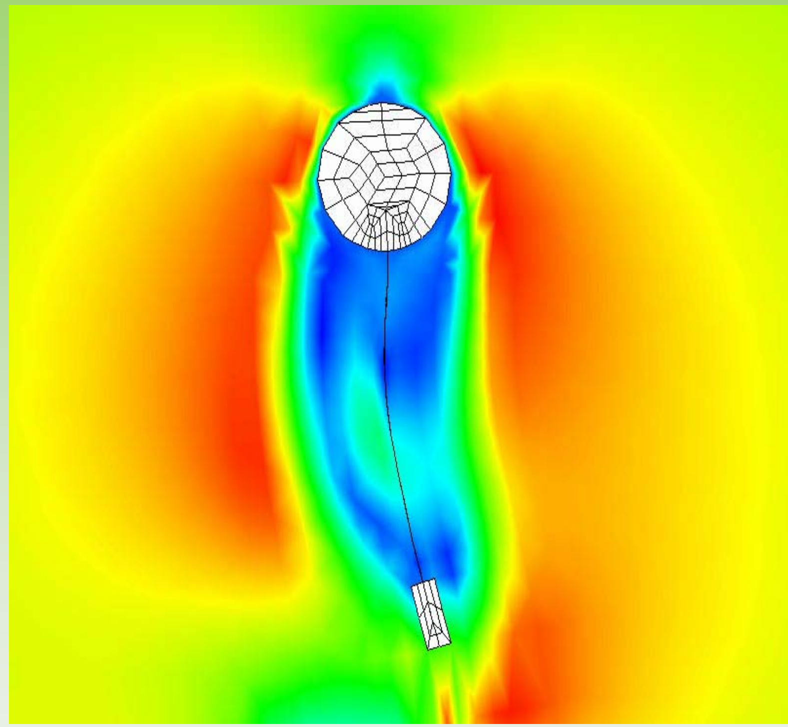
Period of motion
y[mm]
x[mm]
40
30
20
10
0
-10
-20
-30
-40
0
10
20
30
40
50
60
70
80

FSI4: Experiment and numerical simulations

Laminar: Velocity=1.07 m/s, Re=140



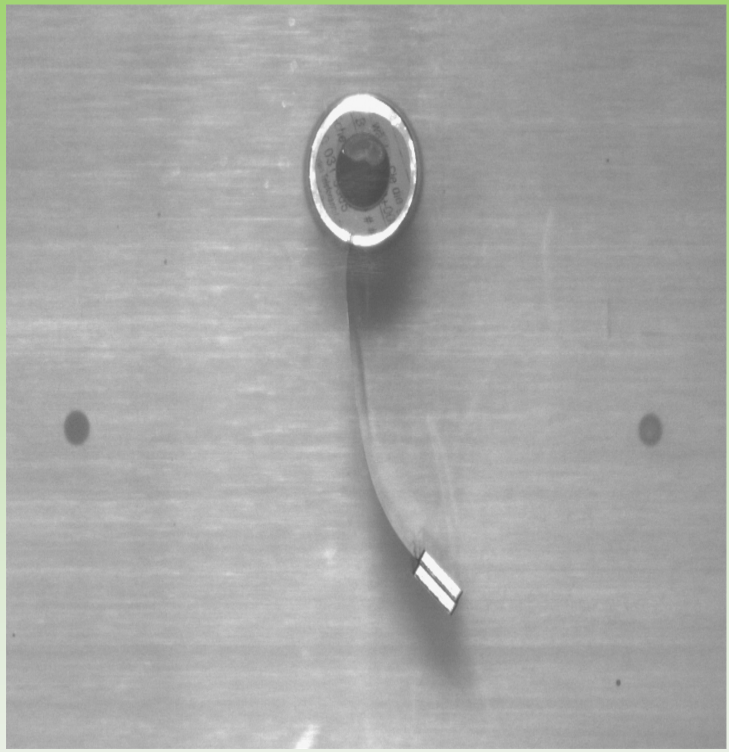
Experiment



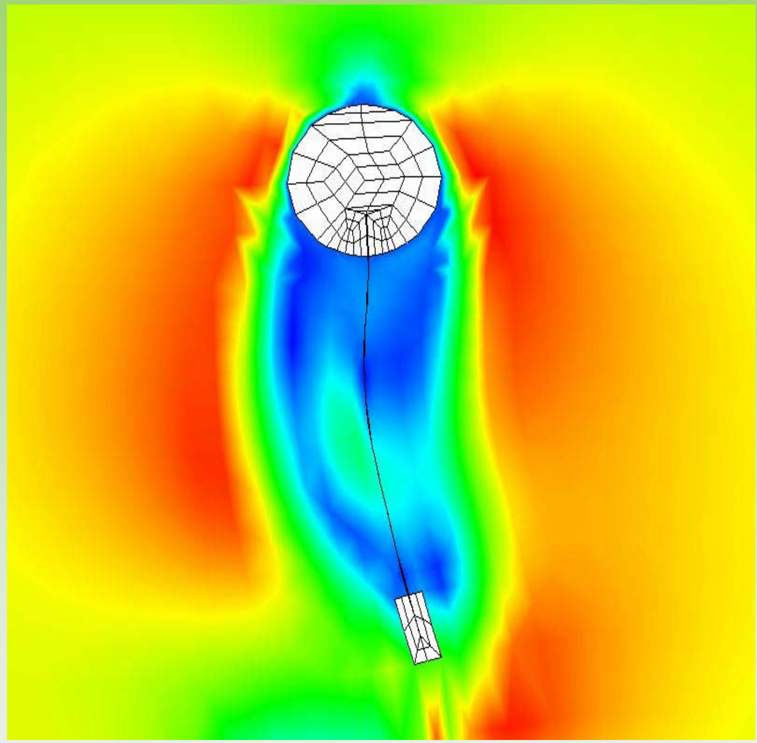
Numerical

FSI4: Experiment and numerical simulations

Laminar: Velocity=1.45 m/s, Re=190



Experiment



Numerical

FSI Optimization

- The main design aims could be
  - **I) Drag/Lift minimization**
  - II) Minimal pressure loss
  - III) Minimal nonstationary oscillations

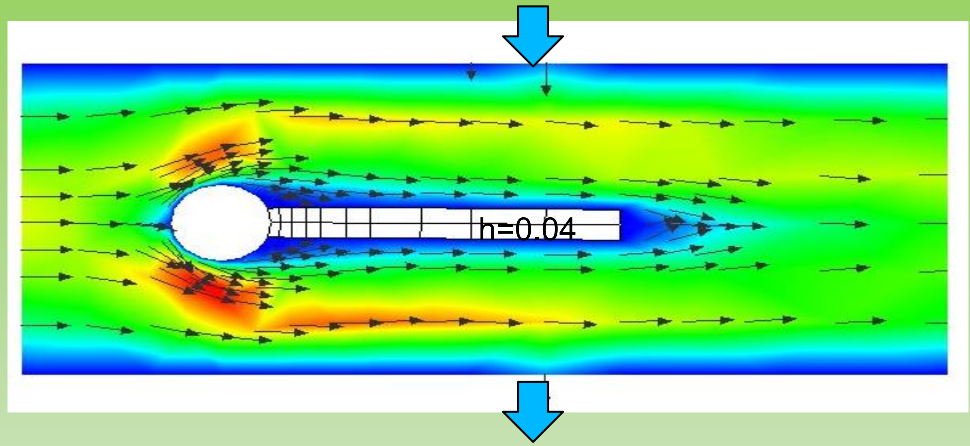
- To reach these aims, we might allow
  1. **Boundary control of inflow section**
  2. Change of geometry: elastic channel walls or length/thickness of elastic beam
  3. Optimal control of volume forces

- Optimal control of nonstationary flow might be hard for the starting

- Results for the moment are combination of **I**)-III) with **1**)-3)

FSI OPT 1

- Uncontrolled flow



	ux of A [$\times 10^{-3}$ m]	uy of A [$\times 10^{-3}$ m]	drag	lift
FSI1	0.0227	0.8209	14.295	0.7638

Lift $\neq$ 0

- Aim: $\text{minimize}\left(lift^2 + \alpha V^2\right)$

w.r.t V1, V2.

V1 velocity from top

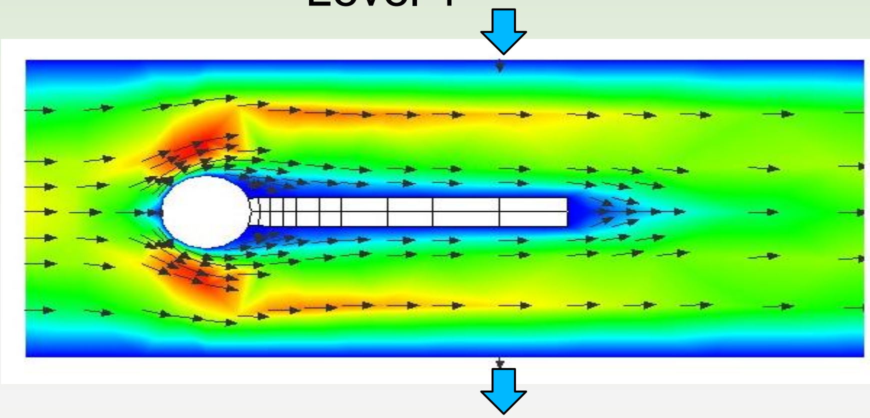
V2 velocity from below

FSI OPT 1

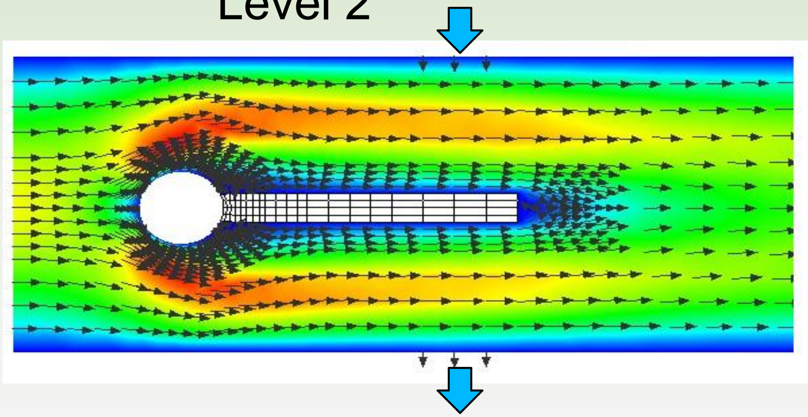
- TESTS for FSI 1 (Boundary control)

	Level 1				Level 2			
α	Iter steps	extreme point	drag	Lift	Iter steps	extreme point	drag	Lift
1e0	57	(3.74e-1,3.88e-1)	1.5471e+01	8.1904e-1	59	(3.66e-1,3.79e-1)	1.5550e+01	7.8497e-1
1e-2	60	(1.04e0,1.06e0)	1.5474e+01	2.2684e-2	59	(1.02e0,1.04e0)	1.5553e+01	2.1755e-2
1e-4	73	(1.06e0,1.08e0)	1.5474e+01	2.3092e-4	71	(1.04e0,1.05e0)	1.5553e+01	2.2147e-4
1e-6	81	(1.06e0,1.08e0)	1.5474e+01	2.3096e-6	86	(1.04e0,1.05e0)	1.5553e+01	2.2151e-6

Level 1

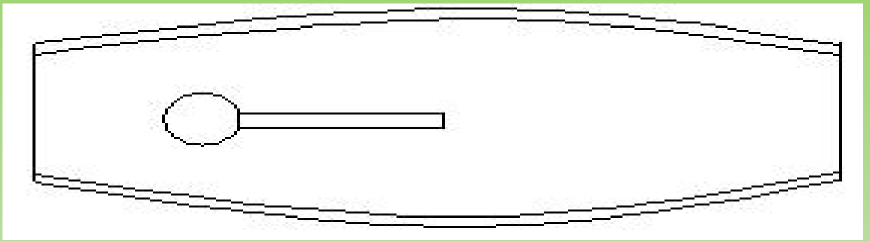


Level 2



Outlook

- **Further examples might be:**



1. $\text{minimize}\left(lift^2 + \alpha V^2\right)$ **for deformed case**
2. **Pressure loss minimize:** $\text{minimize}\left(p_{in} - p_{out}\right)$

w.r.t elastic deformation of the wall

or

w.r.t geometrical and material properties of beam