FOR 493 - P3: A monolithic multigrid FEM solver for fluid structure interaction

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Overview

1. Current status
2. Rotating cylinder
3. 3D
4. FEASTsolid

Overview

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Current status of our FSI solver

- FEATFLOW Q_2/P_1^{disc}
  - monolithic, fully coupled FEM, higher order
  - fully implicit 2nd order discretization in time (Crank-Nicholson)
  - Newton method for the coupled system (Jacobian matrix via divided differences)
  - coupled geometric-MG solver with Vanka-like smother
  - adaptive time step control, a priori space-adapted mesh
  - $\rightarrow$ high accuracy
- FEAST Q_1/Q_1
  - fully coupled, stabilised FEM
  - HPC techniques (tensor product meshes, MPI, co-processor acceleration)
  - generalised MG/DD solvers
  - $\rightarrow$ high numerical and parallel efficiency

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2. **Rotating cylinder**
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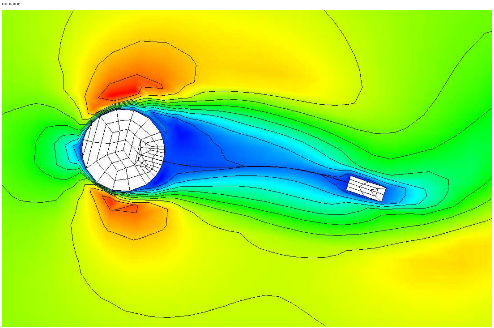
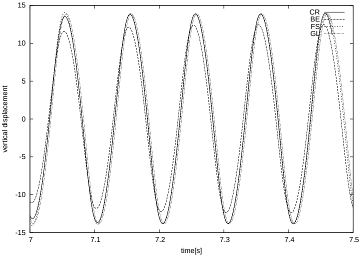
Towards 'rotating cylinder'...

- exact benchmark geometry (beam thickness 0.04mm)
- flow velocity 1450mm/s (Re=195)
- backward Euler time stepping scheme



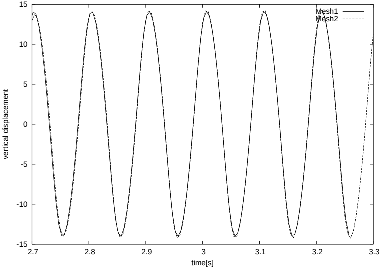
Towards 'rotating cylinder'...

- modified benchmark geometry (beam thickness 1.00mm)
- velocity 1130mm/s (Re=145)
- comparing four time-stepping schemes: 1) backward Euler, 2) Crank-Nicholson, 3) fractional-step-θ, 4) modified fractional-step-θ (Turek,Hron,Glowinski: *Numerical Analysis of a New Time-Stepping θ-Scheme for Incompr. Flow Simulations*, 2005):

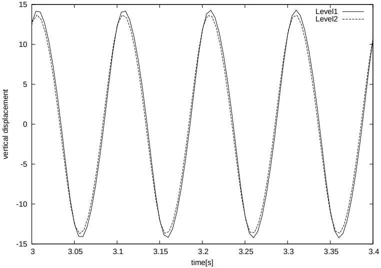


Towards ‘rotating cylinder’...

- modified benchmark geometry (beam thickness 1.00mm)
- velocity 1130mm/s (Re=145)
- Validation of the modified fractional-step-θ scheme:



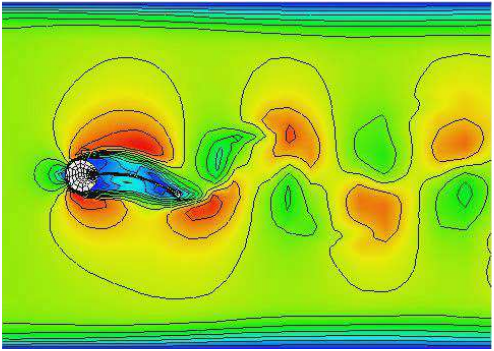
mesh independency



level independency

Towards 'rotating cylinder'...

- modified benchmark geometry (beam thickness 1.00mm)
- velocity 1130mm/s (Re=145)
- Validation of the modified fractional-step-θ scheme:



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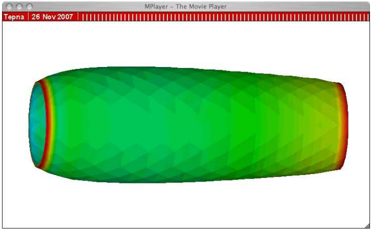
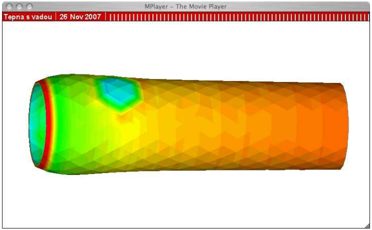
Extension to 3D

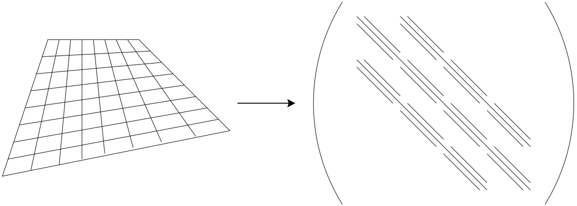
- first test: pulse flow through a soft pipe (homogeneous and inhomogeneous material)
- next step: application to benchmark configuration





'data moving ≫ data processing'

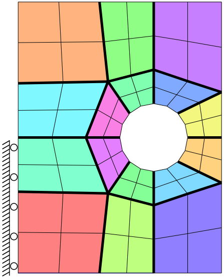
- unstructured meshes ⇒ indirect adressing
  ⇒ expensive memory access ⇒ poor MFLOP/s rates
- FEAST uses generalised tensor product meshes

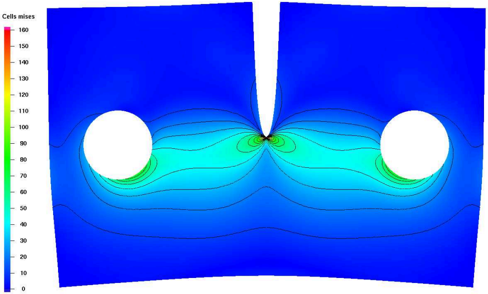


- rowwise numbering
  ⇒ exactly 9 matrix bands for bilinear elements
- direct adressing, caching
- optimised Linear Algebra routines (SPARSE BANDED BLAS)

FEAST's Mesh & Matrix Concept

More complex domains by joining several TP meshes ('macros')



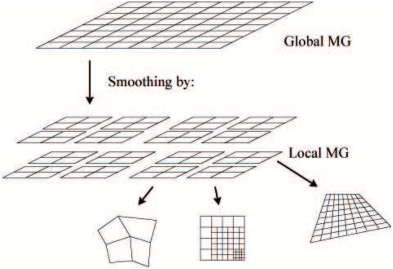


Here:

- 64 macros (=64 local matrices), each macro refined 10x
- distributed over 16 processors
- $\Rightarrow 1.34 \cdot 10^8$ unknowns in total

FEAST uses generalised Multigrid/Domain Decomposition methods

ScaRC
Scalable Recursive Clustering



- global MG smoothed by local MG
- locally adapted solution methods
- recursively hide local mesh irregularities

⇒ high numerical and parallel efficiency

- Basic idea: Reduction to solution of scalar problems
- Example: Linear Elasticity for compressible material
- Separate displacement ordering ⇒ block-structured linear system:

$$\begin{pmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} \\ \mathbf{K}_{12}^{T} & \mathbf{K}_{22} \end{pmatrix} \begin{pmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \end{pmatrix}$$

- $\mathbf{K}_{11}$ and $\mathbf{K}_{22}$ correspond to scalar elliptical operators
- Basic iteration: block-preconditioned Richardson method

$$\mathbf{u}^{k+1} = \mathbf{u}^{k} + \tilde{\mathbf{K}}^{-1}(\mathbf{f} - \mathbf{K}\mathbf{u}^{k}),$$

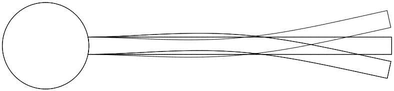
e. g. Block-Jacobi $\quad \tilde{\mathbf{K}}^{-1} = \begin{pmatrix} \mathbf{K}_{11}^{-1} & \mathbf{0} \\ 0 & \mathbf{K}_{22}^{-1} \end{pmatrix}$

Exploit FEAST concepts!

- Acceleration: Krylov-space methods

FEASTsolid: Towards FSI benchmark

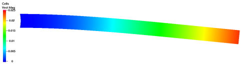
CSM part of Fluid-Solid-Interaction-Benchmark: Beam attached to cylinder in channel (DFG FOR 493)



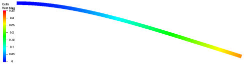
- Comparing three different lengths:

0.08 × 0.02 m

0.32 × 0.02 m

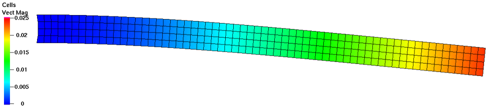


1.28 × 0.02 m

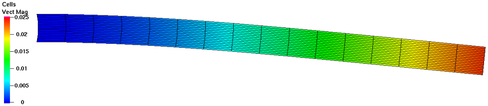


- Comparing two meshings:

isotropic elements:



anisotropic elements:



isotropic elements		0.08 x 0.02 m 4 macros		0.32 x 0.02 m 16 macros		1.28 x 0.02 m 64 macros	
Solver	#elem	it	sec	it	sec	it	sec
BiCG BGS	65 K	13	3.6	28	9.1	132	68.6
	262 K	14	14.9	28	32.5	113	167.3
	1049 K	14	62.3	31	142.7	114	608.8

anisotropic elements		0.08 x 0.02 m 1 macro		0.32 x 0.02 m 1 macro		1.28 x 0.02 m 1 macro	
Solver	#elem	it	sec	it	sec	it	sec
BiCG BGS	65 K	14	3.7	33	8.5	149	38.7
	262 K	16	18.9	35	38.4	154	169.1
	1049 K	14	71.9	34	156.3	210	1078.9

- Outer Krylov-subspace scheme with Block-GS preconditioning not sufficient (strong dependence on global anisotropy)

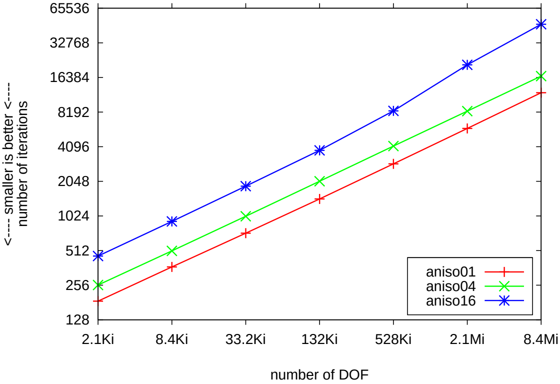
isotropic elements		0.08 × 0.02 m, 4 macros		0.32 × 0.02 m, 16 macros		1.28 × 0.02 m, 64 macros	
Solver	#elem	it	sec	it	sec	it	sec
BiCG BGS	65 K	13	3.6	28	9.1	132	68.6
	262 K	14	14.9	28	32.5	113	167.3
	1049 K	14	62.3	31	142.7	114	608.8
BiCG MG BGS	65 K	4	4.7	5	7.3	6	19.2
	262 K	4	18.5	5	24.2	6	51.8
	1049 K	4	73.7	5	88.9	5	126.5

anisotropic elements		0.08 × 0.02 m, 1 macro		0.32 × 0.02 m, 1 macro		1.28 × 0.02 m, 1 macro	
Solver	#elem	it	sec	it	sec	it	sec
BiCG BGS	65 K	14	3.7	33	8.5	149	38.7
	262 K	16	18.9	35	38.4	154	169.1
	1049 K	14	71.9	34	156.3	210	1078.9
BiCG MG BGS	65 K	4	4.2	5	5.2	9	9.4
	262 K	5	19.5	5	21.8	8	34.8
	1049 K	5	83.4	6	102.9	7	129.1

- Outer Krylov-subspace scheme with Block-GS preconditioning not sufficient (strong dependence on global anisotropy)
- Remedy: Use multigrid (applied to the vector-valued system!) as preconditioner

FEASTsolid: GPU support

- beam configuration is extremely ill-conditioned
- illustration: iterations of a plain CG solver (beams with isotropic elements)



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FEASTsolid: GPU support

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- beam configuration is extremely ill-conditioned
- but: GPU's single precision is not a problem

aniso04 level	Iters. CPU	GPU	y-Displacement CPU	GPU
8	4	4	-2.8083499E-3	-2.8083499E-3
9	4	4	-2.8083628E-3	-2.8083628E-3
10	4.5	4.5	-2.8083667E-3	-2.8083667E-3
aniso16				
8	6	6	-6.6216232E-2	-6.6216232E-2
9	**6**	**5.5**	-6.621655**1E-2**	-6.621655**2E-2**
10	5.5	5.5	-6.621750**1E-2**	-6.621750**2E-2**

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FEASTsolid: GPU support

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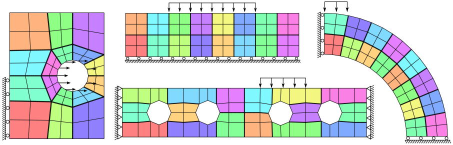
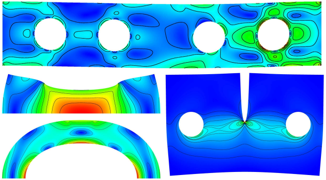
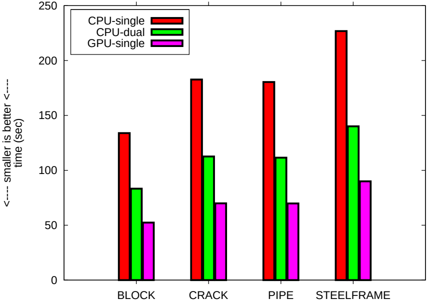
- Speedup tests ($1.34 \cdot 10^8$ unknowns, 16 compute nodes)







Node	Graphics Card
AMD Opteron Santa Rosa dual-core, 1.8 GHz 2 MiB L2 cache 800 W power supply	NVIDIA Quadro FX5600 600 MHz max. 171 W
8 GiB DDR2 667 **12.8 GB/s bandwidth**	1.5 GiB **76.8 GB/s bw**

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Thank you
for your attention!