Benchmark Proposal for Incompressible Two-Phase Flows

FEM techniques for interfacial flows

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Why Benchmarking

Why spend valuable time and effort to establish benchmark test cases?

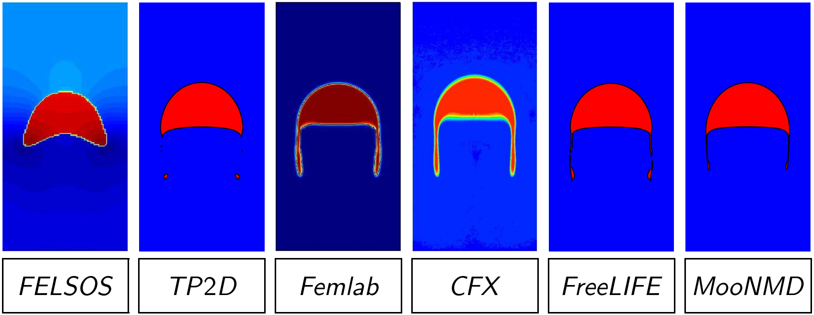
- Validation
- Comparison
- Evaluation

Why Benchmarking

Why spend valuable time and effort to establish benchmark test cases?

What is the CPU cost for achieving a certain accuracy?

Classical Benchmark: 2D Rising Bubble



S. Hysing, S. Turek, D. Kuzmin, N. Parolini, E. Burman, S. Ganesan, and L. Tobiska;
Proposal for quantitative benchmark computations of bubble dynamics,
Submitted to Int. J. Numer. Meth. Fluids.

Quantitative Comparison ?

Benchmark: 2D Rising Bubble

Aim: Development of **quantitative** two-phase flow benchmarks for rigorous evaluation of new and existing methods

Bubble benchmark quantities

- Center of mass
- Circularity
- Rise velocity

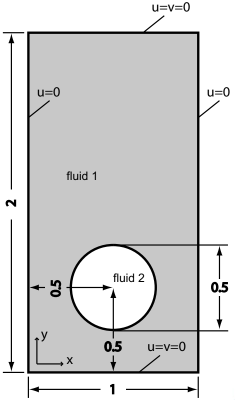


Figure: Initial configuration and boundary conditions for the test cases

Benchmark Quantities

- Center of mass:

$$\mathbf{x}_c = \int_{\Omega_2} \mathbf{x}\ d\mathbf{x} / \int_{\Omega_2} 1\ d\mathbf{x}$$

- Circularity:

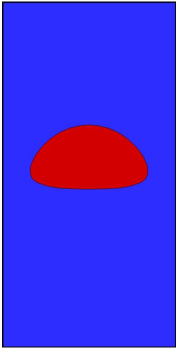
$$\not{c} = \frac{\text{perimeter of area-equivalent circle}}{\text{perimeter of } \Omega_2}$$

- Rise velocity:

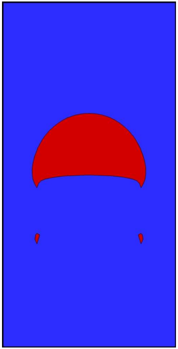
$$\mathbf{U} = \int_{\Omega_2} \mathbf{u}\ d\mathbf{x} / \int_{\Omega_2} 1\ d\mathbf{x}$$

Benchmark Test Cases

Test Case	1	2
ρ_1 (liquid)	1000	1000
ρ_2 (gas)	1	100
μ_1 (liquid)	10	10
μ_2 (gas)	0.1	1
g_y	-0.98	-0.98
σ	1.96	24.5
Re	35	35
Eo	125	10
ρ_1/ρ_2	1000	10
μ_1/μ_2	100	10



Test Case 1

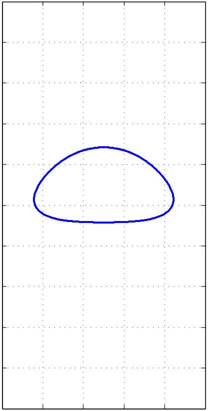


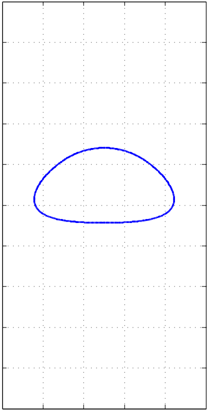
Test Case 2

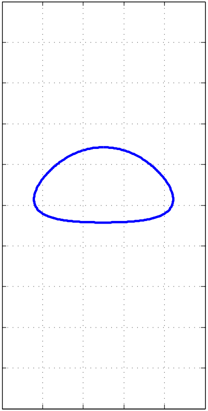
Participating Groups

	Group and Affiliation	Code/Method
1	Uni. Dortmund, Inst. of Applied Math. *S. Turek, D. Kuzmin, S. Hysing*	**TP2D** *FEM-Level Set*
2	EPFL Lausanne, Inst. of Analysis and Sci. Comp. *E. Burman, N. Parolini*	**FreeLIFE** *FEM-Level Set*
3	Uni. Magdeburg, Inst. of Analysis and Num. Math. *L. Tobiska, S. Ganesan*	**MooNMD** *FEM-ALE*

Visual comparison: Test Case 1

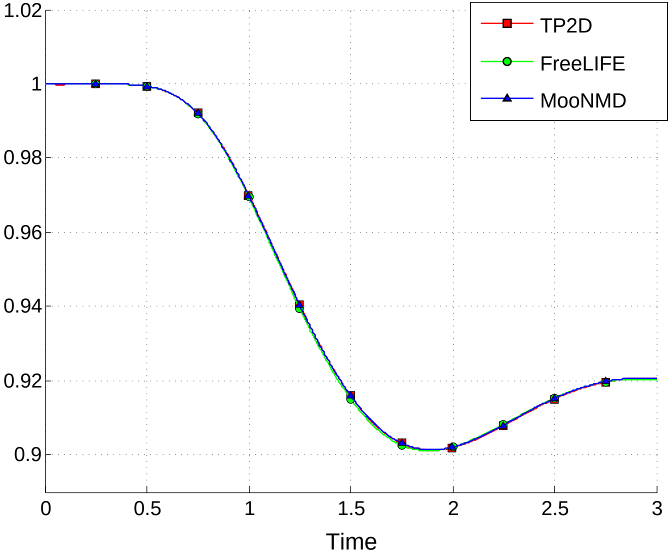






TP2D	FreeLIFE	MooNMD

Benchmark quantities - circularity



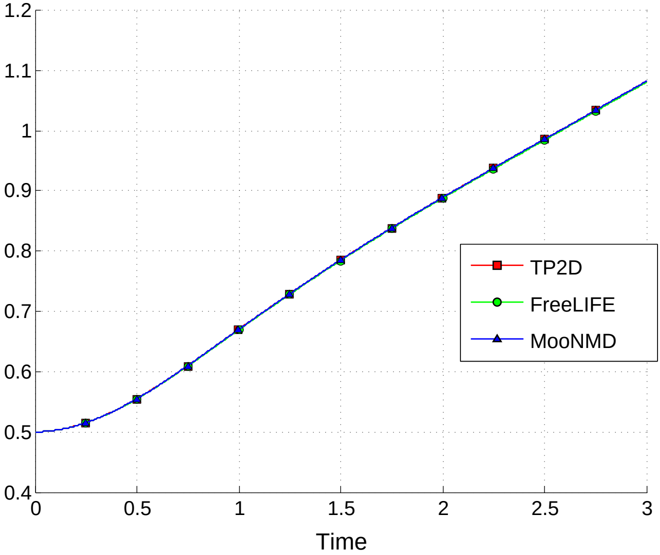
Benchmark quantities - circularity

GridLevel	1	2	3	4
Minimum circularity, $\not{c}_{min}$				
TP2D	0.9016	0.9014	0.9014	0.9013
FreeLIFE		0.9060	0.9021	0.9011
MooNMD	0.9022	0.9018	0.9014	0.9013
Incidence time, $t\|_{\not{c}=\not{c}_{min}}$				
TP2D	1.9234	1.8734	1.9070	1.9041
FreeLIFE		1.8375	1.9125	1.8750
MooNMD	1.8630	1.8883	1.9013	1.9000

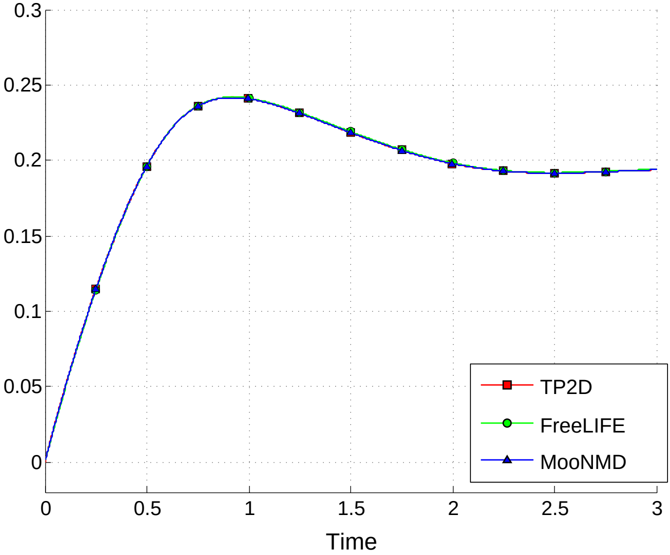
Reference target range

$$\not{c}_{min} = 0.9012 \pm 0.0001, \qquad t|_{\not{c}=\not{c}_{min}} = 1.90 \pm 0.01$$

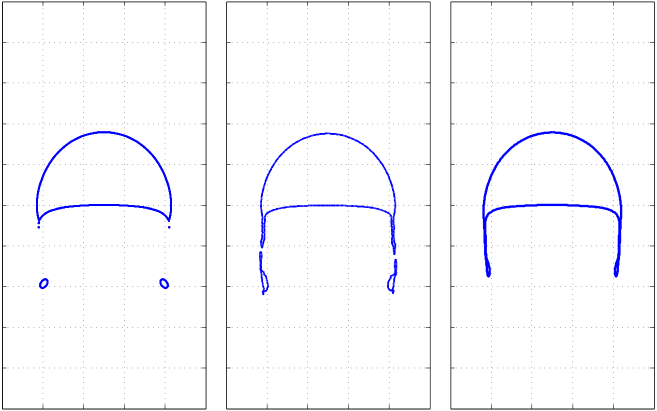
Benchmark quantities - center of mass



Benchmark quantities - rise velocity

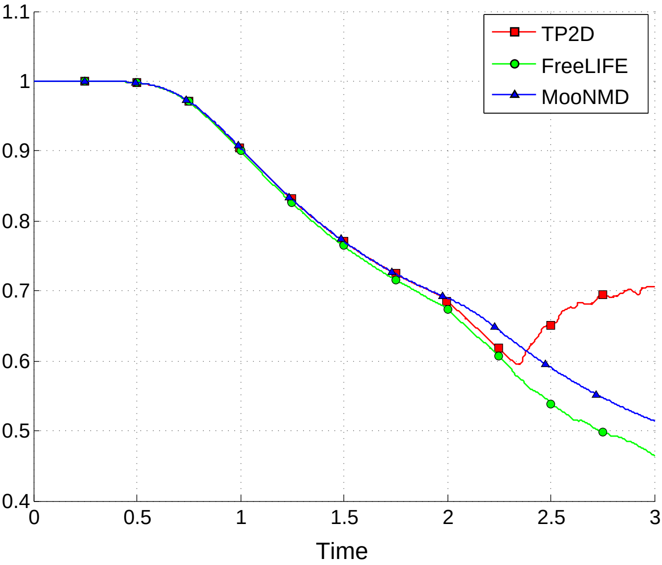


Visual comparison: Test Case 2

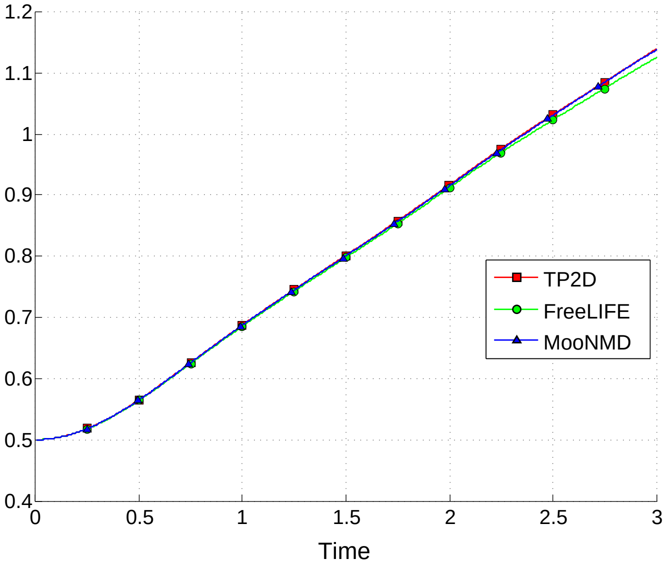


TP2D | *FreeLIFE* | *MooNMD*

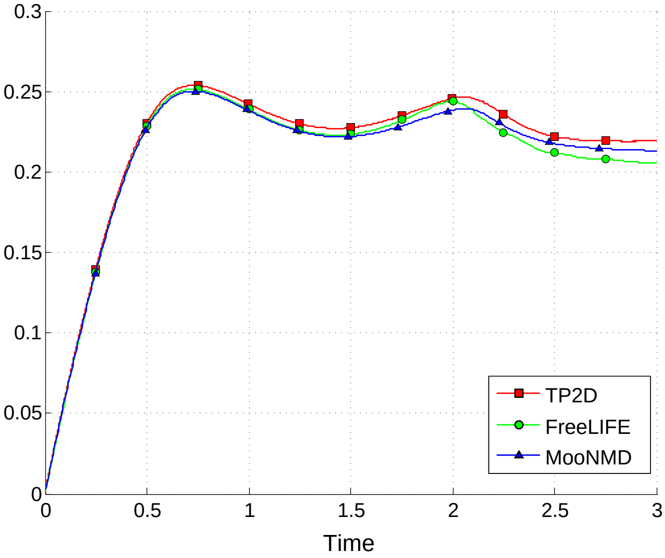
Benchmark quantities - circularity



Benchmark quantities - center of mass



Benchmark quantities - rise velocity



Conclusions

- Proposed two benchmarks for rigorous evaluation (IJNMF)
- Established target reference values for the first benchmark
- Hinted at difficulties during break up in the second benchmark
- New participants: M. Bussmann (FV-VOF); Phase field ???
- Email to `ture@featflow.de` for participation

Interfacial flows: Challenging even in 2D !

Challenges for Interfacial Flows

Numerical simulation of interfacial or two-phase flows with immiscible fluids poses some challenging problems

Issues

- Accurate interface tracking
- Mass conservation
- Resolution of discontinuous fluid properties
- Treatment of interfacial boundary conditions
- **Treatment of surface tension**

Influence of Surface Tension

- Interfacial or two-phase flow where capillary surface tension forces are dominant poses some challenging problems
- Surface tension effects are generally modeled both explicitly in time and space leading to the capillary time step restriction

$$\Delta t_{num}^{(ca)} < \sqrt{\frac{\langle\rho\rangle\, h^3}{2\pi\sigma}}$$

Goal

Remove the capillary time step constraint
while retaining a fully Eulerian interface description

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Goal

Remove the capillary time step constraint
while retaining a fully Eulerian interface description

Definitions

Definition (Tangential gradient)

The tangential gradient *of a function f, which is differentiable in an open neighborhood of Γ, is defined by*

$$\underline{\nabla} f(x) = \nabla f(x) - (\hat{\mathbf{n}}(x) \cdot \nabla f(x))\hat{\mathbf{n}}(x), \quad x \in \Gamma$$

where ∇ denotes the usual gradient in $\mathbb{R}^d$

Definition (Laplace-Beltrami operator)

If f is two times differentiable in a neighborhood of Γ, then we define the Laplace-Beltrami operator *of f as*

$$\underline{\Delta} f(x) = \underline{\nabla} \cdot (\underline{\nabla} f(x)), \quad x \in \Gamma$$

Definitions and Derivation

Theorem

A theorem of differential geometry states that

$$\underline{\Delta}\mathrm{id}_\Gamma = \kappa \hat{\mathbf{n}}$$

where κ is the mean curvature and id_Γ is the identity mapping on Γ

Derivation

First take surface tension force source term, multiply it with the test function space $\mathbf{v}$, and apply partial integration

$$\begin{aligned} \mathbf{f}_{st} &= \int_\Gamma \sigma \kappa \hat{\mathbf{n}} \cdot \mathbf{v} \; d\Gamma = \int_\Gamma \sigma (\underline{\Delta}\mathrm{id}_\Gamma) \cdot \mathbf{v} \; d\Gamma = \\ &= -\int_\Gamma \sigma \underline{\nabla}\mathrm{id}_\Gamma \cdot \underline{\nabla}\mathbf{v} \; d\Gamma + \int_\gamma \sigma \partial_\gamma \mathrm{id}_\Gamma \cdot \mathbf{v} \; d\gamma \end{aligned}$$

Fully Implicit Evaluation in Space

- Boundary integrals can be transformed to volume integrals with the help of a Dirac delta function $\delta(\Gamma, \mathbf{x})$

$$
\begin{aligned}
\mathbf{f}_{st} &= \int_\Omega \sigma \kappa \hat{\mathbf{n}} \cdot \mathbf{v} \; \delta(\Gamma, \mathbf{x}) \; d\mathbf{x} &&= \int_\Omega \sigma (\underline{\Delta} \mathrm{id}_\Gamma) \cdot (\mathbf{v} \delta(\Gamma, \mathbf{x})) \; d\mathbf{x} \\
&= -\int_\Omega \sigma \underline{\nabla} \mathrm{id}_\Gamma \cdot \underline{\nabla}(\mathbf{v} \delta(\Gamma, \mathbf{x})) \; d\mathbf{x} &&= -\int_\Omega \sigma \underline{\nabla} \mathrm{id}_\Gamma \cdot \underline{\nabla} \mathbf{v} \; \delta(\Gamma, \mathbf{x}) \; d\mathbf{x}
\end{aligned}
$$

- **Application of the semi-implicit time integration**

$$
\Gamma^{n+1} = \Gamma^n + \Delta t \; \mathbf{u}^{n+1}
$$

yields

$$
\begin{aligned}
\mathbf{f}_{st} = &- \int_\Omega \sigma \underline{\nabla} (\mathrm{id}_\Gamma)^n \cdot \underline{\nabla} \mathbf{v} \; \delta(\Gamma^n, \mathbf{x}) \; d\mathbf{x} \\
&- \Delta t^{n+1} \int_\Omega \sigma \underline{\nabla} \mathbf{u}^{n+1} \cdot \underline{\nabla} \mathbf{v} \; \delta(\Gamma^n, \mathbf{x}) \; d\mathbf{x}
\end{aligned}
$$

Implicit Surface Tension Force Expression

The surface tension forces are finally given by ...

Implicit Surface Tension Force Expression

$$\mathbf{f}_{st} = -\int_\Omega \sigma\ \delta_\epsilon(dist(\Gamma^n,\mathbf{x}))\ \underline{\nabla}(\widetilde{\mathrm{id}}_\Gamma)^n \cdot \underline{\nabla}\mathbf{v}\ d\mathbf{x}$$
$$-\Delta t^{n+1}\int_\Omega \sigma\ \delta_\epsilon(dist(\Gamma^n,\mathbf{x}))\ \underline{\nabla}\mathbf{u}^{n+1} \cdot \underline{\nabla}\mathbf{v}\ d\mathbf{x}$$

Implicit Surface Tension Force Expression

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Implicit Surface Tension Force Expression

$$\begin{aligned} \mathbf{f}_{st} \quad = \quad & -\int_\Omega \sigma \; \delta_\epsilon(dist(\Gamma^n, \mathbf{x})) \; \underline{\nabla}(\widetilde{\mathrm{id}}_\Gamma)^n \cdot \underline{\nabla}\mathbf{v} \; d\mathbf{x} \\ & -\Delta t^{n+1} \int_\Omega \sigma \; \delta_\epsilon(dist(\Gamma^n, \mathbf{x})) \; \underline{\nabla}\mathbf{u}^{n+1} \cdot \underline{\nabla}\mathbf{v} \; d\mathbf{x} \end{aligned}$$

Level Set Method

A number of key points makes the *level set method* an ideal candidate for interface tracking algorithm when implementing the proposed surface tension force expressions

- Distance functions are in general readily available allowing for simple construction of the regularized Dirac delta functions
- Geometrical quantities such as normal and tangent vectors can be reconstructed globally, eliminating the need to extend these quantities from the interface separately
- The level set method can be coupled with the finite element method giving access to the variational form of the equations

Example, Oscillating Bubble (CSF)

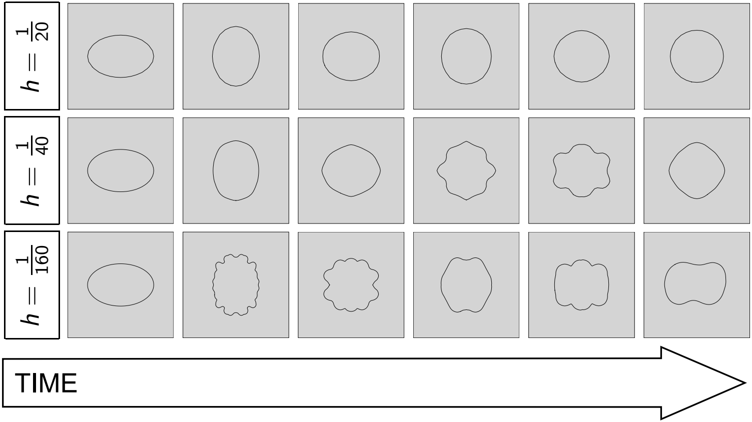


Figure: Evolution of an oscillating bubble; standard explicit CSF method

Example, Oscillating Bubble (CSF-LBI)

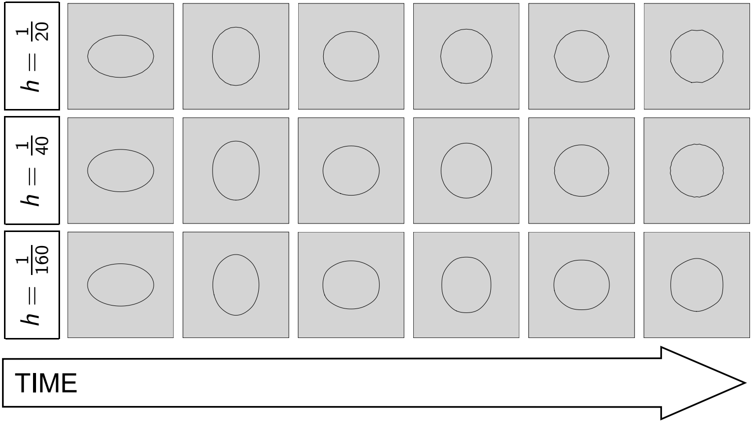


Figure: Evolution of an oscillating bubble; semi-implicit CSF-LBI method

Example, Rising Bubble (CSF)

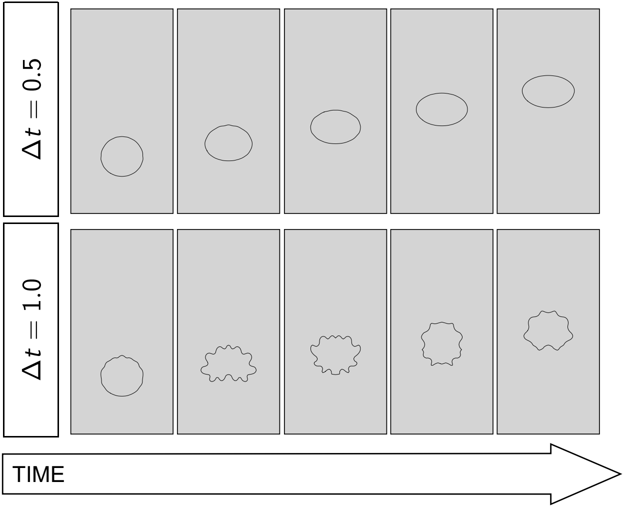


Figure: Evolution of a rising bubble with the standard explicit CSF method

Example, Rising Bubble (CSF-LBI)

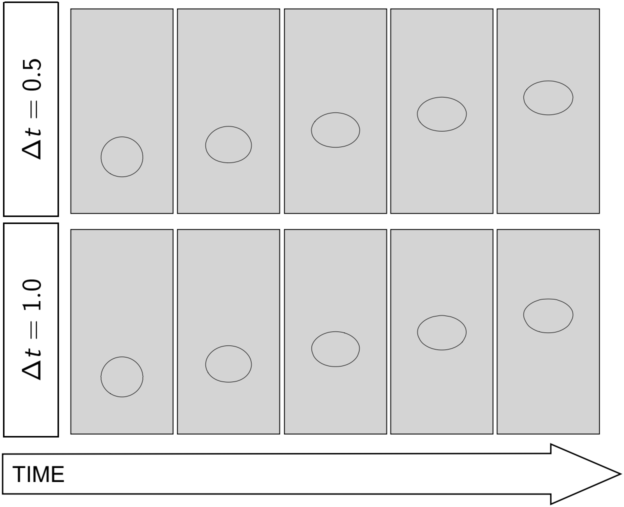


Figure: Evolution of a rising bubble with the semi-implicit CSF-LBI method

Summary

A new implicit surface tension variant has been proposed which relaxes the capillary time step restriction imposed on explicit implementations

Additional advantages

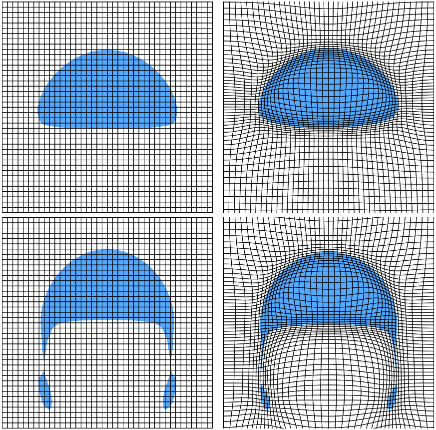
- Fully implicit in space
- Is easy to implement when using the level set method together with finite elements
- Explicit computation of curvature not necessary
- Conceptually identical algorithm in 3D

Future Research Directions

In development...

- ALE techniques coupled with time dependent grid deformation
- Inclusion of *pressure separation* techniques to improve the velocity and pressure
- Linear high order *edge stabilization* for convection of the level set field
- $\mathbb{Q}_2\mathbb{P}_1$ finite element approximation for the NS-equations and $\mathbb{Q}_n$ for the level set equation
- Contact angle, heat transfer, and solidification effects
- 3D + benchmarking

Grid Deformation



Static Bubble Test

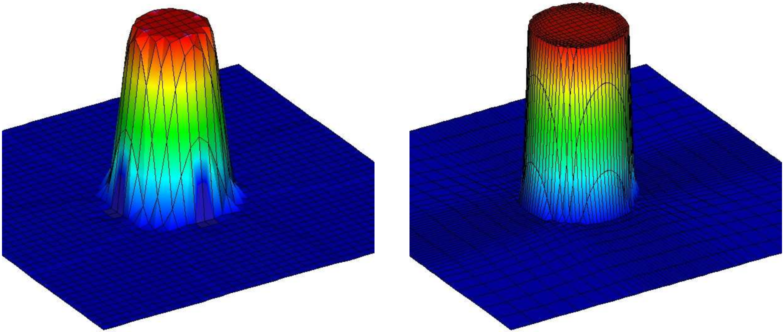


Figure: Pressure field for a static bubble with and without grid deformation