A numerical set-up for benchmarking and optimization of fluid-structure interaction

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Requirements for numerical FSI benchmarking

- *Realistic materials*
  - **incompressible Newtonian fluid**, laminar flow regime
  - **elastic solid**, large deformations
- *Comparative evaluation*
  - setup with periodical oscillations
  - non-graphically based quantities
- *Computable configurations*
  - laminar flow
  - reasonable aspect ratios
  - simple geometry (2D)
- Mainly based on validated CFD benchmarks, but also close to experimental set-up

Key questions

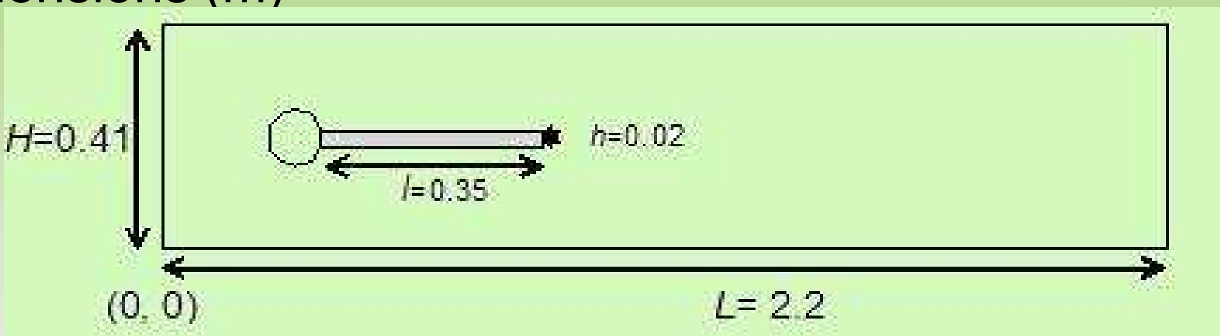
- Accurate and robust description of the interaction mechanisms w.r.t. highly dynamical and nonlinear behaviour and significant geometry changes? That includes:
  - Quality of different discretization techniques (FEM, FV, FD, LBM,resp., beam, shell, volume elements) for FSI?
  - Robustness and numerical efficiency of the integrated solver components?

1st step: *Identification of appropriate **FSI** setting for numerical benchmarking*

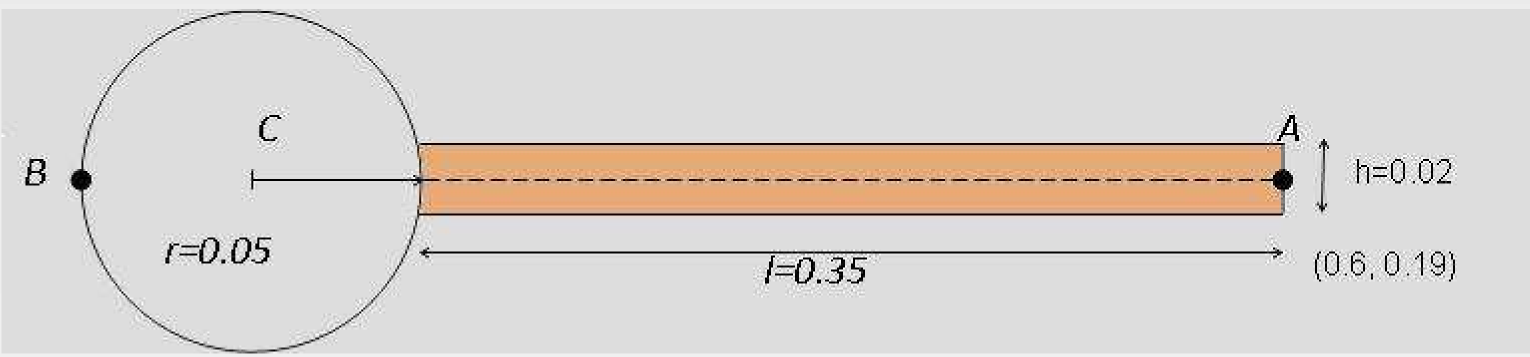
2nd step: ***FSI** benchmark setting due to experimental studies*

Computational domain

- Domain dimensions (m)

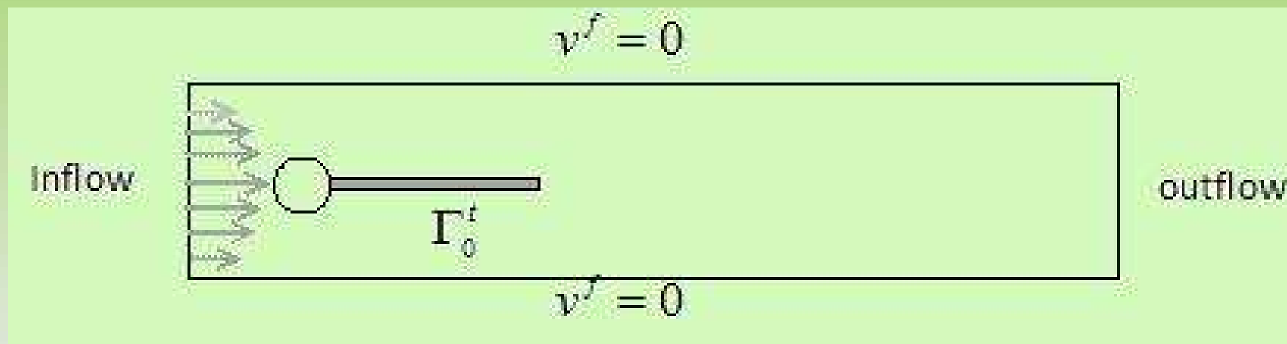


- Detail of the submerged structure



$$A(t=0)=(0.6,0.2),\quad B=(0.15,0.2), C=(0.2,0.2)$$

Boundary and initial conditions



Inflow parabolic velocity profile is prescribed at the left end of the channel

Outflow condition can be chosen by the user, assuming zero reference pressure

Otherwise the *no-slip* condition is prescribed for the fluid on the other boundary parts.

Initial no flow fluid and no deformation + smooth increase of the inflow profile

Fluid and structure properties

- **Incompressible** fluid with density ρ^f

$$\rho^f \frac{\partial \mathrm{v}^f}{\partial t} + \rho^f \left(\nabla \mathrm{v}^f\right)\mathrm{v}^f = \mathrm{div}\ \sigma^f \qquad \text{in } \Omega_t^f$$

$$\mathrm{div}\ \mathrm{v}^f = 0$$

$$\sigma^f = -p^f \mathrm{I} + \rho^f \nu^f \left(\nabla \mathrm{v}^f + \nabla \mathrm{v}^{f^{\mathrm{T}}}\right)$$

- Elastic material with density, $\rho^s, \mathrm{F} = \mathrm{I} + \nabla \mathrm{u}^s, J = \det \mathrm{F}$: **St. Venant -- Kirchhoff** material

$$\rho^s \frac{\partial^2 \mathrm{u}^s}{\partial t^2} = \mathrm{div}\left(\sigma^s \mathrm{F}^{-\mathrm{T}}\right) \qquad \text{in } \Omega^s$$

$$\sigma^s = \frac{1}{J}\mathrm{F}\left(\lambda^s (\mathrm{tr}\mathrm{E})\mathrm{I} + 2\mu^s \mathrm{E}\right)\mathrm{F}^{\mathrm{T}}$$

$$\mathrm{E} = \frac{1}{2}\left(\mathrm{F}^{\mathrm{T}}\mathrm{F} - \mathrm{I}\right)$$

Suggested material parameters

solid

- ρ^s density
- ν^s Poisson ratio
- μ^s shear modulus

fluid

- ρ^f density
- ν^f kinematic viscosity

Parameter	polybutadiene & glycerine	**polypropylene & glycerine**
$\rho^s [10^3 \mathrm{kg/m^3}]$	0.91	1.1
ν^s	0.50	0.42
$\mu^s [10^6 \mathrm{kg/ms^2}]$	0.53	317
$\rho^f [10^3 \mathrm{kg/m^3}]$	1.26	1.26
$\nu^f [10^{-3} \mathrm{m^2}/s]$	1.13	1.13

Parameter	FSI1	FSI2	FSI3
$\rho^s [10^3 \mathrm{kg/m^3}]$	1	1	1
ν^s	0.4	0.4	0.4
$\mu^s [10^6 \mathrm{kg/ms^2}]$	0.5	0.5	2.0
$\rho^f [10^3 \mathrm{kg/m^3}]$	1	1	1
$\nu^f [10^{-3} \mathrm{m^2}/s]$	1	1	1
$\bar{U} [\mathrm{m}/s]$	0.2	1	2

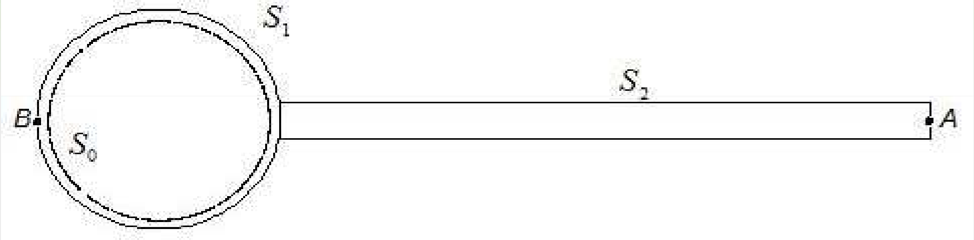
Parameter	FSI1	FSI2	FSI3
$\beta = \frac{\rho^s}{\rho^f}$	1	1	1
ν^s	0.4	0.4	0.4
$\mathrm{Ae} = \frac{E^s}{\rho^f \bar{U}^2}$	3.5×10^4	1.4×10^3	1.4×10^3
$\mathrm{Re} = \frac{\bar{U} d}{\nu^f}$	20	100	200
$\bar{U} [\mathrm{m/s}]$	0.2	1	2

Quantities of interest

- The position $A(t) = (x(t), y(t))$ of the end of the structure
- Pressure difference between the points *A(t)* and *B*

$$\Delta p^{AB}(t) = p^{B}(t) - p^{A(t)}(t)$$

- Forces exerted by the fluid on the *whole body*, i.e. lift and drag forces acting on the cylinder and the structure together



$$(F_D, F_L) = \int_S \sigma \, ndS = \int_{S_1} \sigma^f \, ndS + \int_{S_2} \sigma^{f|S} ndS = \int_{S_0} \sigma \, ndS$$

- Frequency and maximum amplitude
- Compare results for *one* full period and 3 different levels of spatial discretization *h* and 3 time step sizes Δt

FSI1: steady, small deformations

Parameter	FSI1	FSI2	FSI3
$\rho^s [10^3 \mathrm{kg/m^3}]$	1	1	1
ν^s	0.4	0.4	0.4
$\mu^s [10^6 \mathrm{kg/ms^2}]$	0.5	0.5	2.0
$\rho^s [10^3 \mathrm{kg/m^3}]$	1	1	1
$\nu^s [10^{-3} \mathrm{m^2/s}]$	1	1	1
$\bar{U} [\mathrm{m/s}]$	0.2	1	2

Parameter	FSI1	FSI2	FSI3
$\beta = \frac{\rho^s}{\rho^f}$	1	1	1
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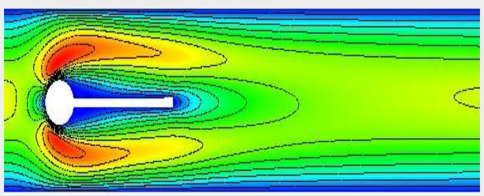
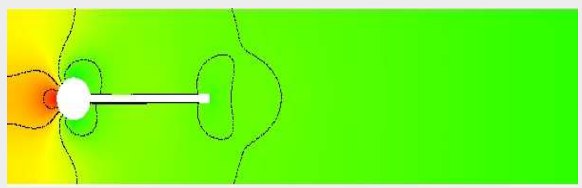
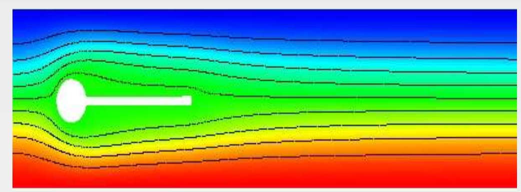
$\mathrm{Ae} = \frac{E^s}{\rho^f \bar{U}^2}$	3.5×10^4	1.4×10^3	1.4×10^3
$\mathrm{Re} = \frac{\bar{U} d}{\nu^f}$	20	100	200
$\bar{U} [\mathrm{m/s}]$	0.2	1	2

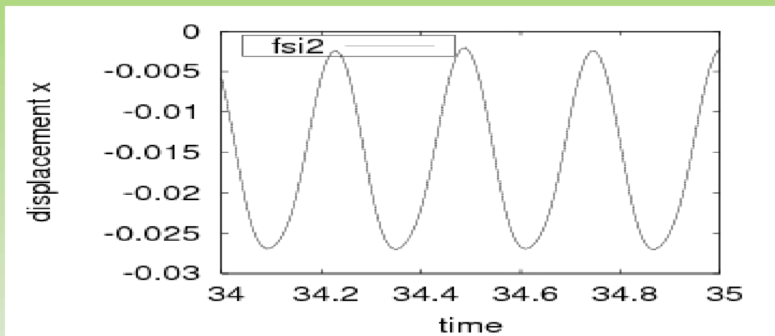


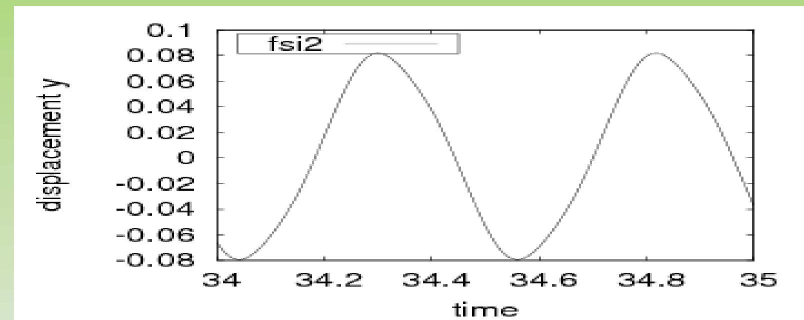


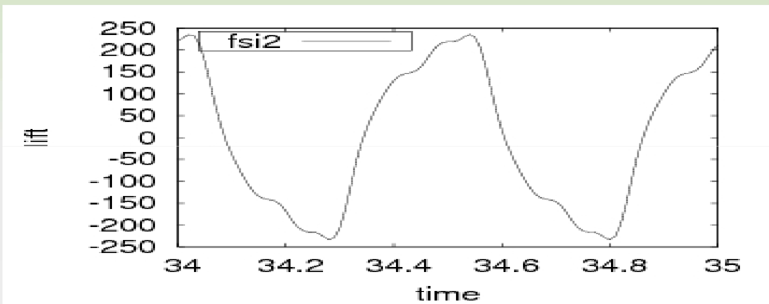


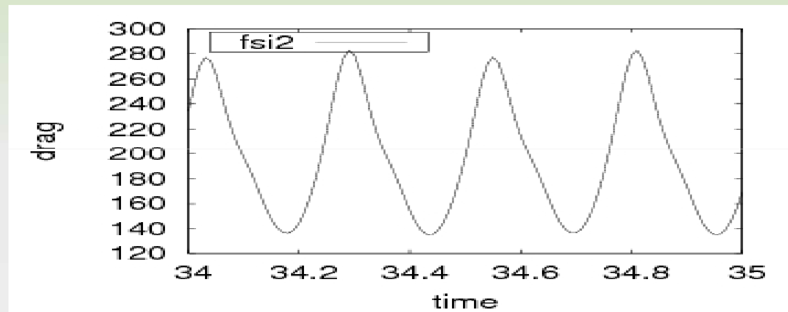
	ux of A $[\times 10^{-3} \mathrm{m}]$	uy of A $[\times 10^{-3} \mathrm{m}]$	drag	lift
FSI1	0.0227	0.8209	14.295	0.7638

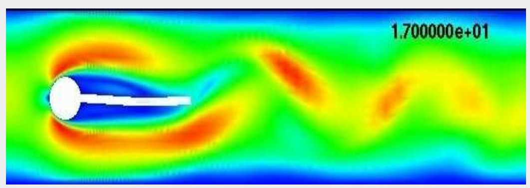
FSI2: large deformations, periodical oscillations

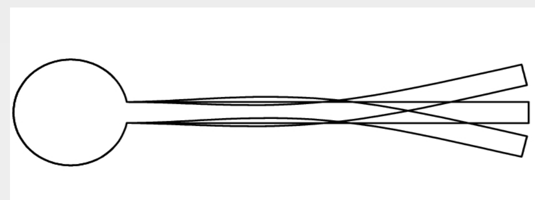






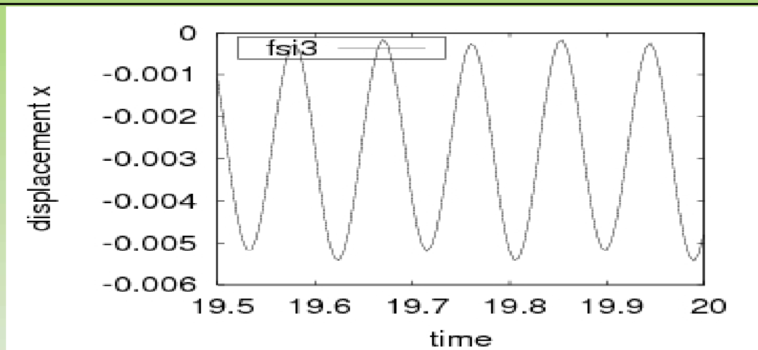


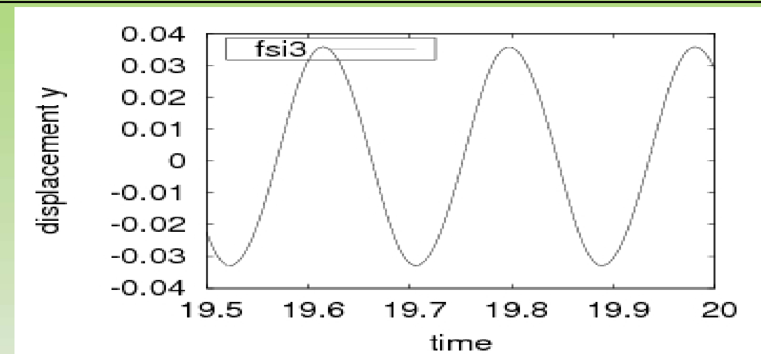


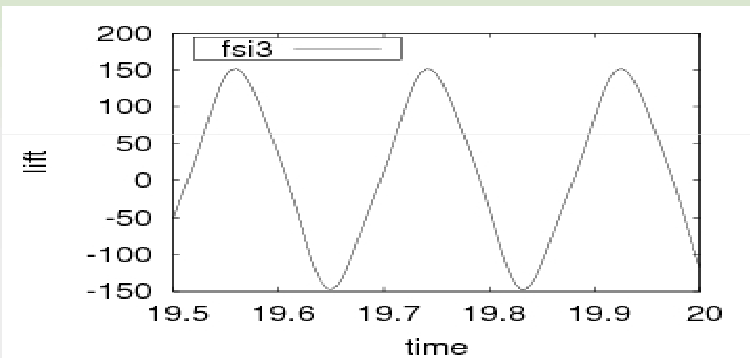


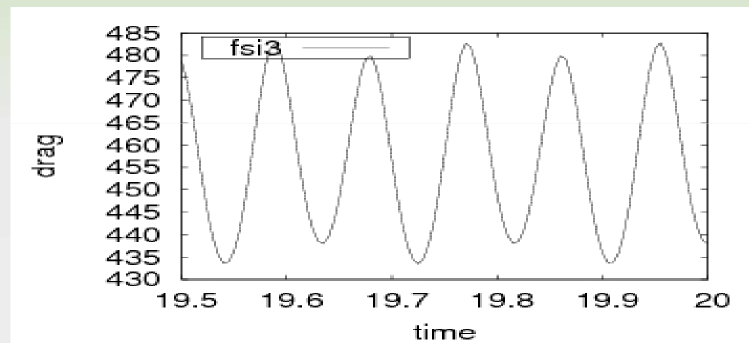
Test	ux of A $[\times 10^{-3}\text{m}]$	uy of A $[\times 10^{-3}\text{m}]$	drag	lift
FSI2	$-14.58 \pm 12.44[3.8]$	$1.23 \pm 80.6[2.0]$	$208.83 \pm 73.75[3.8]$	$0.88 \pm 234.2[2.0]$

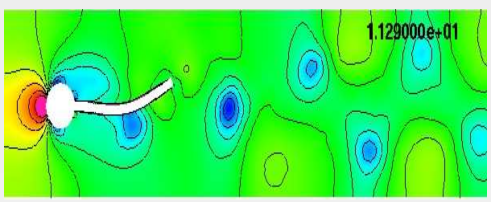
FSI3: large deformations, complex oscillations

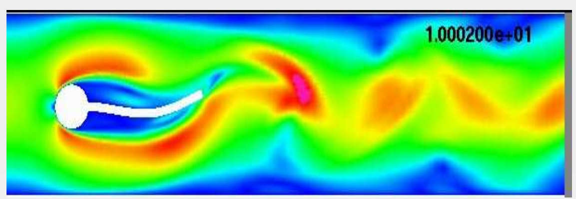


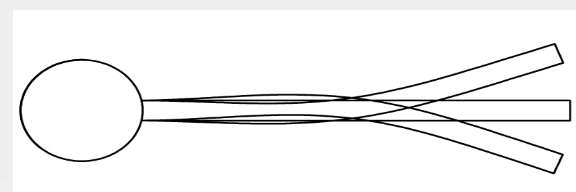












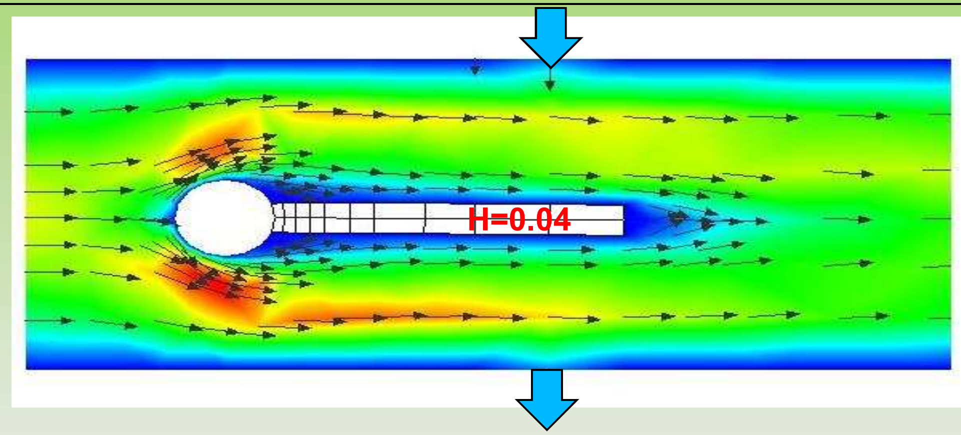
Test	ux of A $[\times 10^{-3}\text{m}]$	ux of A $[\times 10^{-3}\text{m}]$	drag	lift
FSI3	$-2.69 \pm 2.53[10.9]$	$1.48 \pm 34.38[5.3]$	$457.3 \pm 22.66[10.9]$	$2.22 \pm 149.78[5.3]$

FSI Optimization

- Optimization problem
  - Associated design or control variable
- The main design aims could be
  - I) Drag/Lift minimization
  - II) Minimal pressure loss
  - III) Minimal nonstationary oscillations
- To reach these aims, we might allow
  1. Boundary control of inflow section
  2. Change of geometry: elastic channel walls or length/thickness of elastic beam
  3. Optimal control of volume forces
- Optimal control of nonstationary flow might be hard for the starting
- Results for the moment are combination of I)-III) with 1)-3).

FSI Optimization: Example 1

- uncontrolled flow



	ux of A $[\times 10^{-3}$m]	uy of A $[\times 10^{-3}$m]	drag	lift
FSI1	0.0227	0.8209	14.295	0.7638

lift $\neq$ 0

- Aim: $\text{minimize}\left(lift^2 + \alpha V^2\right)$

w.r.t V1, V2.

V1 velocity from top

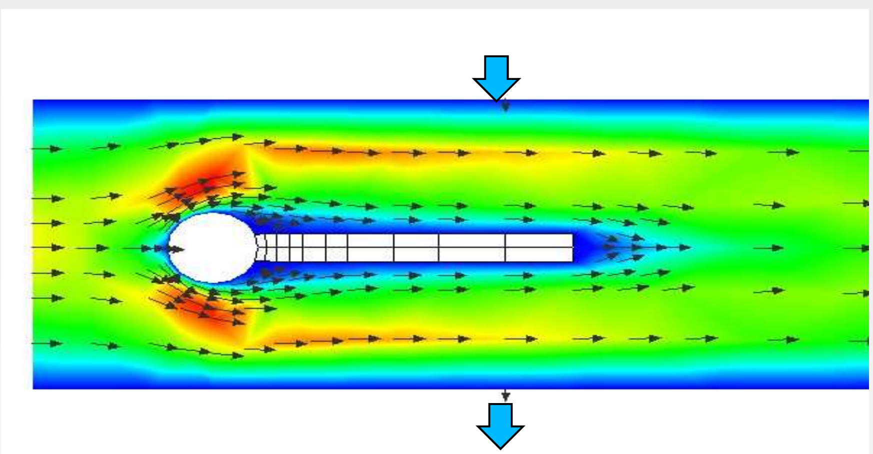
V2 velocity from below

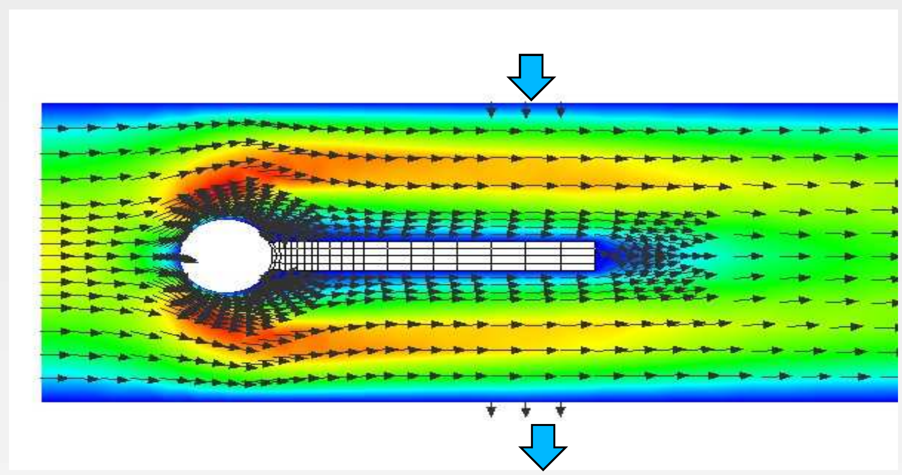
FSI Optimization: Example 1

- TESTS for FSI 1 (Boundary control)

level 1 | level 2

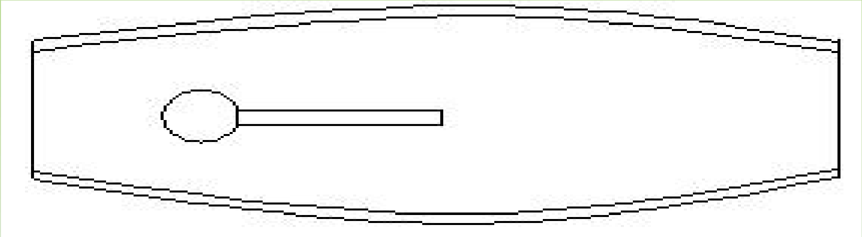
α	Iter steps	extreme point	drag	Lift	Iter steps	extreme point	drag	Lift
1e0	57	(3.74e-1,3.88e-1)	1.5471e+01	8.1904e-1	59	(3.66e-1,3.79e-1)	1.5550e+01	7.8497e-1
1e-2	60	(1.04e0,1.06e0)	1.5474e+01	2.2684e-2	59	(1.02e0,1.04e0)	1.5553e+01	2.1755e-2
1e-4	73	(1.06e0,1.08e0)	1.5474e+01	2.3092e-4	71	(1.04e0,1.05e0)	1.5553e+01	2.2147e-4
1e-6	81	(1.06e0,1.08e0)	1.5474e+01	2.3096e-6	86	(1.04e0,1.05e0)	1.5553e+01	2.2151e-6





Outlook

- further examples might be:



1. $\text{minimize}\left(lift^2 + \alpha V^2\right)$ for deformed case
2. pressure loss minimize: $\text{minimize}\left(p_{in} - p_{out}\right)$

w.r.t elastic deformation of the wall

or

w.r.t geometrical and material properties of beam