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A numerical set-up for benchmarking and optimization of fluid-structure interaction

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Integrated Multiphysics Simulation and Design Optimization

March 10-12, 2010

Aim

1st step: *Identification of appropriate FSI settings for numerical benchmarking and calculate*

If done ⇒ *2nd step*

2nd step: *Extension to FSI-Optimisation benchmark settings*

- Data files are available on

http://featflow.de/beta/en/benchmarks.html

Key question: FSI benchmarking

- *Accurate and robust description of the interaction mechanisms*

  w.r.t.

  highly dynamical ,

  nonlinear behavior ,

  and significant geometry changes?

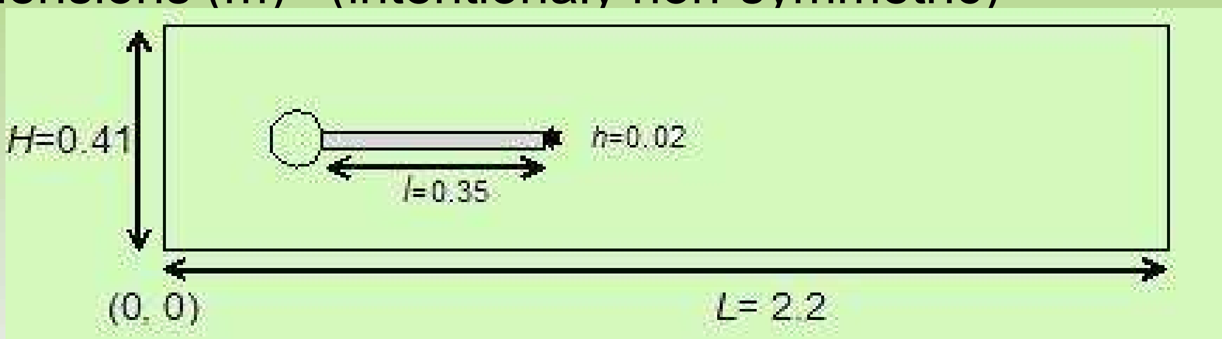
- That includes:
  - Quality of different discretization techniques (FEM, FV, FD, LBM, resp., beam, shell, volume elements) for FSI?
  - Robustness and numerical efficiency of the integrated solver components?
  - Coupling mechanisms (partitioned/monolithic, weak/strong)?

Requirements for numerical FSI benchmarking

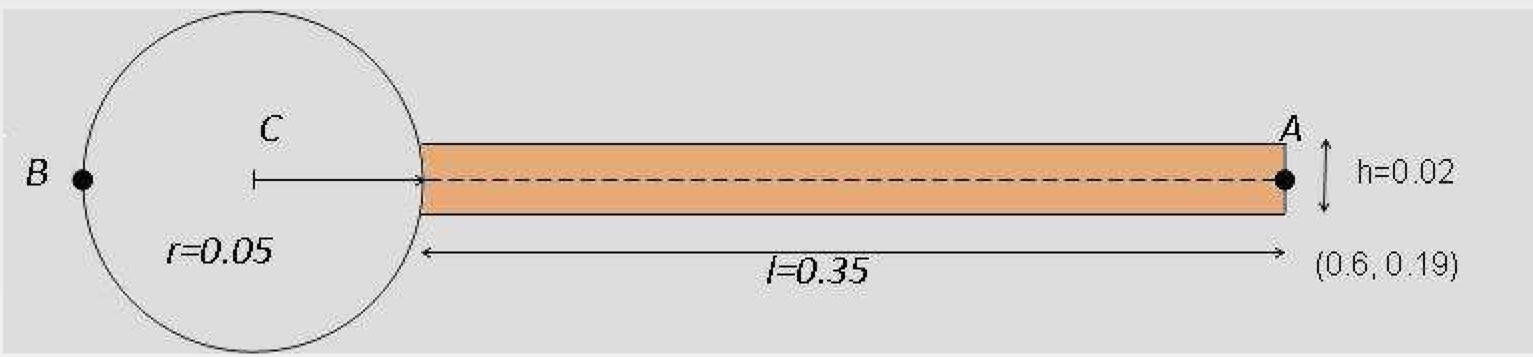
- *Realistic materials*
  - **incompressible Newtonian fluid**, laminar flow regime
  - **elastic solid**, large deformations
- *Comparative evaluation*
  - setup with periodical oscillations
  - non-graphically based quantities
- *Computable configurations*
  - laminar flow
  - reasonable aspect ratios
  - simple geometry (2D)
- Mainly based on validated CFD benchmarks, but also close to experimental set-up

Computational domain

- Domain dimensions (m)   (intentionaly non-symmetric)

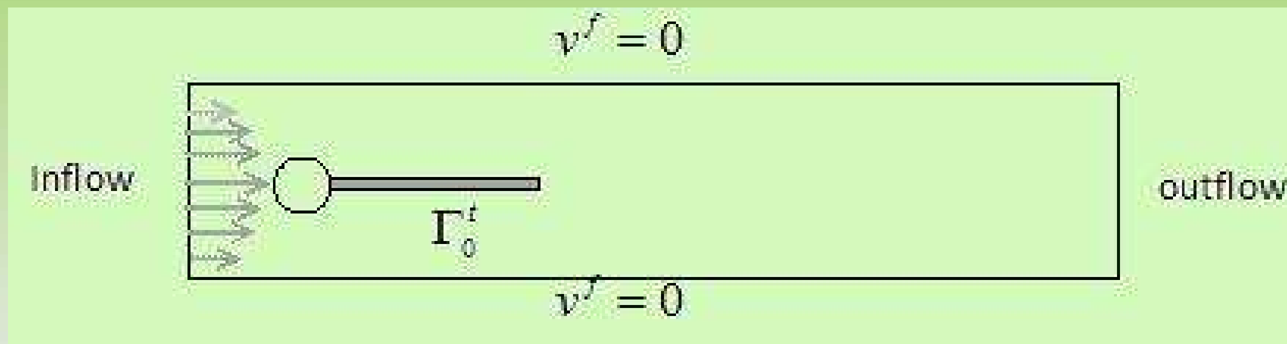


- Detail of the submerged structure



$$A(t=0)=(0.6,0.2),\quad B=(0.15,0.2), C=(0.2,0.2)$$

Boundary and initial conditions



Inflow parabolic velocity profile is prescribed at the left end of the channel

Outflow condition can be chosen by the user, assuming zero reference pressure (*stress free* or *do nothing*)

Otherwise the *no-slip* condition is prescribed for the fluid on the other boundary parts.

Initial no fluid flow and no deformation + smooth increase of the inflow profile

Fluid and structure properties

- **Incompressible Newtonian** fluid with density ρ^f

Momentum balance $$\rho^f \frac{\partial \mathrm{v}^f}{\partial t} + \rho^f \left(\nabla \mathrm{v}^f\right)\mathrm{v}^f = \operatorname{div} \sigma^f$$

in Ω_t^f

Mass balance $$\operatorname{div} \mathrm{v}^f = 0$$

Cauchy stress tensor $$\sigma^f = -p^f \mathrm{I} + \rho^f \nu^f \left(\nabla \mathrm{v}^f + \nabla \mathrm{v}^{f\,\mathrm{T}}\right)$$

- Elastic material with density, $\rho^s, \mathrm{F} = \mathrm{I} + \nabla \mathrm{u}^s, J = \det \mathrm{F}$: **St. Venant -- Kirchhoff** material

$$\rho^s \frac{\partial^2 \mathrm{u}^s}{\partial t^2} = \operatorname{div}\left(\sigma^s \mathrm{F}^{-\mathrm{T}}\right)$$

in Ω^s

$$\sigma^s = \frac{1}{J} \mathrm{F}\left(\lambda^s \left(\mathrm{tr}\mathrm{E}\right)\mathrm{I} + 2\mu^s \mathrm{E}\right)\mathrm{F}^{\mathrm{T}}$$

Green-lagrange strain tensor $$\mathrm{E} = \frac{1}{2}\left(\mathrm{F}^{\mathrm{T}}\mathrm{F} - \mathrm{I}\right)$$

A neo-Hookean material model can also be used for compressible/incompressible

Suggested material parameters

solid

- ρ^s density
- ν^s Poisson ratio
- μ^s shear modulus

fluid

- ρ^f density
- ν^f kinematic viscosity

Oriented materials

Parameter	polybutadiene & glycerine		**polypropylene & glycerine**	
$\rho^s [10^3 \mathrm{kg/m^3}]$ ν^s $\mu^s [10^6 \mathrm{kg/ms^2}]$	incompressible	0.91 0.50 0.53	compressible	1.1 0.42 317
$\rho^f [10^3 \mathrm{kg/m^3}]$ $\nu^f [10^{-3} \mathrm{m^2}/s]$		1.26 1.13		1.26 1.13

Parameter	FSI1	FSI2	FSI3
$\rho^s [10^3 \mathrm{kg/m^3}]$ ν^s $\mu^s [10^6 \mathrm{kg/ms^2}]$	1 0.4 0.5	1 0.4 0.5	1 0.4 2.0
$\rho^f [10^3 \mathrm{kg/m^3}]$ $\nu^f [10^{-3} \mathrm{m^2}/s]$	1 1	1 1	1 1
$\bar{U} [\mathrm{m}/s]$	0.2	1	2

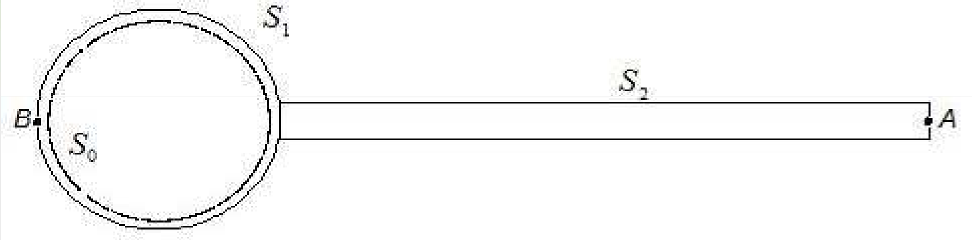
Parameter	FSI1	FSI2	FSI3
$\beta = \frac{\rho^s}{\rho^f}$ ν^s $\mathrm{Ae} = \frac{E^s}{\rho^f \bar{U}^2}$	1 0.4 3.5×10^4	1 0.4 1.4×10^3	1 0.4 1.4×10^3
$\mathrm{Re} = \frac{\bar{U} d}{\nu^f}$ $\bar{U} [\mathrm{m/s}]$	20 0.2	100 1	200 2

Quantities of interest

- The position $A(t)=(x(t),y(t))$ of the end of the structure
- Pressure difference between the points *A(t)* and *B*

$$\Delta p^{AB}(t) = p^{B}(t) - p^{A(t)}(t)$$

- Forces exerted by the fluid on the *whole body*, i.e. lift and drag forces acting on the cylinder and the structure together



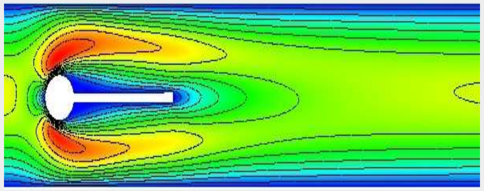
$$(F_D, F_L) = \int_S \sigma\, n dS = \int_{S_1} \sigma^{f} n dS + \int_{S_2} \sigma^{f|S} n dS = \int_{S_0} \sigma\, n dS$$

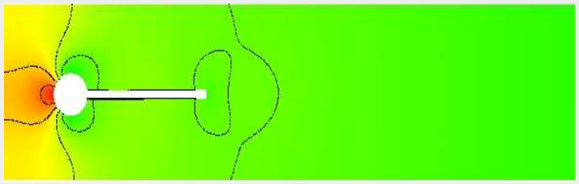
- Frequency and maximum amplitude
- Compare results for *one* full period and 3 different levels of spatial discretization *h* and 3 time step sizes Δt

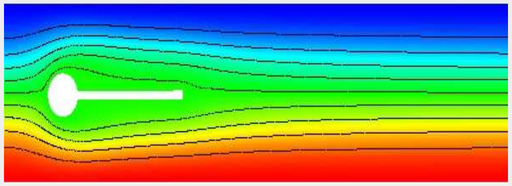
FSI1: steady, small deformations

Parameter	FSI1	FSI2	FSI3
$\rho^s [10^3 \mathrm{kg/m^3}]$ ν^s $\mu^s [10^6 \mathrm{kg/ms^2}]$	1 0.4 0.5	1 0.4 0.5	1 0.4 2.0
$\rho^s [10^3 \mathrm{kg/m^3}]$ $\nu^s [10^{-3} \mathrm{m^2/s}]$	1 1	1 1	1 1
$\bar{U} [\mathrm{m/s}]$	0.2	1	2

Parameter	FSI1	FSI2	FSI3
$\beta = \frac{\rho^s}{\rho^f}$ ν^s $\mathrm{Ae} = \frac{E^s}{\rho^f \bar{U}^2}$	1 0.4 3.5×10^4	1 0.4 1.4×10^3	1 0.4 1.4×10^3
$\mathrm{Re} = \frac{\bar{U} d}{\nu^f}$ $\bar{U} [\mathrm{m/s}]$	20 0.2	100 1	200 2

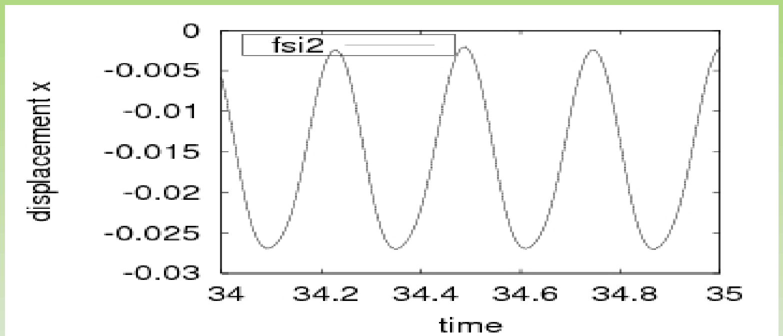


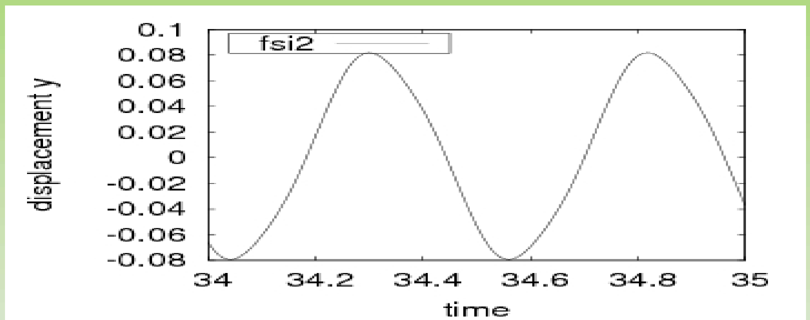


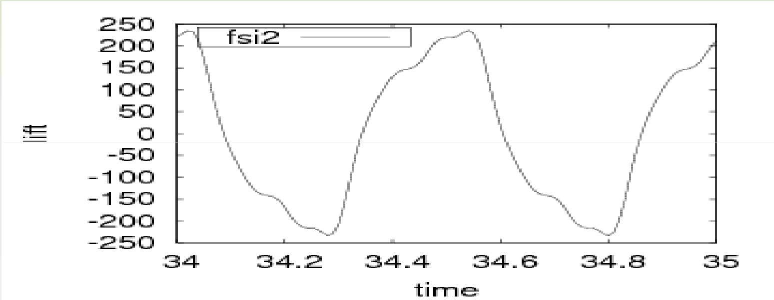


	ux of A $[\times 10^{-5} \mathrm{m}]$	uy of A $[\times 10^{-4} \mathrm{m}]$	drag	lift
FSI1	2.270493	8.208773	14.2942	0.76374

FSI2: large deformations, periodical oscillations







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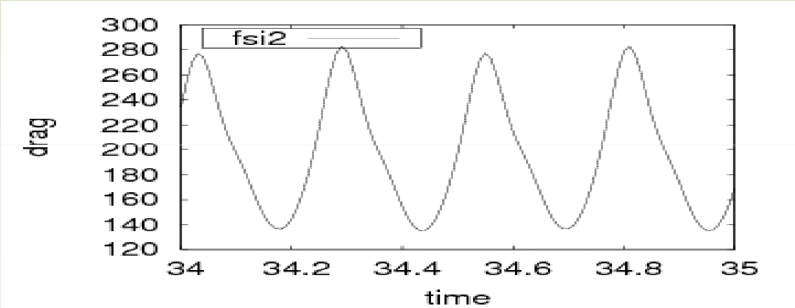
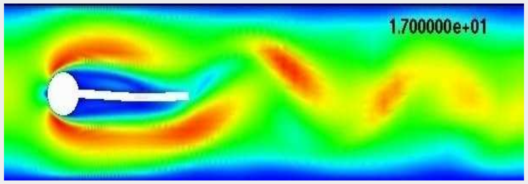
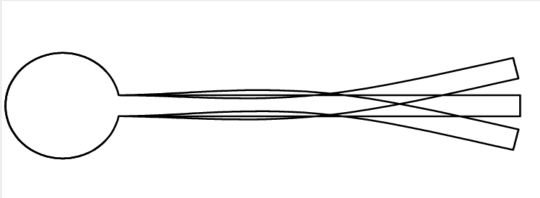


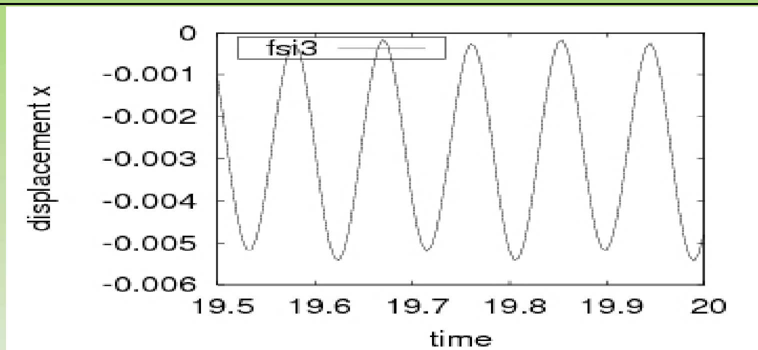


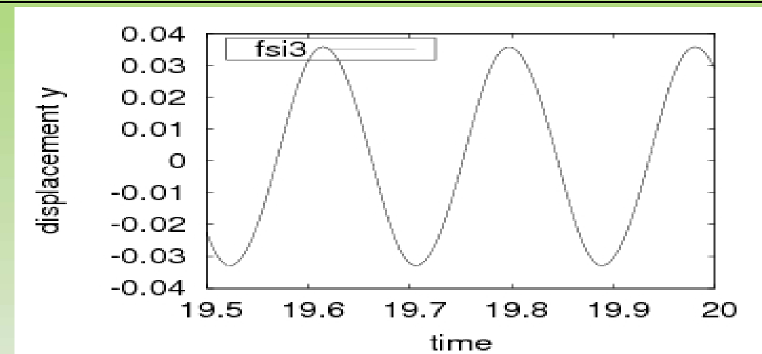


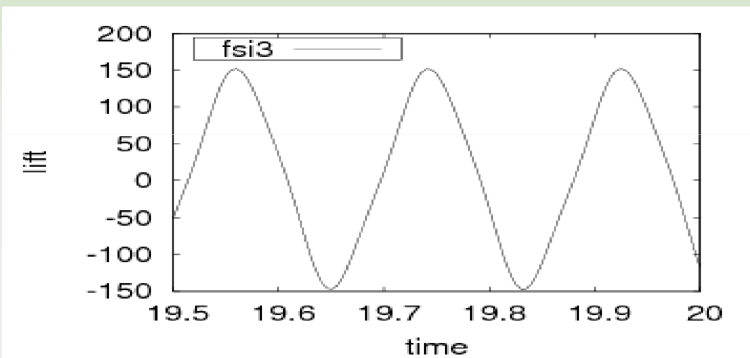
Test	ux of A $[\times 10^{-3}\text{m}]$	uy of A $[\times 10^{-3}\text{m}]$	drag	lift
FSI2	$-14.85\pm 12.70[3.86]$	$1.30\pm 81.7[1.93]$	$215.06\pm 77.65[3.86]$	$0.61\pm 237.8[1.93]$

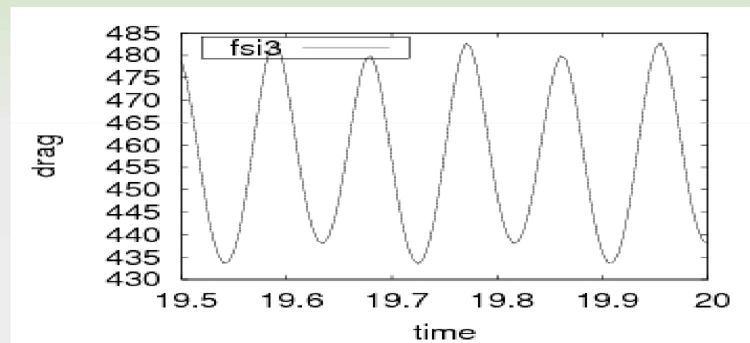
Mean ± amplitude[frequency]

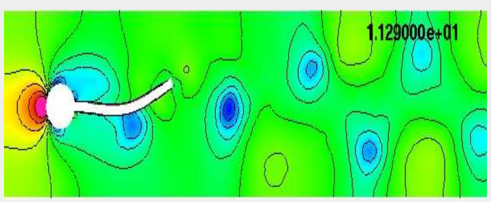
FSI3: large deformations, complex oscillations

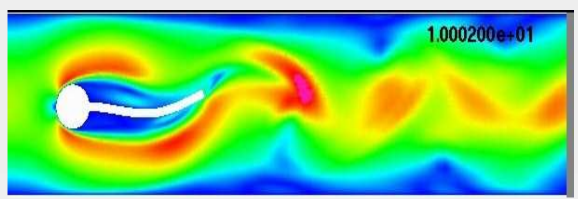


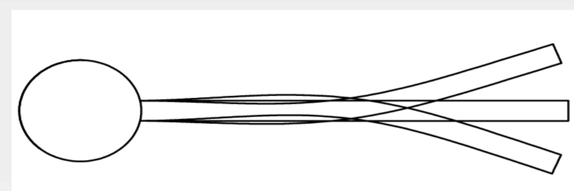












Test	ux of A $[\times 10^{-3}$m]	ux of A $[\times 10^{-3}$m]	drag	lift
FSI3	$-2.88 \pm 2.72[10.93]$	$1.47 \pm 34.99[5.46]$	$460.5 \pm 27.74[10.93]$	$2.50 \pm 153.91[5.46]$

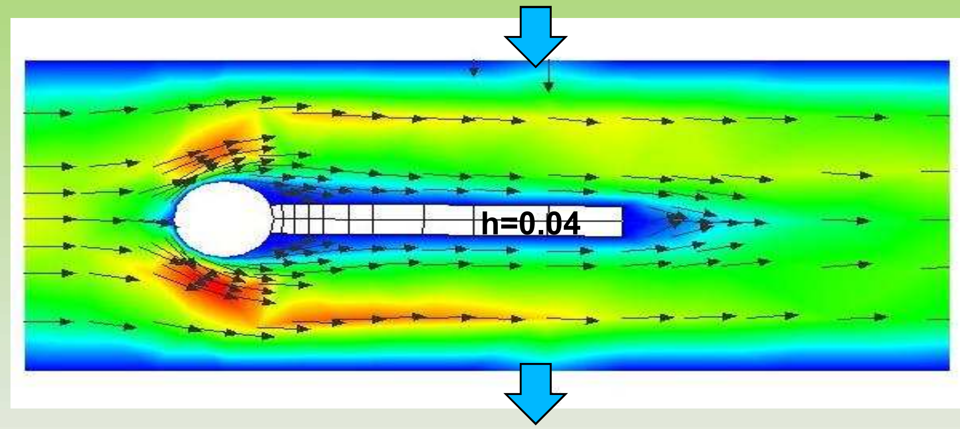
Mean ± amplitude[frequency]

FSI Optimization

- The main design aims could be
  I) Drag/Lift minimization
  II) Minimal pressure loss
  III) Minimal nonstationary oscillations
- To reach these aims, we might allow
  1. Boundary control of inflow section
  2. Change of geometry: elastic channel walls or length/thickness of elastic beam
  3. Optimal control of volume forces
- Optimal control of nonstationary flow might be hard for the starting
- Results for the moment are combination of I)-III) with 1)-3).

FSI-Opti1

- uncontrolled flow



	ux of A $[\times 10^{-3}$ m]	uy of A $[\times 10^{-3}$ m]	drag	lift
FSI1	0.0227	0.8209	14.295	0.7638

lift $\neq$ 0

- Aim: $\text{minimize}\left(lift^2 + \alpha V^2\right)$

w.r.t V1, V2.

V1 velocity from top
V2 velocity from below

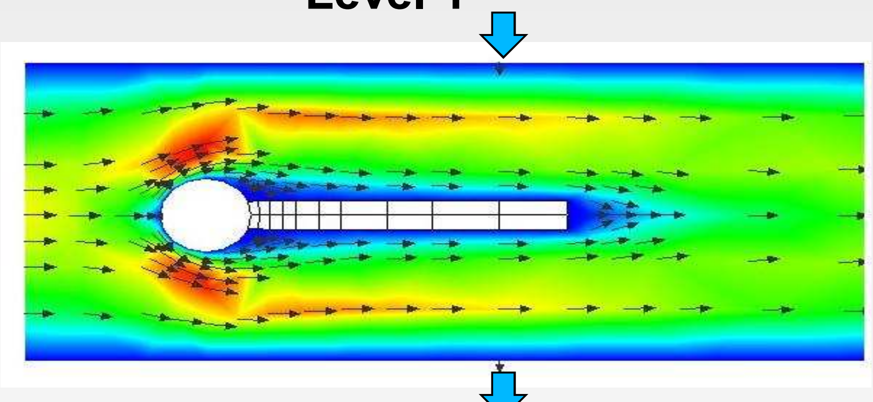
Nelder Mead

FSI-Opt 1

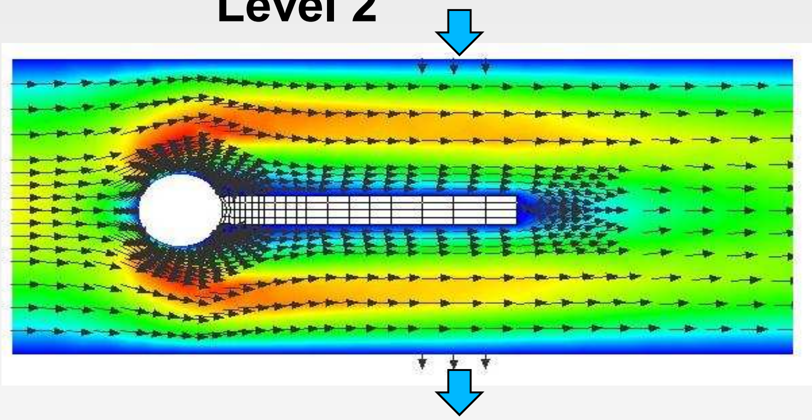
- TESTS for FSI 1 (Boundary control)

	level 1				level 2			
α	Iter steps	extreme point	drag	Lift	Iter steps	extreme point	drag	Lift
1e0	57	(3.74e-1,3.88e-1)	1.5471e+01	8.1904e-1	59	(3.66e-1,3.79e-1)	1.5550e+01	7.8497e-1
1e-2	60	(1.04e0,1.06e0)	1.5474e+01	2.2684e-2	59	(1.02e0,1.04e0)	1.5553e+01	2.1755e-2
1e-4	73	(1.06e0,1.08e0)	1.5474e+01	2.3092e-4	71	(1.04e0,1.05e0)	1.5553e+01	2.2147e-4
1e-6	81	(1.06e0,1.08e0)	1.5474e+01	2.3096e-6	86	(1.04e0,1.05e0)	1.5553e+01	2.2151e-6

Level 1

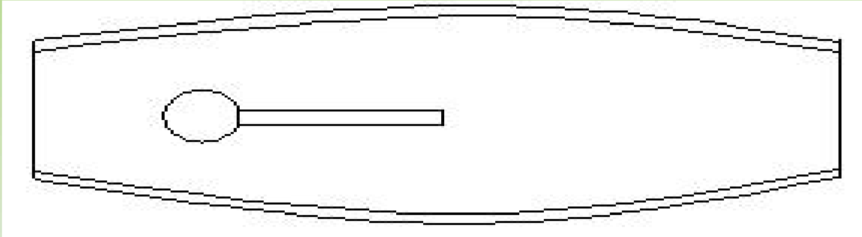


Level 2



Outlook

- Future examples might be:
  - complete walls elastic
  - portion of elastic



1. $\text{minimize}\left(lift^2 + \alpha V^2\right)$ for deformed case
2. pressure loss minimize: $\text{minimize}\left(p_{in} - p_{out}\right)$

w.r.t elastic deformation of the wall

or

w.r.t geometrical and material properties of beam