Croatian German Meeting on

Analysis and Mathematical Physics

March 21 – March 25

2021

Meeting is organised under COST Action CA18232 *Mathematical models for interacting dynamics on networks* and the DAAD PPP project *Robust optimal control of parabolic equations*

technische universität dortmund

TECHNISCHE UNIVERSITÄT CHEMNITZ

Mathematical models for interacting dynamics on networks

cost EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY

DAAD Deutscher Akademischer Austauschdienst German Academic Exchange Service

Contents

About

General information

With this Workshop we continue a tradition of events that took place in Chemnitz, Murter and Dortmund in recent years. The purpose of this event is to bring together scientists working in various areas of applied mathematics, analysis, operator theory, mathematical physics, and differential equations, with an emphasis on the following fields of research:

- General operators and spectral theory, structured operators and matrices,
- Elliptic PDEs including Schrödinger operators,
- Evolution equations, semigroups, control theory,
- Numerical methods related to any of the above.

Technical information

The workshop is organized purely online. The talks will be held in Zoom:

Direct link:	https://tu-dortmund.zoom.us/j/96...
Zoom ID:	969 3485 2855
Password:	012558

Organizing committee

Luka Grubišić	University of Zagreb
Ivica Nakić	University of Zagreb
Martin Tautenhahn	Technische Universität Chemnitz
Ivan Veselić	Technische Universität Dortmund

Schedule

Monday, March 22, 2021

Morning	Chair: Ivan Veselić	
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11:00–11:45	**Marko Vrdoljak** Zagreb, Croatia	Analysis of optimality conditions by shape calculus for optimal designs in conductivity problems
Afternoon	**Chair: Luka Grubišić**	
14:00–14:45	**Ninoslav Truhar** Osijek, Croatia	Eigenvector-Dependent Nonlinear Eigenvalue Problem
15:00–15:45	**Zoran Tomljanović** Osijek, Croatia	Sampling-free model reduction of systems with low-rank parameterization
16:00–16:45	**Matthias Langer** Glasgow, United Kingdom	Discrete coagulation-fragmentation equations

Tuesday, March 23, 2021

Morning	Chair: Matthias Täufer	
10:00–10:45	**Peter Benner** Magdeburg, Germany	Numerical Computation of Robust Controllers for Incompressible Flow Problems
11:00–11:45	**Boris Muha** Zagreb, Croatia	Existence and regularity of weak solutions for a fluid interacting with a non-linear shell in 3D
Afternoon	**Chair: Boris Muha**	
15:00–15:45	**Martin Lazar** Dubrovnik, Croatia	Distributed optimal control of parabolic equations by spectral decomposition
16:00–16:45	**Matthias Täufer** Hagen, Germany	On fully supported eigenfunctions on metric tree graphs
17:00–17:45	**Igor Mezić** Santa Barbara, USA	Ergodic theory and Koopman operator theory for dissipative systems

Wednesday, March 24, 2021

Morning	Chair: Martin Tautenhahn	
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11:00–11:45	**Albrecht Seelmann** Dortmund, Germany	On a minimax principle in spectral gaps
Afternoon	**Chair: Albrecht Seelmann**	
14:00–14:45	**Max von Renesse** Leipzig, Germany	Molecules as metric measure spaces with Kato-bounded Ricci curvature
15:00–15:45	**Delio Mugnolo** Hagen, Germany	Spectral surgery and heat kernels on quantum graphs
16:00–16:45	**Matko Ljulj** Zagreb, Croatia	Nonlinear Naghdi type shell model

Thursday, March 25, 2021

Morning	Chair: Josip Tambača	
10:00–10:45	**Cristhian Montoya** Dubrovnik, Croatia	Robust control and Stackelberg strategy for a forth–order parabolic equation
11:00–11:45	**Vjekoslav Kovač** Zagreb, Croatia	Density theorems for Euclidean point configurations
Afternoon	**Chair: Ivica Nakić**	
14:00–14:45	**Marko Erceg** Zagreb, Croatia	Existence of strong traces of degenerate parabolic equations via velocity averaging
15:00–15:45	**Jussi Behrndt** Graz, Austria	Spectral theory for Dirac operators with singular potentials
16:00–16:45	**Peter Stollmann** Chemnitz, Germany	A new uncertainty principle at low energies

List of Abstracts

Spectral theory for Dirac operators with singular potentials

Jussi Behrndt, Technische Universität Graz, Austria

Thursday, 15:00-15:45

In this talk we discuss qualitative spectral properties of self-adjoint Dirac operators. We first briefly review some of the standard results for regular potentials from the literature and turn to more recent developments afterwards. Our main objective in this lecture is to discuss singular potentials supported on curves or hyperplanes, where it is necessary to distinguish the so-called non-critical and critical cases for the strength of the singular perturbation. In particular, it turns out that Dirac operators with singular potentials in the critical case have some unexpected spectral properties.

This talk is based on joint some recent works with P. Exner, M. Holzmann, V. Lotoreichik, T. Ourmieres-Bonafos, and K. Pankrashkin.

Numerical Computation of Robust Controllers for Incompressible Flow Problems

Peter Benner, Max Planck Institute for Dynamics of Complex Technical Systems, Magdeburg, Germany

Tuesday, 10:00-10:45

We consider the stabilization of incompressible fluid flow using linearized and spatially discretized models. In order to potentially work in applications, the designed controller must stabilize the discrete model with a robustness margin that covers linearization, discretization, and modeling errors. We show that a linearization error in the infinite-dimensional model amounts to a coprime factor uncertainty and show that H_∞-robust controllers can compensate this in the discrete approximation. To compute such controllers, standard software to solve algebraic Riccati equations cannot be used due to the potential indefiniteness of the Hessian. We report on an iterative scheme that overcomes this problem and can be applied to large-scale problems by emplyoing low-rank techniques. In numerical experiments, we quantify the robustness margins and show that the H_∞-robust controller, unlike the LQG-controller, is capable of stabilizing nonlinear incompressible Navier-Stokes equations with an inexact linearization.

This is joint work with Jan Heiland and Steffen Werner (MPI Magdeburg).

Existence of strong traces of degenerate parabolic equations via velocity averaging

Thursday,
14:00-14:45

Marko Erceg, University of Zagreb, Croatia

In this talk we study solutions to the degenerate parabolic equation

$$\partial_t u + \operatorname{div}_x f(u) = \operatorname{div}_x (a(u)\nabla u)\,,$$

subject to the initial condition $u(0,\cdot) = u_0$. Here the degeneracy appears as the matrix $a(\lambda)$ is only positive semi-definite, i.e. it can be equal to zero in some directions. Moreover, the directions can depend on λ. Equations of this form often occur in modelling flows in porous media and sedimentation-consolidation processes.

As a consequence of the degeneracy, solutions could be singular, so one needs to justify the meaning of the initial condition. A standard way is to show that u_0 is the strong trace of a solution u at $t = 0$. The notion of strong traces proved to be very useful in showing the uniqueness of the solution to scalar conservation laws with discontinuous flux.

We prove existence of strong traces for entropy solutions to the equation above. No non-degeneracy conditions on f and a are required. The proof is based on the blow-up techniques and the velocity averaging result for ultra-parabolic equations, applied to the kinetic formulation. In some special situations, we can allow even for heterogeneous fluxes and diffusion matrices, i.e. f and a dependent of (t, x) as well.

This is joint work with Darko Mitrović.

Density theorems for Euclidean point configurations

Monday,
11:00-12:00

Vjekoslav Kovač, University of Zagreb, Croatia

Several mathematical areas search for patterns in large, but otherwise arbitrary structures. Euclidean density theorems seek for dilated, rotated, and translated copies of a fixed finite point configuration within a "large" subset of the Euclidean space of appropriate dimension. Of particular interest are results that identify the configuration dilated by all sufficiently large scales. Since the seminal paper by Bourgain, which handled configurations forming vertices of a non-degenerate simplex, the proof strategy has always been a certain (explicit or implicit) decomposition of the counting form into structured, error, and uniform pieces, much in the spirit of regularity lemmas from arithmetic combinatorics and graph theory. However, a significant extra input from harmonic analysis is also needed in the proofs, often from the subfields of multilinear singular and oscillatory integrals. We will present several recent results on this topic, discussing: configurations standing in an arithmetic progression, three point corners, vertices of boxes with three-term progressions “attached”, configurations coming from anisotropic dilations, and configurations in sets of density close to 1, and leaving a few open problems. We will also always try to emphasize singular integral operators relevant to the discussed proof. The talk will be based on several recent papers coauthored with P. Durcik, K. Falconer, L. Rimanić, and A. Yavicoli.

Discrete coagulation-fragmentation equations

Matthias Langer, University of Strathclyde, United Kingdom

Monday, 16:00-16:45

In many situations in nature and industrial processes clusters of particles can combine into larger clusters or fragment into smaller ones. In the absence of any spatial variation, the evolution of the cluster size distribution can be described by an integro-differential equation (when the size of the clusters is arbitrary) or an infinite system of differential equations (when the cluster size assumes only discrete values). In this talk I shall discuss how operator semigroups can be applied to study the discrete version of the coagulation-fragmentation equation. This talk is based on joint work with Lyndsay Kerr and Wilson Lamb.

Distributed optimal control of parabolic equations by spectral decomposition

Martin Lazar and Cesare Molinari, University of Dubrovnik (Croatia), and Istituto Italiano di Tecnologia (Italy)

Tuesday, 15:00-15:45

We consider the constrained minimisation problem

$$(\mathcal{P}) \qquad \min_{u \in L^2_{T,\mathcal{U}}} \left\{ J(u) : \quad x(T) \in \overline{B_\varepsilon\left(x^T\right)} \right\}, \tag{0.1}$$

where x^T is some given target state, J is a given cost functional and x is the solution of

$$(\mathcal{E}) \qquad \begin{cases} \frac{d}{dt}x(t) + \mathcal{A}x(t) = \mathcal{B}_t u(t) & \text{for } t \in (0,T) \\ x(0) = 0. \end{cases} \tag{0.2}$$

Here $\mathcal{A}$ is an unbounded linear operator allowing for spectral decomposition, while $\mathcal{B}_t$ is a (time dependent) control operator.

If the cost functional J is given by $J(u) = \|u\|_{L^2}$ the problem (P) is reduced to a classical minimal norm control problem which can be solved by Hilbert uniqueness method (HUM). In [LM21] we allow for a more general cost functional and analyse examples in which, apart from the target state and the control norm, one considers a desired trajectory and penalise a distance of the state from it. Such problem requires a more general approach, and it has been addressed by different methods throughout last decades.

In this paper we suggest another method based on the spectral decomposition in terms of eigenfunctions of the operator $\mathcal{A}$. Surprisingly, the problem reduces to a non-linear equation for a scalar unknown, representing a Lagrangian multiplier. The same approach has been recently introduced in [LMP17] for an optimal control problem of the heat equation in which the control was given through the initial datum. This paper generalises the method to the distributed control problems. As can be expected, in this case one has to consider the associated dual problem which makes the calculation more complicated, although the algorithm steps follow a similar structure as in [LMP17].

In the talk basic steps of the method will be explained, followed by numerical examples demonstrating its efficiency.

[LM21] M. Lazar and C. Molinari. "Optimal distributed control of linear parabolic equations by spectral decomposition". In: *Optimal Control Appl. Methods* (2021). doi: 10.1002/oca.2708.

[LMP17] M. Lazar, C. Molinari, and J. Peypouquet. "Optimal control of parabolic equations by spectral decomposition". In: *Optimization* 66.8 (2017), pp. 1359–1381.

Nonlinear Naghdi type shell model

Wednesday, 16:00-16:45

Matko Ljulj, University of Zagreb, Croatia

In this talk a new nonlinear shell model of Naghdi type is proposed. It is well defined for shells whose middle surface is parameterised by a Lipschitz function. Behaviour of the shell is described by deformation of the middle surface and the matrix function with values in rotations which describes the rotation of the shell cross–section. The model is formulated as the minimization problem for the total energy functional that includes flexural, membrane, shear and drill energies.

We describe its properties: restrictions to a particular subset of admissible functions gives as a model of flexural or Koiter type, it is frame indifferent, its linearization gives a linear model of Naghdi type, its differential formulation implies that it is a Cosserat model with one director... After that we will analyse its asymptotic behaviour as the thickness of the shell tends to zero, in various regimes depending on the elastic properties of the material. We compare limit models (obtained by using Gamma-convergence) with rigorously derived nonlinear models obtained from 3d elasticity. This model has similar asymptotic properties when coupled to a thick 3d elastic body.

Ergodic theory and Koopman operator theory for dissipative systems

Igor Mezić, University of California, Santa Barbara, USA

Tuesday, 17:00-17:45

Classical Koopman operator theory provides a fruitful template for studies in ergodic theory of measure-preserving systems. It is of interest to extend its use to studies of dissipative systems - those that are globally attractive (in Birkhoff sense) to an invariant ergodic measure that is singular with respect to a predefined measure of interest (e.g. Lebesgue measure). The issue that arises is that of appropriate functional spaces to work in (pure L^2 theory has some issues), and we define these using an internal tensor product. As a consequence of the nature of the functional space, the spectrum of the operator is simplified. The extension also leads to new geometric objects related to state space partitions, such as isostables, and enables efficient analysis of physical models, but also of data.

Robust control and Stackelberg strategy for a forth–order parabolic equation

Cristhian Montoya, University of Dubrovnik, Croatia

Thursday, 10:00-10:45

In recent works, the notion of searching for a robust control system is developed simultaneously with a strategy on hierarchic control. From a mathematical point of view, the robustness of a system is equivalent to find a saddle point because we are looking for maximizing the perturbation and simultaneously minimizing the control which stabilizes the system. In addition, a hierarchic control strategy appears on the system, namely, a Stackelberg strategy, which establishes a game between two forces (called follower and leader) into the system. The scheme shows a robust control problem for the *follower control* and its associated disturbance function. Afterwards, we consider a Stackelberg optimization (which is associated to the *leader control*) in order to deduce a controllability result for the Kuramoto–Sivashinsky equation.

Spectral surgery and heat kernels on quantum graphs

Wednesday, 15:00-15:45

Delio Mugnolo, FernUniversität in Hagen, Germany

Quantum graphs are collections of intervals glued at their endpoints in a network-like fashion, along with differential operators acting upon them: here, I will focus on the Laplacian with natural or Dirichlet vertex conditions. In the first part of my talk, I will review old and new results about surgery of graphs and how they can be used to derive estimates for eigenvalues. In the second part, I will turn to the topic of how the heat content depends on the graph topology. In particular, I will apply spectral surgery methods to a variational problem originally proposed by Pólya, thus deducing an isoperimetric-type inequality for an integrated version of the heat content.

Existence and regularity of weak solutions for a fluid interacting with a non-linear shell in 3D

Tuesday, 11:00-11:45

Boris Muha, University of Zagreb, Croatia

We study the unsteady incompressible Navier-Stokes equations in three dimensions interacting with a non-linear flexible shell of Koiter type. This leads to a coupled system of non-linear PDEs where the moving part of the boundary is an unknown of the problem. The known existence theory for weak solutions is extended to non-linear Koiter shell models. We introduce a-priori estimates that reveal higher regularity of the shell displacement beyond energy estimates. These are essential for non-linear Koiter shell models, since such shell models are non-convex (w.r.t. terms of highest order). The estimates are obtained by introducing new analytical tools that allow to exploit dissipative effects of the fluid for the (non-dissipative) solid. The regularity result depends on the geometric constitution alone and is independent of the approximation procedure; hence it holds for arbitrary weak solutions. The developed tools are further used to introduce a generalized Aubin-Lions type compactness result suitable for fluid-structure interactions.

This is a joint work with S. Schwarzacher.

Existence of weak solutions to a cross-diffusion Cahn-Hilliard type system

Monday, 10:00-10:45

Jan-Frederik Pietschmann, Technische Universität Chemnitz, Germany

We will study a Cahn-Hilliard model for a multicomponent mixture with cross-diffusion effects, degenerate mobility and where only one of the species does separate from the others. We define a notion of weak solution adapted to possible degeneracies and our main result is (global in time) existence. Due to their low regularity, the Cahn-Hilliard terms require a special treatment.

This talk is based on a joint work with Virginie Ehrlacher and Greta Marino.

Molecules as metric measure spaces with Kato-bounded Ricci curvature

Max von Renesse, Universität Leipzig, Germany

Wednesday, 14:00-14:45

Via ground state transform we establish a connection between the analysis of the Schrödinger operator of certain molecules to the recently developing theory of metric measure spaces with generalized lower Ricci curvature bounds. Using some probabilistic methods like the Bismut derivative formula we can thus establish some new results on the smoothing property of the initial Schrödinger semigroup. (Joint work with Batu Güneysu, Humboldt-Universität Berlin)

On a minimax principle in spectral gaps

Albrecht Seelmann, Technische Universität Dortmund, Germany

Wednesday, 11:00-11:45

This talk deals with a minimax principle for eigenvalues in gaps of the essential spectrum of perturbed self-adjoint operators. This builds upon an abstract minimax principle devised by Griesemer, Lewis, and Siedentop and recents developments on block diagonalization of operators and forms in the off-diagonal perturbation setting. The Stokes operator is revisited as an example.

A new uncertainty principle at low energies

Peter Stollmann, Technische Universität Chemnitz, Germany

Thursday, 16:00-16:45

This talk is about a new *uncertainty principle at low energies* for graphs and more general spaces and its very simple proof. Based on earlier joint works with Anne Boutet de Monvel, Daniel Lenz, Marcel Schmidt and Gunter Stolz and on work in progress with Daniel Lenz, Marcel Schmidt, Gunter Stolz and Martin Tautenhahn.

On fully supported eigenfunctions on metric tree graphs

Tuesday, 16:00-16:45

Matthias Täufer, FernUniversität in Hagen, Germany

We prove that the Laplacian on all compact metric tree graphs has an infinite family of eigenfunctions of full support. This implies in particular Pjejel's Theorem on nodal domains on these graphs without genericity assumptions. Our proofs use a new technical approach exploiting pseudoperiodicity in a reformulation of the eigenvalue problem.

This is joint work with Marvin Plümer (FernUniversität in Hagen).

Modeling of heat transfer through conductive pipe

Wednesday, 10:00-10:45

Josip Tambača, University of Zagreb, Croatia

The standard engineer's model for heat transfer between the fluid flowing in the pipe and the exterior medium neglects the effects of the pipe's wall. Our goal is to prove that they are not always negligible. Depending on the ratio between diffusivities of the fluid and the thin wall thickness, using rigorous asymptotic analysis, we find five different models for effective description of the heat exchange. In all cases the effective equation is the same heat conduction equation written only on the fluid domain with difference in the effective boundary condition describing the heat exchange through the pipe's wall. Furthermore we present an unified model, that is also given only on the fluid domain, which has the same asympotics as the original problem and thus approximates the physical situation independently of the particular regime.

This is a joint work with Matko Ljulj, Eduard Marušić-Paloka and Igor Pažanin.

Sampling-free model reduction of systems with low-rank parameterization

Monday, 15:00-15:45

Zoran Tomljanović, University of Osijek, Croatia

We consider the reduction of parametric families of linear dynamical systems having an affine parameter dependence that allow for low-rank variation in the state matrix. Usual approaches for parametric model reduction typically involve exploring the parameter space to identify representative parameter values and the associated models become the principal focus of model reduction methodology. These models are then combined in various ways in order to interpolate the response. The initial exploration of the parameter space can be a forbiddingly expensive task. A different approach is proposed here that requires neither parameter sampling nor parameter space exploration. Instead, we represent the system response function as a composition of four subsystem response functions that are non-parametric with a purely parameter-dependent function. One may apply any one of a number of standard (non-parametric) model reduction strategies to reduce the subsystems independently, and then conjoin these reduced models with the underlying parameterization to obtain the overall parameterized response. Our approach offers flexibility and potentially control over accuracy. In particular, a data-driven variation of our approach is described that exercises this flexibility through the use of limited frequency-sampling of the underlying non-parametric models. The parametric structure of our system representation allows for a priori guarantees of system stability in the resulting reduced models across the full range of parameter values. Incorporation of system theoretic error bounds allows us to determine appropriate approximation orders for the non-parametric systems sufficient to yield uniformly high accuracy across the parameter range. The parametric structure of our reduced system representation lends itself very well to the development of optimization strategies making use of efficient cost function surrogates. We illustrate our approach on a class of structural damping optimization problems and on a benchmark model of thermal conduction in a semiconductor chip.

This is joint work with Christopher Beattie and Serkan Gugercin from the Department of Mathematics, Virginia Polytechnic Institute and State University, Blacksburg, VA, USA.

Eigenvector-Dependent Nonlinear Eigenvalue Problem

Ninoslav Truhar, University of Osijek, Croatia

Monday,
14:00-14:45

We consider the eigenvector-dependent nonlinear eigenvalue problem (NEPv) $H(V)V = V\Lambda$, where $H(V) \in \mathbb{C}^{n\times n}$ is an Hermitian matrix-valued function of $V \in \mathbb{C}^{n\times k}$ with orthonormal columns, i.e., $V^H V = I_k$, $k \leq n$ (usually $k \ll n$). We present the conditions on existence and uniqueness for the solvability of NEPv.

The presented results are obtained by two approaches. The first one is based on well-known standard perturbation theory, and the second one on a relative perturbation theory for Hermitian matrices. The differences between the standard and the relative perturbation theory approaches, have been illustrated in several numerical examples.

Analysis of optimality conditions by shape calculus for optimal designs in conductivity problems

Monday, 11:00-11:45

Petar Kunštek and Marko Vrdoljak, University of Zagreb, Croatia

We consider conductivity optimal design problems for two isotropic phases, possibly with several state equations. Our aim is to find a distribution of materials which maximizes the energy functional.

By relaxing the problem via homogenization method, an application of classical methods of calculus of variations is enabled, leading to optimality conditions and various numerical methods. However, usually in spherically symmetric problems classical solutions occur, so it is reasonable to compare these methods to ones which are based on shape derivative analysis.

Various numerical methods (both the first and the second order methods) show nice convergence properties, but we are here interested in theoretical analysis of optimality conditions obtained by shape derivatives. For problems on a ball, the first order optimality condition easily leads to few critical shapes. Thanks to symmetry assumptions, we are able to further analyse these critical shapes. Similar techniques are applied for classical isoperimetric problem, or similar questions of eigenfrequency optimization, where one is able, by the Fourier analysis, to check second order optimality condition for simple spherical candidates.

Our problem is much more technically demanding, but we are able to express second order optimality conditions by Fourier analysis, not only in the case of simple spherical interface between two given materials, but also in the case where this interface is made of two spheres.