The Heat Equation with a Potential

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Abstract

For a positive, measurable potential V on an open set $\Omega \subseteq \mathbb{R}^n$, we consider the following perturbed heat equation

$$\begin{cases} \partial_t u = \Delta u + V u & \text{on } (0,\infty) \times \Omega\,, \\ u(0,\cdot) = f \in L_2(\Omega)_+ \end{cases} \tag{1}$$

with zero Dirichlet boundary conditions. The question of interest is under what conditions on V there exist non-trivial positive solutions to (1). For bounded Ω this was already investigated in [Cabré und Martel, 1999]. For $\Omega = \mathbb{R}^n$, we get an unexpected answer. It turns out that one can find positive classical solutions with the potential $V = |\cdot|^2$. We define the notion of approximated solution to (1) and show that it exists for any positive potential $V \leq |\cdot|^2$. On the other hand, if an approximated solution to (1) exists, we show that V is in some sense less than $|\cdot|^2$, i.e. $|\cdot|^2$ is the limit case. The tools we use are absorption semigroups and the form method.
This is a joint work with Hendrik Vogt.

References

[Cabré und Martel 1999] Cabré, Xavier ; Martel, Yvan: Existence versus explosion instantanée pour des équations de la chaleur linéaires avec potentiel singulier. In: *Comptes Rendus de l'Académie des Sciences-Series I-Mathematics* 329 (1999), Nr. 11, S. 973–978