Angewandte Harmonische Analysis

Lösung zu Aufgabe 34

geg.:

- $\alpha > 0$
- $g \in W(\mathbb{R}) = \left\{ f \in L^\infty(\mathbb{R}) \,:\, \sum_{n\in\mathbb{Z}} \operatorname*{ess\,sup}_{x\in[0,1]} |g(x+n)| < \infty \right\}$
- $\mathcal{Z}_\alpha g(x,\omega) = \sum_{k\in\mathbb{Z}} g(x+k\alpha)e^{-2k\pi i\alpha\omega}$

z.z.:

a) $\int_0^\alpha \mathcal{Z}_\alpha g(x,\omega)e^{-2\pi i x\omega}\,dx = \hat{g}(\omega)$ f.ü.

b) $\alpha \int_0^{1/\alpha} \mathcal{Z}_\alpha g(x,\omega)\,d\omega = g(x)$ f.ü.

Bew.:

a)

$$\begin{aligned}
\int_0^\alpha \mathcal{Z}_\alpha g(x,\omega)e^{-2\pi i x\omega}\,dx &= \int_0^\alpha \sum_{k\in\mathbb{Z}} g(x+k\alpha)e^{-2k\pi i\alpha\omega}e^{-2\pi i x\omega}\,dx \\
&= \sum_{k\in\mathbb{Z}} \int_0^\alpha g(x+k\alpha)e^{-2(x+k\alpha)\pi i\omega}\,dx \\
&= \int_{\mathbb{R}} g(x)e^{-2\pi i x\omega}\,dx \quad = \quad \hat{g}(\omega)
\end{aligned}$$

b)

$$\begin{aligned}
\alpha \int_0^{1/\alpha} \mathcal{Z}_\alpha g(x,\omega)\,d\omega &\overset{7.4c)}{=} \int_0^{1/\alpha} \mathcal{Z}_{1/\alpha}\hat{g}(\omega,-x)e^{2\pi i x\omega}\,d\omega \\
&= \int_0^{1/\alpha} \sum_{k\in\mathbb{Z}} \hat{g}(\omega+k/\alpha)e^{2k\pi i x/\alpha}e^{2\pi i x\omega}\,d\omega \\
&= \sum_{k\in\mathbb{Z}} \int_0^{1/\alpha} \hat{g}(\omega+k/\alpha)e^{2\pi i x(\omega+k/\alpha)}\,d\omega \\
&= \int_{\mathbb{R}} \hat{g}(x)e^{2\pi i x\omega}\,d\omega \quad = \quad g(x)
\end{aligned}$$

□