Wachmacher-Aufgabe 3

$$\frac{\sqrt{27}}{3^{0.5}} = \frac{\sqrt{27}}{\sqrt{3}} = \frac{\sqrt{9}\cdot\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}} = 3 \qquad \text{Alternativ: } \frac{\sqrt{27}}{\sqrt{3}} = \sqrt{\frac{27}{3}} = \sqrt{9} = 3.$$

Frage: Gilt $x<y \Rightarrow x^2<y^2$?

Lsg: Nein, auch nicht, wenn x, y das gleiche Vorzeichen haben

Bsp: $x=-4$, $y=-2$, also $x<y$, aber $\underbrace{(-4)^2}_{=16=x^2} > \underbrace{(-2)^2}_{=4=y^2}$

Bsp: Für welche $x\in\mathbb{R}$ gilt $\frac{1}{x+3} < -2$?

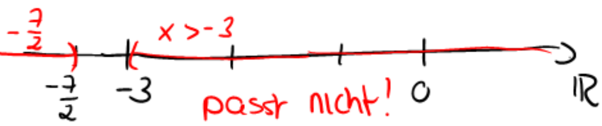
Lsg:

1 Fall: $x+3>0 \Leftrightarrow x>-3$. Dann gilt

$$\frac{1}{x+3} < -2 \overset{\cdot(x+3)>0}{\Leftrightarrow} 1 < \underbrace{-2(x+3)}_{-2x-6} \overset{+6}{\Leftrightarrow} 7 < -2x \overset{:(-2)}{\Leftrightarrow} -\frac{7}{2} > x$$

$\Rightarrow \mathbb{L}_1 = \{\ \}$

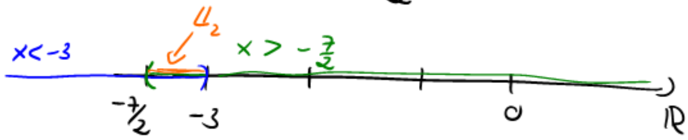
Alternativ: Für $x+3>0 \Rightarrow \frac{1}{x+3} > 0 > -2$ ↯



2 Fall $x+3<0 \Leftrightarrow x<-3$. Dann gilt

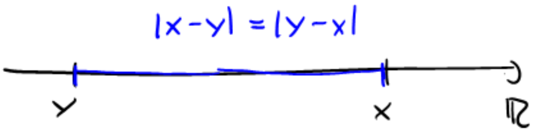
$$\frac{1}{x+3} < -2 \overset{(x+3)<0}{\Leftrightarrow} 1 > \underbrace{-2(x+3)}_{-2x-6} \Leftrightarrow 7 > -2x \Leftrightarrow -\frac{7}{2} < x$$

$\Rightarrow \mathbb{L}_2 = (-\frac{7}{2}, -3)$



Gesamtlösung: $\mathbb{L} = \mathbb{L}_1 \cup \mathbb{L}_2 = (-\frac{7}{2}, -3)$

zu 4.6



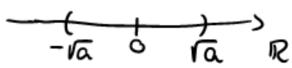
Bsp $|5-3| = |2| = 2$

$|3-5| = |-2| = 2$

zu 4.7 5) $|\underset{x}{5} + (\underset{y}{-3})| = |2| = 2 < \underbrace{|5|}_{5} + \underbrace{|-3|}_{=3} = 8$

6) $\big|\, |\underset{x}{5}| - |\underset{=y}{-3}| \,\big| = |5-3| = |2| = 2 < |5-(-3)| = |8| = 8$

Sei $a>0$. Dann ist



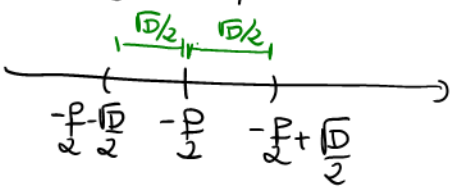
1) $x^2 < a \overset{\sqrt{\ }}{\Leftrightarrow} \sqrt{x^2} < \sqrt{a} \Leftrightarrow |x| < \sqrt{a} \Leftrightarrow x\in(-\sqrt{a}, \sqrt{a})$

2) $x^2 > a \Leftrightarrow \sqrt{x^2} > \sqrt{a} \Leftrightarrow |x| > \sqrt{a} \Leftrightarrow x\in(-\infty, -\sqrt{a}) \cup (\sqrt{a}, \infty)$

Bem: Für $a \leq 0$ hat die Ungl. $x^2 < a$ keine reelle Lsg.

zu 4.8:

Fall $D>0$: $\left(x+\frac{p}{2}\right)^2 < \frac{D}{4} \overset{\sqrt{\ }}{\iff} \left|x+\frac{p}{2}\right| < \frac{\sqrt{D}}{2} \iff \left|x-\left(-\frac{p}{2}\right)\right| < \frac{\sqrt{D}}{2}$



$\iff x \in \left(-\frac{p}{2}-\frac{\sqrt{D}}{2}, -\frac{p}{2}+\frac{\sqrt{D}}{2}\right)$

Bsp 5.1

- $D=[1,2]$, $W=\mathbb{R}$, $f:[1,2]\to\mathbb{R}$, $x\mapsto x^2$

Bsp: $f:[1,2]\to \underbrace{\boxed{\mathbb{R}}}_{=W_1}$, $x\mapsto x^2$

$g:[1,2]\to \underbrace{\boxed{[0,\infty)}}_{W_2}$, $x\mapsto x^2$

Dann ist $f\neq g$!! (da $W_1 \neq W_2$)

Bsp $\lfloor 3.2\rfloor = 3$, $\lfloor -4.6\rfloor = -5$, $\lfloor -2\rfloor = -2$

Bsp $f:\mathbb{R}\to\mathbb{R}$ $\quad f(x) = \begin{cases} \frac{1}{x-2}, & \text{falls } x\neq 2 \\ 0, & \text{falls } x=2 \end{cases}$

Bsp zu 5.3

1. $f:\mathbb{R}\to\mathbb{R}$ $\quad x\mapsto x^2$

   $f([1,3]) = [1,9]$

2. $f^{-1}([1,9]) = [-3,-1] \cup [1,3]$