Wachmacher - Aufgabe 5

Für welche $x \in \mathbb{R}$ gilt $\frac{1}{3-x} > 1$?

Lsg:

1Fall: $x<3$, also $3-x>0$. Dann

$$\frac{1}{3-x} > 1 \overset{\cdot(3-x)}{\Leftrightarrow} 1 > 3-x \overset{+x-1}{\Leftrightarrow} x > 2$$

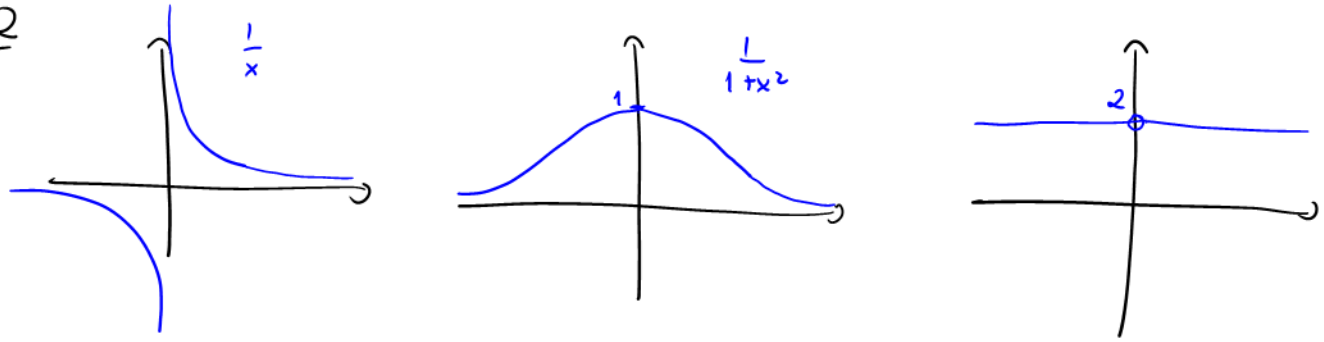
$$\Rightarrow \mathbb{L}_1 = (2,3)$$

2Fall $x>3$, also $3-x<0$. Dann gilt

$$\underbrace{\frac{1}{3-x}}_{<0} < 0 < 1, \text{ also } \mathbb{L}_2 = \emptyset$$

Gesamtlösung: $\mathbb{L} = \mathbb{L}_1 \cup \mathbb{L}_2 = (2,3)$

zu 5.2



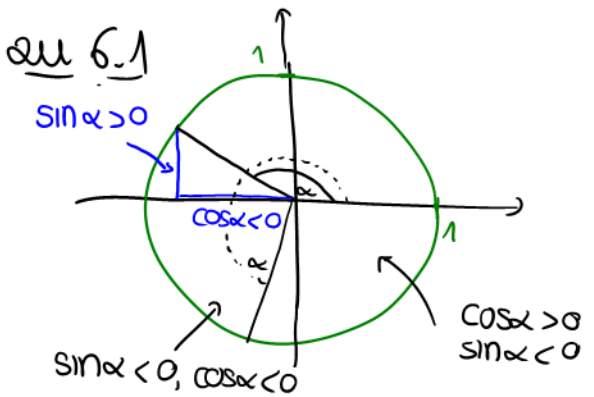
6 Trigonometrie

- $20° \triangleq \frac{20^{1}}{\cancel{360}_{18}} \cdot 2\pi = \frac{1}{\cancel{18}_{9}} \cdot \cancel{2}\pi = \frac{\pi}{9}$

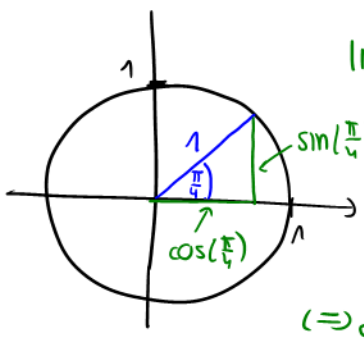
Man kann am Bild herleiten:

- $\sin^2\alpha + \cos^2\alpha = 1$
- $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$ (Strahlensatz)

zu 6.1



Spezialfall: $\alpha = \frac{\pi}{4} \triangleq 45°$



In diesem Fall:

$\sin(\frac{\pi}{4}) = \cos(\frac{\pi}{4}) > 0$

$\sin^2(\frac{\pi}{4}) + \underbrace{\cos^2(\frac{\pi}{4})}_{=\sin^2(\frac{\pi}{4})} = 1$

$\Leftrightarrow 2\sin^2(\frac{\pi}{4}) = 1$

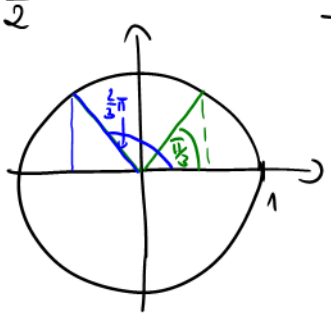
$\Leftrightarrow \sin^2(\frac{\pi}{4}) = \frac{1}{2}$

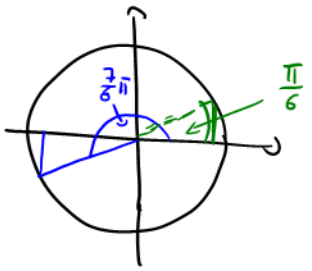
$\overset{\sin(\frac{\pi}{4})>0}{\Leftrightarrow} \sin(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$

Werte/tabelle sin, cos:

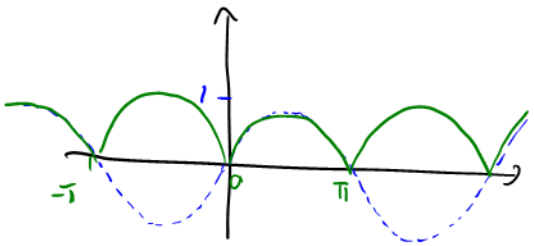
$\sin\left(\frac{7}{6}\pi\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$

$\sin\left(\frac{2}{3}\pi\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

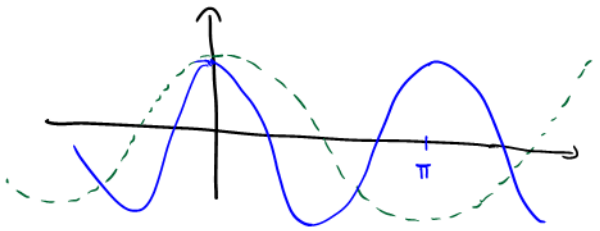




zu 6.3 Die Fkt $f: \mathbb{R} \to \mathbb{R}$, $f(x) = |\sin x|$ ist π-periodisch



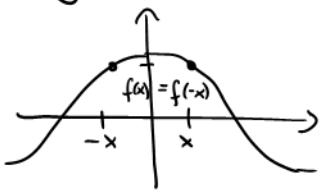
Die Fkt $f: \mathbb{R} \to \mathbb{R}$, $f(x) = \cos(2x)$ ist

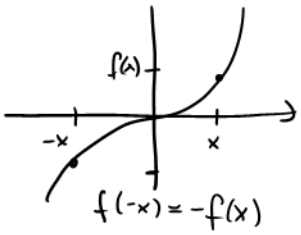


--- cos(x)

zu 6.4

Symmetrisches Intervall: $I = (-a, a)$ oder $[-a, a]$, oder $(-\infty, \infty) = \mathbb{R}$





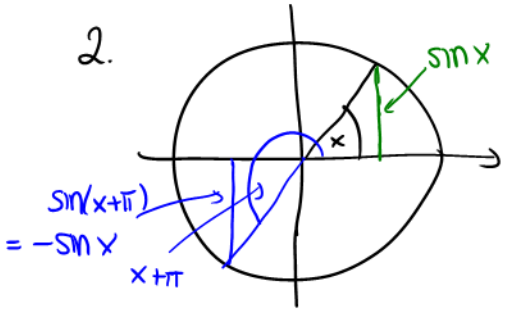
Bsp:
- $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$ ist gerade, denn
  $f(-x) = (-x)^2 = x^2 = f(x)$
- $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^3$ ist ungerade, denn
  $f(-x) = (-x)^3 = \underbrace{(-1)^3}_{=-1} \cdot x^3 = -x^3 = -f(x)$
- f gerade, g ungerade $\Rightarrow$ f·g ist ungerade

Bem: Für jede ungerade Fkt gilt $f(0) = 0$, denn

$f(0) = f(-0) = -f(0) \Rightarrow 2f(0) = 0 \Rightarrow f(0) = \frac{0}{2} = 0$.

Aufg 6.5

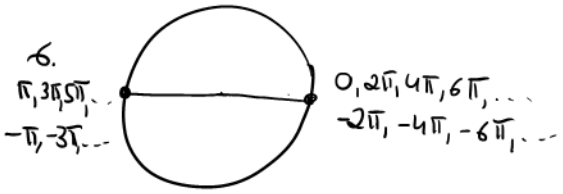
2.



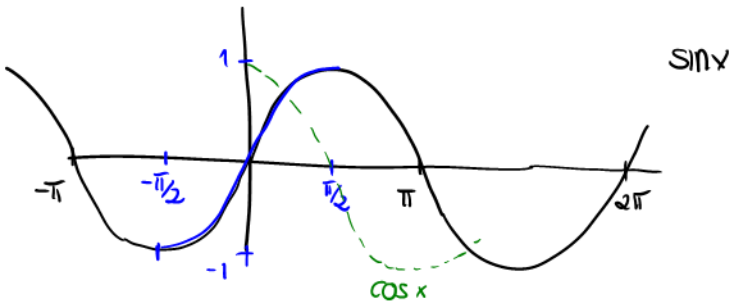
5. tan ist ungerade, da

$$\tan(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin(x)}{\cos(x)} = -\frac{\sin x}{\cos x} = -\tan x$$

6.



Aufg 6.6



Aufg 6.7

$\arcsin(\sin(\frac{\pi}{4})) = \frac{\pi}{4}$, da $\frac{\pi}{4} \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$\arcsin(\sin(\frac{5}{4}\pi)) = -\frac{\pi}{4}$, da $\sin(-\frac{\pi}{4}) = \sin(\frac{5}{4}\pi)$ und $-\frac{\pi}{4} \in [-\frac{\pi}{2}, \frac{\pi}{2}]$