Wachmacher-Aufgabe 7

$$\frac{\sqrt{3}+\sqrt{12}}{\sqrt{3}-\sqrt{12}} = ?$$

Lösung:

$$\frac{\sqrt{3}+\sqrt{12}}{\sqrt{3}-\sqrt{12}} = \frac{(\sqrt{3}+\sqrt{12})^2}{(\sqrt{3}-\sqrt{12})(\sqrt{3}+\sqrt{12})} = \frac{(\sqrt{3})^2+2\sqrt{3}\cdot\sqrt{12}+(\sqrt{12})^2}{(\sqrt{3})^2-(\sqrt{12})^2} = \frac{3+2\cdot\overbrace{\sqrt{36}}^{6}+12}{3-12} = \frac{27}{-9} = -3$$

zu 7.8

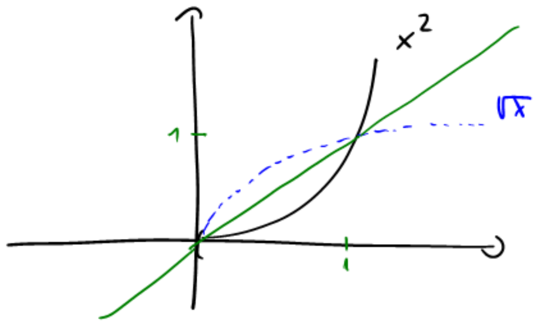
$f: (0,\infty) \to (0,\infty)$, $f(x)=x^2$

hat die Umkehrfkt

$f^{-1}: (0,\infty) \to (0,\infty)$, $f^{-1}(y) = \sqrt{y}$

Es gilt $f'(x) = 2x$ für $x \in (0,\infty)$, also

$$(f^{-1}(y))' = (\sqrt{y})' \underset{\substack{x=f^{-1}(y)\\=\sqrt{y}}}{=} \frac{1}{f'(\sqrt{y})} = \frac{1}{2\sqrt{y}}$$

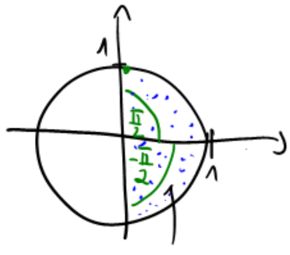


zum Bsp unter 7.8

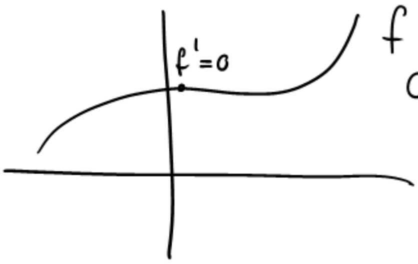
$\cos^2 x + \sin^2 x = 1 \Rightarrow \cos^2 x = 1 - \sin^2 x \Rightarrow \cos x = \pm\sqrt{1-\sin^2 x}$

Hier: $x = \arcsin(y)$ mit $y \in (-1,1)$, also $\arcsin(y) \in (-\frac{\pi}{2}, \frac{\pi}{2})$

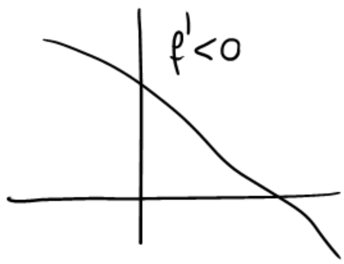
Damit folgt $\cos(\underbrace{\arcsin y}_{\in(-\frac{\pi}{2},\frac{\pi}{2})}) > 0$, also $\cos x = +\sqrt{1-\sin^2 x}$



zu 8.1



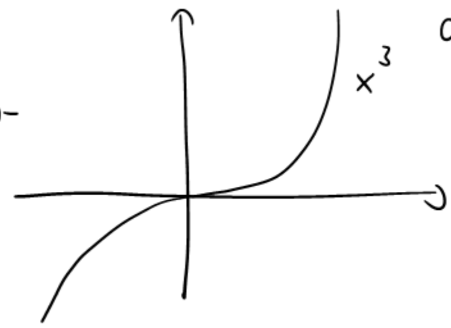
f mon. wachsend, da $f' \geq 0$



f streng mon. fallend, da $f' < 0$.

Bsp: $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^3$ ist streng monoton wachsend (damit automatisch mon. wachsend)

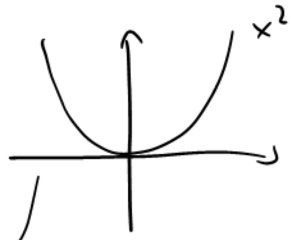
Aber: $f'(x) = 3x^2$, also $f'(0) = 0$

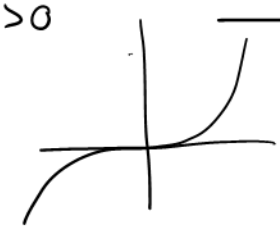


zu 8.2

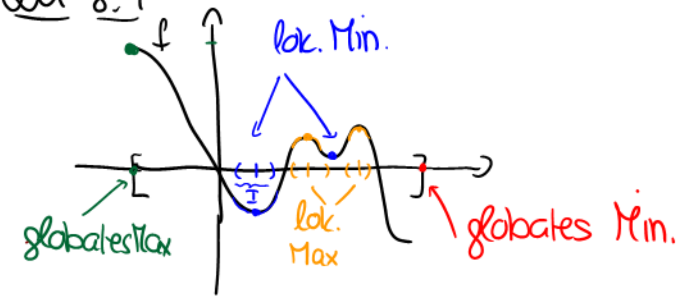
- $f: \mathbb{R} \to \mathbb{R}$, $x \mapsto x^2$ ist linksgekrümmt, da $f''(x) = (f'(x))' = (2x)' = 2 > 0$
- $f: \mathbb{R} \to \mathbb{R}$, $x \mapsto -x^2$ ist rechtsgekrümmt, da $f''(x) = -2 < 0$.

zu 8.3 $f:\mathbb{R}\to\mathbb{R}$, $x \mapsto x^3$ hat in $x_0=0$ eine Wendestelle, da $f''(x) = (3x^2)' = 6x \begin{cases} <0, & x<0, \\ 0, & x>0 \end{cases}$



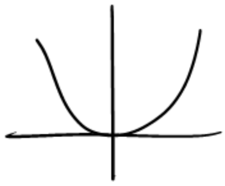


zu 8.4



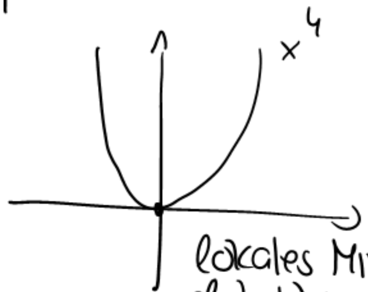
Bem: Randpunkte von D sind als lokale Extrema nicht erlaubt!

zu 8.5



$g:\mathbb{R}\to\mathbb{R}$, $x \mapsto x^2$
g hat in $x_0=0$ lokales (und globales) Minimum,
also $f'(0)=0$ (tatsächlich: $f'(x)=2x$, also $f'(0)=0$)

zu 8.6



lokales Min (auch global) in $x_0=0$

$h:\mathbb{R}\to\mathbb{R}$, $h(x) = x^4$ hat in $x_0=0$ ein lok. Min.

Also: $h'(x) = 4x^3$. und $h'(0) = 4\cdot 0 = 0$ ✓

Aber: $h''(x) = 12x^2$ und $h''(0) = 12 \cdot 0 = 0$ ✗