

Bachelor Seminar Analysis/PDG

Winter Semester 2022/2023

Prof. Dr. Peter Bella

Vorkenntnisse: Analysis I,II,III

Ort und Zeit: Siehe Vorlesungsverzeichnis

Themen:

1. Calderon-Zygmund Theorie: Für u eine Lösung von $-\Delta u = f$ und $f \in L^p(\mathbb{R}^d)$ gilt $D^2u \in L^p(\mathbb{R}^d)$, $p = 2$. Gilt das für andere p ? ([9, Kap 7.1,7.2], oder auch [3])
2. BMO Raum - Interpolation und John-Nirenberg Ungleichung [9, Kap 7.4]
3. De Giorgi-(Nash)-Moser Theorie [5, Kap 4]
4. Moving-Plane Method [4, Kap 9.5.2], [5, Kap 2.6]
5. Einführung in Optimal Transport [2, Kap 1 und Teil von Kap 2]
6. Mountain Pass Theorem [4, Kap 8.5]
7. Rearrangement Inequalities [6, Kap 3 bis Satz 3.7]
8. Variational Models for Phase Transition [1]
9. Gradient Estimates and Liouville Theorem for nonlinear Poisson Equation [7]
10. Singular perturbations as a selection criterion for periodic minimizing sequences [8]

References

- [1] G. Alberti, *Variational models for phase transitions, an approach via Γ -convergence*, Calculus of variations and partial differential equations (Pisa, 1996), Springer, Berlin, 2000, pp. 95–114. MR 1757697
- [2] Luigi Ambrosio, *Lecture notes on optimal transport problems*, Mathematical aspects of evolving interfaces (Funchal, 2000), Lecture Notes in Math., vol. 1812, Springer, Berlin, 2003, pp. 1–52. MR 2011032
- [3] Javier Duoandikoetxea, *Fourier analysis*, Graduate Studies in Mathematics, vol. 29, American Mathematical Society, Providence, RI, 2001, Translated and revised from the 1995 Spanish original by David Cruz-Urbe. MR 1800316
- [4] Lawrence C. Evans, *Partial differential equations*, second ed., Graduate Studies in Mathematics, vol. 19, American Mathematical Society, Providence, RI, 2010. MR 2597943

- [5] Qing Han and Fanghua Lin, *Elliptic partial differential equations*, second ed., Courant Lecture Notes in Mathematics, vol. 1, Courant Institute of Mathematical Sciences, New York; American Mathematical Society, Providence, RI, 2011. MR 2777537
- [6] Elliott H. Lieb and Michael Loss, *Analysis*, second ed., Graduate Studies in Mathematics, vol. 14, American Mathematical Society, Providence, RI, 2001. MR 1817225
- [7] Luciano Modica, *A gradient bound and a Liouville theorem for nonlinear Poisson equations*, Comm. Pure Appl. Math. **38** (1985), no. 5, 679–684. MR 803255
- [8] Stefan Müller, *Singular perturbations as a selection criterion for periodic minimizing sequences*, Calc. Var. Partial Differential Equations **1** (1993), no. 2, 169–204. MR 1261722
- [9] Camil Muscalu and Wilhelm Schlag, *Classical and multilinear harmonic analysis. Vol. I*, Cambridge Studies in Advanced Mathematics, vol. 137, Cambridge University Press, Cambridge, 2013. MR 3052498