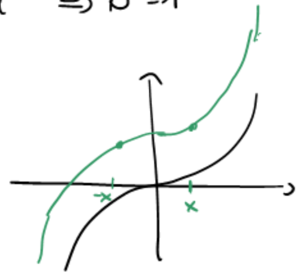
Aufgabe: $\frac{1}{a}\sqrt{a^4} = \frac{1}{a}(a^4)^{\frac{1}{2}} = \frac{1}{a}\, a^{4\cdot\frac{1}{2}} = \frac{a^{\cancel{2}\,1}}{\cancel{a}} = a = a^1 \Rightarrow b = 1$



zu 6.4: Für jede ungerade Funktion $f: I \to \mathbb{R}$ gilt
$f(0) = 0$, denn für $x = 0$ gilt

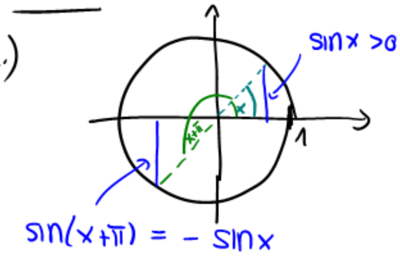
$$f(0) = f(-0) \overset{f \text{ ungerade}}{=} -f(0) \Rightarrow 2\cdot f(0) = 0 \Rightarrow f(0) = 0$$

Bsp: Sei f gerade, g ungerade $\Rightarrow f\cdot g$ ist ungerade, denn

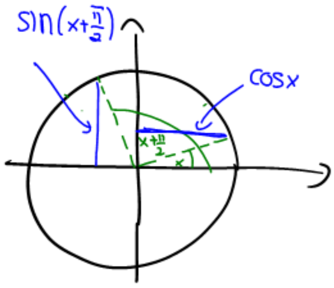
$$(f\cdot g)(-x) = \underbrace{f(-x)}_{f \text{ gerade } = f(x)}\underbrace{g(-x)}_{g \text{ ungerade } = -g(x)} = f(x)\cdot(-g(x)) = -(f\cdot g)(x)$$

zu 6.5

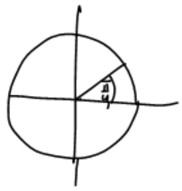
2.)



3.)

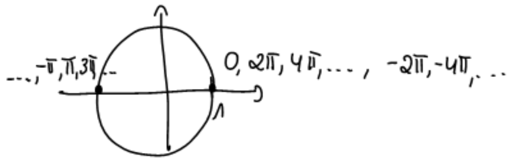


$\sin(\frac{\pi}{4}) = \cos(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$

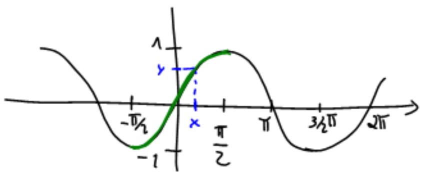


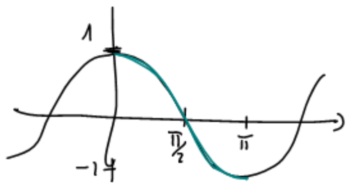
4.) $\tan\left(\frac{81\pi}{4}\right) = \tan\left(20\pi + \frac{\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) = \frac{\sin(\frac{\pi}{4})}{\cos(\frac{\pi}{4})} = 1$

7.)



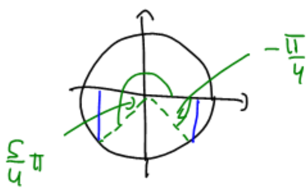
zu Bem 6.6:





Bsp zu 6.7 $\quad \arcsin\left(\sin\left(\frac{\pi}{4}\right)\right) = \frac{\pi}{4}$

$\arcsin\left(\sin\left(\frac{5}{4}\pi\right)\right) = \arcsin\left(\sin\left(-\frac{\pi}{4}\right)\right) = -\frac{\pi}{4}$



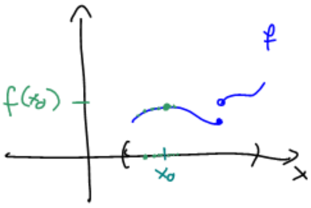
zu 6.8: $\cos(\underset{x}{\pi} + \underset{y}{\tfrac{\pi}{4}}) = \underbrace{\cos\pi}_{=-1}\cdot\cos\frac{\pi}{4} - \underbrace{\sin\pi}_{=0}\cdot\sin\frac{\pi}{4} = -\cos\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

zu 6.9: $\sin(3x) = \sin(2x + x) = \sin(2x)\cos x + \sin x\cos(2x)$

$\overset{1.,2.}{=} 2\sin x\cos x\cdot\cos x + \sin x\left(\cos^2 x - \sin^2 x\right)$

$= 2\sin x\cdot\cos^2 x + \sin x\cos^2 x - \sin^3 x = 3\sin x\cos^2 x - \sin^3 x$

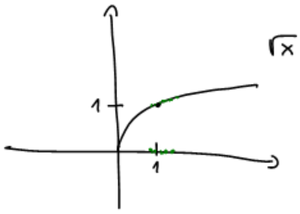
zu 7.1



$$\lim_{x\to x_0} f(x) = \underbrace{f(x_0)}_{=a}$$

Bsp: $f: [0,\infty) \to \mathbb{R}$, $f(x) = \sqrt{x}$

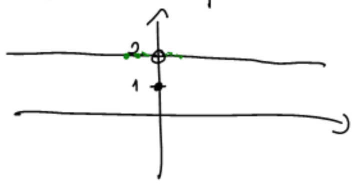
$$\lim_{x\to 1} f(x) = 1$$



Allgemein: Für jedes $x_0 \in [0,\infty)$ gilt $\lim_{x\to x_0} \sqrt{x} = \sqrt{x_0}$, also ist f auf $[0,\infty)$ stetig.

zur Heaviside-Fkt: $\lim_{x\to 0} H(x)$ existiert nicht.
Insbesondere ist H in $x_0 = 0$ nicht stetig.

Bsp: $f: \mathbb{R} \to \mathbb{R}$, $f(x) = \begin{cases} 2 & , x \neq 0 \\ 1 & , x = 0 \end{cases}$



$\lim_{x\to 0} f(x) = 2 \neq 1 = f(0)$. Daher ist f in $x_0 = 0$ nicht stetig.

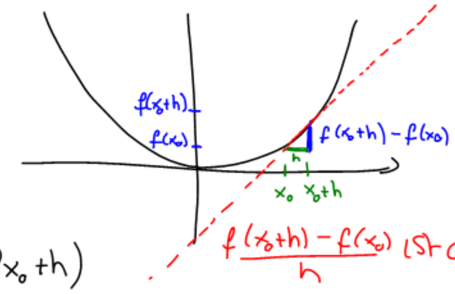
zu 7.2 $f: \mathbb{R} \to \mathbb{R}$, $x \mapsto x^2$

Sei $x_0 \in \mathbb{R}$ beliebig. Dann ist

$$\lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h} = \lim_{h\to 0} \frac{(x_0+h)^2 - x_0^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x_0^2} + 2x_0 h + h^2 - \cancel{x_0^2}}{h} = \lim_{h\to 0} \frac{2x_0 \cdot h + h^2}{h} = \lim_{h\to 0} (2x_0 + h)$$

$$= 2x_0 = f'(x_0)$$



Sekante durch die Punkte $(x_0, f(x_0))$ und $(x_0+h, f(x_0+h))$

$\frac{f(x_0+h) - f(x_0)}{h}$ ist die Sekantensteigung

Gerade:

$T(x) = ax + b$

hier

$$\underbrace{f'(x_0)}_{a} \cdot x + \underbrace{f(x_0) - x_0 f'(x_0)}_{b}$$