Development of Personal Constructs about mathematical tasks –

A qualitative study using Repertory Grid Methodology

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Abstract: We can fill the gap between theory (of ambitious teacher education) and practice (in mathematics classrooms) more easily, when we take into account that teacher education is not only explicit acquisition of knowledge and methods but also development of individual personal constructs of mathematics and mathematics education. But how can we get knowledge about the development of individual implicit theories of teacher students? In this article, we propose a method to explore teacher students' individual conceptions about learning and teaching mathematics. It is based on Kelly's personal construct theory and uses repertory grid methodology for data collection as well as formal concept analysis for data analysis. The method was applied in a qualitative evaluation study of a pre-service teacher course in Darmstadt.

Various research studies have unanimously shown that classroom activities of teachers are heavily influenced by implicit subjective theories (operationalized as cognitions, beliefs, attitudes, or personal constructs, respectively). These implicit theories relate to all aspects of mathematics classrooms, especially mathematics, methods, teaching, learning, the teachers' role, and the pupils (see Thompson 1992, or Törner 1997 for overviews). The fundamental insight in the importance of implicit theories should therefore have consequences in teacher education. Sophisticated teacher training conceptions should not only focus on the explicit transmission of information, conceptions and methods for learning and teaching mathematics but also on the development of the implicit subjective theory of each teacher student. It was in 1983 already that Alan Bishop emphasized that the main aim of teacher education should be the "broadening and development of teacher's personal constructs." (Bishop 1983, p. 21). In the course of his argumentation, he refers to the theory of personal constructs developed by the clinical psychologist Kelly (1955). This approach was theoretically and methodologically deepened by McQualter who analyzed the process of "becoming a mathematics teacher" with personal construct theory (1986).

In this article, we follow the idea that it is a main task for teacher education to develop teachers' personal constructs. As a consequence, we claim for including this aspect into the evaluation of teacher training modules. Therefore, qualitative evaluation studies should also focus on the individual development of personal constructs.

Investigating Personal Constructs

The constructivist key message of Kelly's personal construct theory is that the world is 'perceived' by a person in terms of whatever 'meaning' that person applies to it (Kelly 1955, Fransella/Bannister 1977). The basis on which persons develop their personality, attitudes, and concepts and perceive reality are their systems of personal constructs.

> "It is important to note that constructs are not only names, or concepts, or attitudes, or opinions. Constructs have a function for the individual. They serve as tools to replicate events in our imagination, and to make up our view of the world by continuous confirmation or disconfirmation, thus 'to construe reality' (the *construction* corollary). Constructs are organized in systems, often hierarchical in structure, there are superordinate constructs, core constructs, peripheral constructs, according to their importance to the individual's life (the *organisation* corollary). We have different construct systems for different areas and realms which may even be partially incompatible or at least contradictory when involved at the same time (the *fragmentation* corollary). Constructs in principle can be changed through experience (the *experience* corollary). […] Constructs are significant characteristics of the individual (the *individuality* corollary), i. e. 'personal'." (Scheer 1996).

We agree with Bishop (1982) and McQualter (1986) that mathematics teachers' perceiving and acting is highly determined by their personal constructs about teaching and learning. Hence, a qualitative evaluation of teacher training modules should also focus on these implicit elements. But how can they be investigated?

In the methodology of qualitative social research, a large variety of methods have been developed to identify implicit theories and belief systems. The test procedures and methods for knowledge elicitation mainly differ in their degree of standardization. One extreme is the completely standardized questionnaire offering multiple choice answers only. The advantage of this kind of data collection is that it can be compared directly and is analysed qualitatively as well as quantitatively. On the other hand, the missing possibility for test persons to express their thoughts in their own language produces reductive results which sometimes cannot adequately explore the probands' implicit theories. The other extreme is the free interview without any structured guidelines. This kind of knowledge elicitation is not reductive but the results are not easily comparable and the processes of interpretative analysis are too sophisticated for evaluation of teacher education.

Kelly himself developed a methodology for exploring systems of personal constructs by so-called repertory grids (1955). A repertory grid guides a form of highly structured interview, formalizing the interactions of interviewer and interviewee and putting into relations personal constructs and given objects of discourse (more details in the next section).

In a procedure described below, the person first defines the area that the test is to be applied to ("elements"), then he develops the items ("constructs"), then the test person completes the grid that is made up by the two dimensions, elements and constructs. Repertory grids try to combine the advantages of both extremes: having a structured way of data collection in order to simplify the analysis afterwards while no language in which the test persons are supposed to express their implicit theories and personal constructs is imposed.

The method of repertory grids has been applied increasingly in clinical psychology and psychodiagnostics concerned with self conceptions and social relations (cf. Fransella/Bannister 1977, Spangenberg 1990).

In mathematics and science education research, there is an increasing number of studies using repertory grids, for example to explore teachers' beliefs about educational principles and aims (in science education Fischler 1996), teachers' views of mathematics (Williams/ Pack 1997) and pupils' beliefs on being good or poor in mathematics (Hoskonen 1999; more studies are cited in his paper). The closest to ours is McQualter (1986) who analysed the *development* of mathematics teacher's role conceptions within a teacher education program. Non of the existing studies used our approach of starting with the mathematical tasks as an important tool to design learning environments, and all of them used different methods for data analysis.

Design of the Study

We activated repertory grid methodology to evaluate the course "diagnostics of learning efficiency" for prospective mathematics teachers held by Regina Bruder in Summer 2002 in the department of mathematics education at Darmstadt University of Technology. The course focused on the design and the assessment of mathematical tasks for multi-faceted diagnostics of learning effects (for all details concerning the study see Bruder/Lengnink/Prediger 2003). We started out our investigation with the hypothesis that the course would not only increase the teacher students' explicit knowledge about mathematical tasks and their didactical functions but that it would also change and develop their implicit personal constructs about mathematical tasks. In order to evaluate this assumption, we have interviewed 16 students by means of repertory grids in the beginning and at the end of the course.

In a *first step* of these repertory grid sessions, the probands were supposed to become familiar with a set of mathematical tasks which we proposed as the elements of our study (see Table 1 on the next page). All of these tasks are belonging to the algebraic treatment of linear equations but vary in nature and content.

In a *second step*, the interviewer proposed three pairs of tasks. Then the interviewees were asked to find at least one pair of attributes which separates the tasks. In this step, personal constructs are specified which are relevant for the implicit theory of the probands. In a *third step* we organized the tasks and the attributes in a two-dimensional grid, the so-called repertory grid (see Fig. 1 for an example). The students were asked to add other attributes which they considered to be important for talking about mathematical tasks. Then they were supposed to fill the grid with crosses between all tasks and attributes which they considered to be related in their view.

The main idea of this approach is that the probands are free to talk about the mathematical tasks in their categories and their language.

1. Solve the following equations. The variable for solution is x.

   18x-25-11x+49=3 3x + 12a = 0

   3,7x+0,8=6,1-5,3x+0,7 5(x-a)=0

2. Find a linear equation that has the solution – 5.
3. Can you find any linear equations with more than one solution and linear equations without any solution?
4. Chris has tried to find three succeeding natural numbers with the sum 81. For this problem, he formulated the following equation: (n-1) + n + (n+1) =81. What does the n signify?
   - the smallest of the three natural numbers
   - the middle of the three natural numbers
   - the greatest of the three natural numbers
   - the difference between the smallest and the greatest of the three natural numbers.
5. Write down a problem which can be formalized by the following equation: 3(0,5x-7)=5-1,5x
6. In two boxes are 54kg apples. The second box has 12kg more weight than the first box. How much kg apples are in each box?
7. Give examples of applications, for which it is worth to formulate and solve a linear equation. Or: For what do we need linear equations?

Table 1: Set of tasks used as the elements in the interviews

By doing so, we can investigate how teacher students think and speak about a main instrument of mathematics classrooms without a filter of prefixed possibilities of answers, i.e. we want to identify their personal constructs without imposing given constructs.

Notwithstanding this openness, the process of the construct elicitation is strongly structured: sequence of steps, the focus on separating attributes resulting from the comparison of two given tasks, the restriction to the relation "element has the attribute", the demand to appraise all tasks with respect to their attributes. These restrictions serve as structural aids to make personal constructs explicit which the test persons are usually not conscious of.

	comprehensive task	computational task	find a problem for a solution	find a solution for a problem	closed	open	formal/algorithmic task	"heuristic" task	comprehend a solution	find a solution	metacognitive task	isolated task	geometrical understanding	reality-orientation
T1		X			X		X					X		
T2	X		X		X									
T3	X					X		X						
T4					X				X					
T5	X		X		X			X						
T6		X		X	X					X				
T7						X					X			X

Fig. 1: First Repertory Grid of Test Person 1

Data Analysis

For analysing such repertory grids as ours, different methods of data analysis can be used. Often, complexity is reduced by using factor or cluster analysis. We decided to take a more cautious instrument of data analysis which has been used for repertory grids in psychoanalysis by Spangenberg/Wolff (1988) for the first time: Formal Concept Analysis (see Ganter/Wille 1999 for the well developed mathematical background). It allows to visualize the structures of small grids in a line diagram without loss of information. Hence, the data can be represented without artefacts being produced by the analysis itself which is very important for psychodiagnostic contexts like ours.

We can read off the repertory grid of Fig. 1 from the line diagram in Fig. 2 in the following way: every task is assigned to a circle, and the related attributes can be found by following ascending lines to the attributes. For example, task 5 has the attributes "comprehensive task", "closed", "find a problem for a solution" and "heuristic task". Dually, an attribute is assigned to all tasks which can be reached by descending lines.

The line diagram does not only represent the grid but it also makes its logical structure explicit. We can find logical dependencies between attributes: e.g. "find a problem for a given solution" implies "computational task" and "closed" since these attributes can be reached from "find a problem for a solution" via ascending lines. Even incompatibility become visible: In the context of the given grid, there is no task which is considered to be a "computational task" and to have "reality-orientation" at the same time.

The line diagram represents the landscape of the implicit theory of the test person and allows its systematic investigation. On this basis, the test person's understanding of constructs like "open" and "closed" can be explored and clarified.

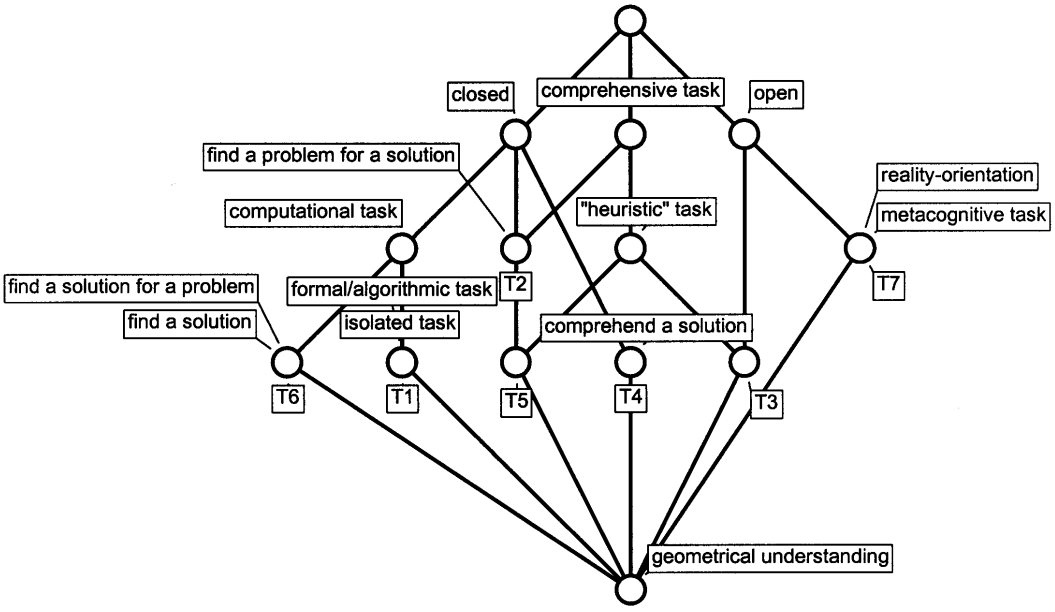

Fig. 2: Line Diagram for the first Repertory Grid of Test Person 1

Research Questions and selected Results

In the evaluative study of our course, we were interested in comparisons between individuals' grids and in the development of their implicit theories during the course. Therefore, our analysis was guided by the following research questions:

- What attributes are taken by a student, how do they differ in the second interview and how do they differ from other students? Can we identify a development towards the didactical terminology offered in the course? How is it integrated into the individual system of personal constructs?
- On what categories do the different persons focus?
- How does the focus change from the first to the second investigation?
- Which degree of generality and which degree of differentiation do the given attributes have and how does it change?
- Can we specify typical patterns of development for the identified systems of personal constructs?

Due to restricted space, we can only give some hints on selected results concerning the last research question, namely the patterns of development. Our study has not only shown that the students strongly differ in their respective focus which is expressed through differing linguistic levels and categories about the tasks. Students also vary in their ways how they integrate the learned vocabulary and theories about mathematical tasks into their individual system of personal constructs. We have found essentially four patterns of developments:

- replacement
- extension
- differentiation
- exactification

All of them can be found in the development of Test person 1 (see table 2), whereas Test person 2 shows essentially patterns of replacement.

Table 2: Categorization of Test person 1's constructs

Categories	Test Person 1's constructs in first interview	Test Person 1's constructs in second interview
demanded activities	heuristic task find a problem for a solution find a solution find a solution for a problem comprehend solution	working forward working backward applying/doing describing/reasoning
degree of difficulty		solvable in one step
format and structure	closed task open task formal, algorithmic task	closed task open task word problem solvable in one step broadened basic task xx- basic task -xx inverse task x— purely mathematical task/completely formalized
aim and content dimension	reality-orientation geometrical understanding comprehensive task computational task isolated task	
didactical function	meta-cognitive task comprehensive task computational task	meta-cognitive task problem solving continuation / task at the end of a subject task in the beginning of a new subject: basic task

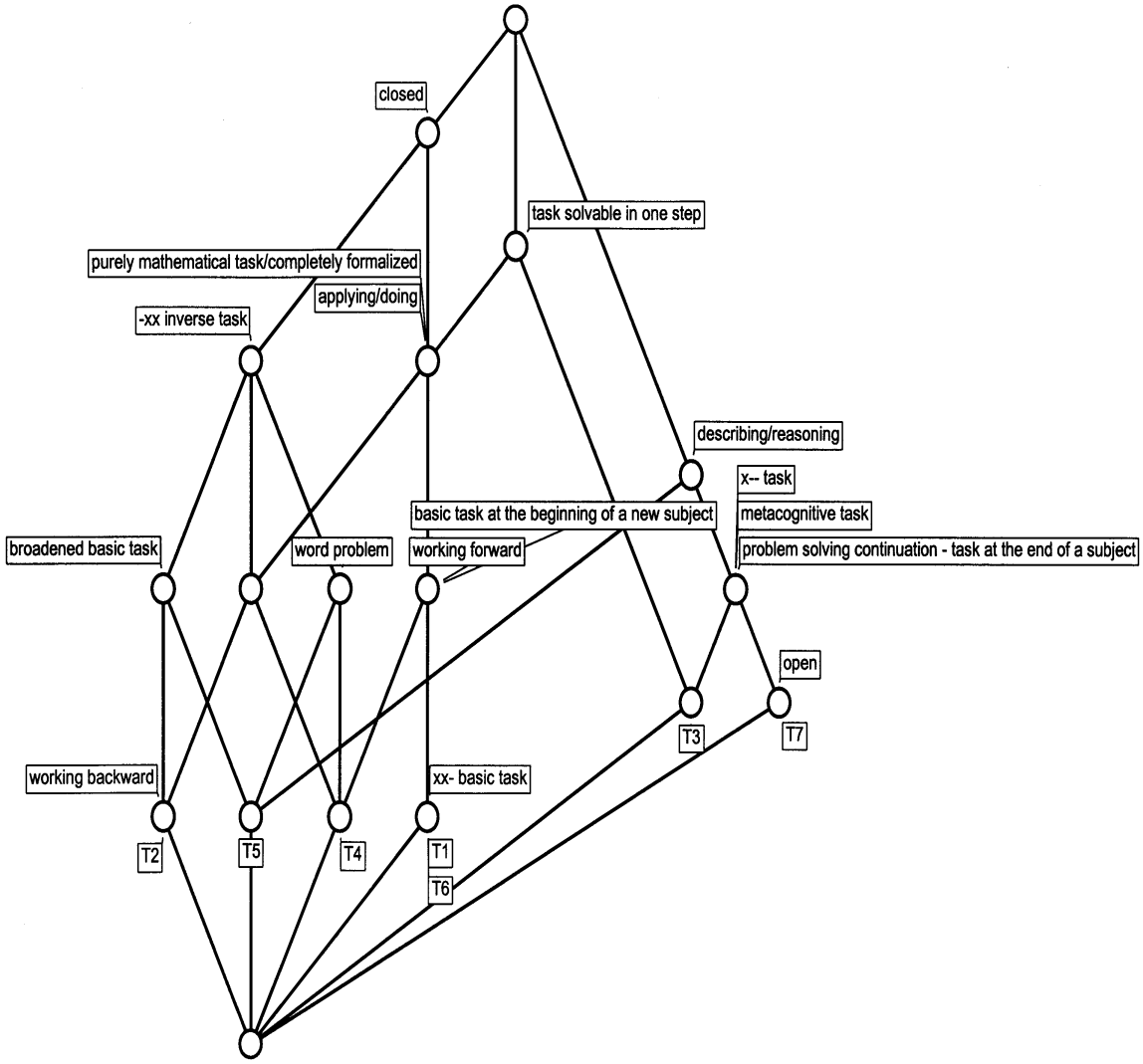

Fig 3: Line Diagram for the second Repertory Grid of Test Person 1

One typical pattern of development can be seen by comparing the systems of personal constructs of Test Person 1 (see Fig. 2 and 3). His first diagram shows a clear focus on attributes concerning the categories "task-format" and "requested learning-activities". It is significant that this remains constant in the second evaluation, although the vocabulary is professionalized and the didactical terms have been integrated in the students' subjective theory.

For illustration, consider the attributes "find a solution for a problem", "find a problem for a solution" and "comprehend a solution" of the first diagram. In the second investigation those attributes have been replaced by a complete characterization of task-formats: In a triple consisting of (precondition, way of solution, solution) for every task it is indicated which part is given and which is searched for. This language was introduced in the course and has been integrated in this personal construct, probably because of its utility to describe the former intuitive distinction of tasks with respect to their format.

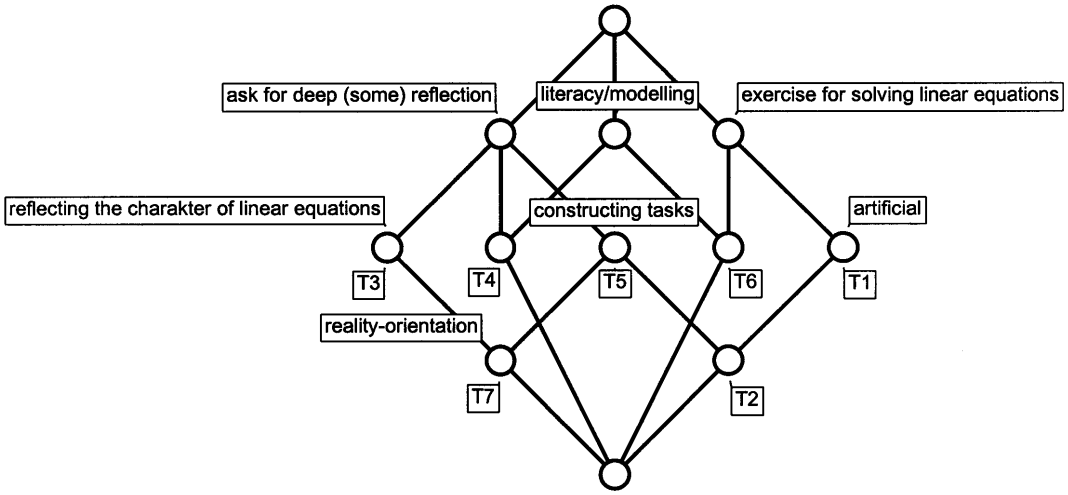

Fig 4: Line Diagram for the first Repertory Grid of Test Person 2

In contrast to Test Person 1 there is another type of student, who constructs an almost completely new theory about tasks during the course. As an example see the development of repertory-grid line diagrams constructed by Test Person 2 (Fig. 4 and 5). This person has dropped his former analysis of the tasks and adopted the new language used in the course. The attributes chosen for tasks in the second evaluation are certainly more general than the ones of the first evaluation. The diagram is smaller and the structure is simpler.

But even though the diagram of the second system of personal constructs is less complex than the one of the first evaluation, a learning effect can be demonstrated by the analysis. In the second evaluation, the test person focuses on three interesting categories of attributes for tasks, namely “the intention of tasks”, “the task-format” and “the requested learning activities”. Discussion of these diagrams could help this student to analyze his own learning level and to show the necessity of having a more differentiated language.

Besides these two types of learners, there is at least one other interesting type of student which enriches his language by some professional concepts, whereas the basic language remains almost constant. Although this categorization into three different patterns of development is idealized and some students may belong to more than one of the described types, it helped us to understand the different learning effects in didactical courses.

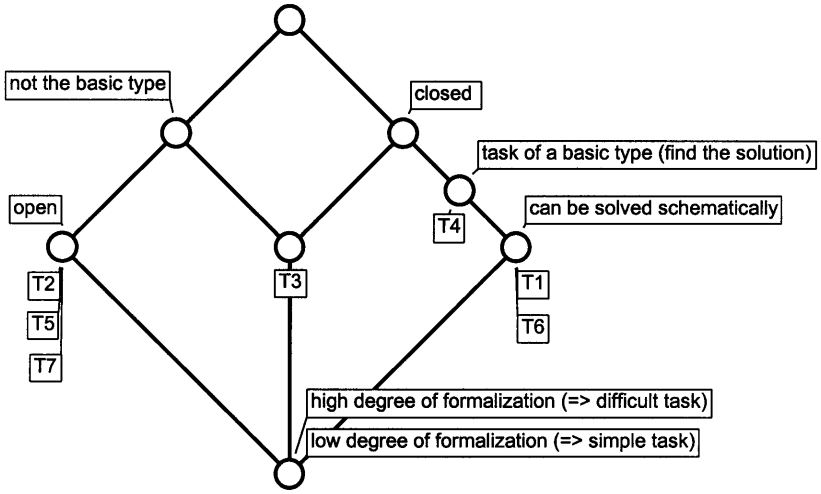

Fig 5: Line Diagram for the second Repertory Grid of Test Person 2

Conclusion

Repertory grid methodology, combined with formal concept analysis, has proved to be a promising tool to investigate the development of personal constructs of the teacher students. The line diagrams help to understand how teacher students think and talk about mathematical tasks in their own language. Whereas tests like classical evaluation tools usually assess the increase of explicit knowledge within the course, the comparison of repertory grids has given us the opportunity to explore the patterns of individual development on the implicit level. The gap between the official learning content and the individual way to integrate it into the system of personal constructs gives us a clue about reasons for the gap between theory (of ambitious teacher education) and the prospective practice in mathematics classrooms: Although the teacher students have learned their lessons of explicit knowledge quite well, they have only fragmentarily integrated it into their individual thinking. This is why we should pay more attention to the evolution of individuals' implicit theories.

For this aim, we have proposed a methodology which proved to be useful, but not without difficulties. According to our experience, the most critical step in this method is the specification of mathematical tasks which serve as elements for the grids. These choices heavily influence the resulting landscapes of constructs.

In our future research, the method will be complemented by direct discussion about the constructed repertory grids. This will clarify open questions of interpretation and, at the same time, it offers an opportunity for consulting the teacher students about their individual thinking and their perspectives of developing their personal constructs.

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