DG 4: Philosophy of mathematics education

Team Chairs: ***Susanne Prediger***, University of Dortmund, Germany
Maria Viggiani-Bicudo, State University of São Paulo, Brazil
Team Member: ***Paul Ernest***, University of Exeter, United Kingdom

Aims and focus

The aim of Discussion Group 4 was to explore the nature, role and state of Philosophy of Mathematics Education (PhoME) and particular themes focused on the perspective of PhoME.

The group met three times. The initial part of the first session was dedicated to an orientation with an introductory overview "What is philosophy of mathematics education?" (see Ernest, 2004, for a written version). The second session, "Strands and issues for discussion within PhoME", took place in smaller groups addressing different questions, followed by a synthesizing session. The prepared questions were: What are the conceptions of mathematics and mathematical knowledge underlying different learning theories? What roles do philosophies of mathematics play in the teaching and learning of mathematics? How do they relate to mathematics curriculum, teaching reforms and classroom practices?

Different perspectives – a first metaphoric approach

The group agreed on Paul Ernest's suggestion to consider the PhoME not as one single (perhaps dominant) position, but as an area of investigation (2004, p. 1). Beyond this most general description, there is a great variety of possible approaches. The discussion in the group was dominated by the experience that the question for the core of PhoME can be answered in many different ways, each of them interesting and with a totally different perspective.

In order to give a first intuition about the different possible perspectives, we will start the section by quoting the story told by Jean Paul Bendegem as his report from one subgroup. It illustrates the range of important questions in PhoME:

"Rather than presenting here a faithful reproduction of the discussion itself, I, as reporter, have taken an option to summarize our findings in the form of a story. The story runs like this.

As we know this conference started out from building 101. Now suppose that someone, let us call her the teacher, explained to us how to get to building 208. We, the pupils, are given a set of instructions to find 208. So we all wander out: some of us get their straight away, some get absolutely lost, some ended up in another building, some in wild nature and, actually, there was one person who found his way, although he did not hear the instructions.

Let us analyse this short story on the premises that we want to understand what is going on. What can you do? Well, you can start "bottom-up", of course. What one studies are the (specific) relations between teachers and pupils and one ends up with questions like:

- What are the philosophical ideas of the teacher?
- What are good road indications and how does his or her philosophy help to determine (if so) the instructions?
- How did the pupils understand the instructions? How does their cultural background interfere, or, if you like, what are the philosophical views of the pupils?

In a way what one does in answering these questions is to render the implicit explicit. This in turn raises a two-fold question:

- Are there several ways to make the implicit explicit, how does one justify a choice of methodology?
- At the same time, making the invisible visible can trouble one's view. In concrete terms: will it actually help teachers and/or pupils to know explicitly this implicit background? Will it necessarily constitute an improvement?

If one does not feel all too happy with this approach, a different route can be tried out: let us look at the problem how the instructions relate to the actual (I prefer not to use "real") situation. This perspective creates a different set of questions:

- Is there in fact a road from building 101 to 208? Or are there many roads and do we simply prefer (for whatever reasons) one particular road over all the other possibilities?
- Or, quite the opposite: as it turns out, there is no road. The instructions are in a sense an invitation to wander out and make or construct a road. Subquestions here would be what kind of roads we make in this way and, of course, how we do the constructing.

This metaphorical way of speaking refers to a large part of our discussion about the opposition between a structuralist view (sloganesque: "The language of road instructions is the language of set theory") and a more or less radical constructivism ("Roads are created collectively by people moving about in the open field".) However there is no need to stop here!

So far there has been no questioning of the fact why we all have to go to building 208 in the first place. And, of course, this leads us into a new set of "big" questions:

- Why should we all know where building 208 is?
- Who built the damn thing in the first place?
- What other buildings (if that is what they are supposed to be?) are possible, desirable, and accessible to few, many or all?

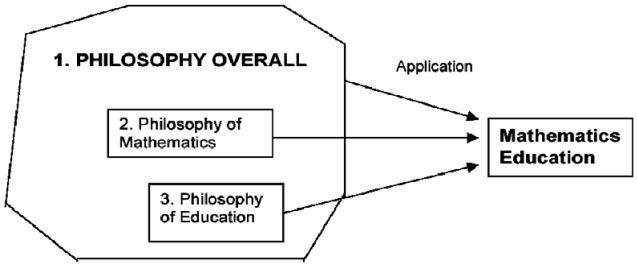
These questions invite us to philosophise about societal issues about mathematics education. And, for that matter, to be openly critical about it. And, finally, to wonder why all of a sudden we became so critical?

The story leads me quite easily to some observations in the guise of conclusions:

- At all levels and from all perspectives mentioned philosophy does enter into the picture. However the role philosophy has to play is quite different in each case. Perhaps part of the complexity of the problem of what a philosophy of mathematics education can or should be, resides in this fact.

- Is there a kind of "division of labour" imaginable? Can, e.g., questioning the existence of building 208 be done more or less independently from questioning the quality of the instructions? The view that "all is connected to all" is perhaps a philosophically pleasing view, but often one is left unable to act.
- And, finally, perhaps the most important one: who will listen to whom? Or better still: who is prepared and willing to listen and be listened to? To see the complexity of the issues involved, just ponder the following question: imagine that a philosopher somehow manages to show convincingly that building 208 does not exist, what is the poor teacher in building 101 supposed to do? If you know the answer to that, an important problem would have been solved." (Jean Paul van Bendegen at ICME-10)

Picking up the story systematically, we can see that the different questions to be posed are influenced each by a different understanding of the term 'philosophy' in 'philosophy of mathematics education'. It is obvious in theory but a challenge for communication in practice that the notion of philosophy and its relation to practice is understood quite differently by the different participants of the discussion group. Each understanding is one the one side influenced by the participants' culture and tradition in each country, and on the other side by the question on what exactly the philosophical focus is. This last question has been raised by Stephen Brown (1995) by posing a trichotomy.



Is the philosophical focus or dimension: Philosophy applied to or of mathematics education? Philosophy of mathematics applied to mathematics education or to education in general? Philosophy of education applied to mathematics education? The figure illustrates these alternatives diagrammatically in a simplified way. Each of these three possible 'applications' of philosophy to mathematics education represents a different focus, and might very well foreground different issues and problems. Far from trying to give a survey about all possibilities, we specify some issues that were dominant in our discussions.

Philosophy of mathematics and its impacts on mathematics education

René Thom's statement that "all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics." (Thom, 1973, p. 204) is the classical starting point for studying impacts that views of mathematics can have on mathematics teaching. Steiner (1987) has also emphasized the other direction: every philosophy of mathematics includes implicit implications on instructional practices. Various empirical studies

have provided evidence for both directions, even though the connections are not unicausal dependencies (e.g. Thompson, 1984).

This has raised the normative question for the desired instructional practices and desirable views on mathematics (e.g., Ernest, 1994). In the course of these discussions, a simplistic opposition appeared between "the absolutist view" on mathematics and "the fallibilist" one, and these were also too directly connected with "transmission practices" versus "constructivist practices" in classrooms. Meanwhile, important contributions have been made to elaborate such (too) over-simplistic pictures into well-founded and multi-faceted accounts for the nature of mathematics. However, this can still be called a major task for PhoME (to which e.g. Meneghetti, 2004, contributed in the DG).

Envisaging the importance of personal philosophies for classroom practices, many authors in PhoME have concluded that changing instructional practices in mathematics classrooms can not only be a matter of new curricula or of providing materials, but also a matter of challenging traditional personal philosophies of teachers. That is why philosophical reflections become more and more part of teacher education programs (Lindgren, 2004, has contributed one example to the DG).

Beyond these activities is the conviction that if we acknowledge the impact of even implicit philosophies, the most important strategy is to make the underlying assumptions explicit. This idea of *making explicit the implicit* has become a leading idea for the whole discussion group.

The idea of making explicit the implicit philosophies is not restricted to teacher education or professional discussions of reform curricula. Instead, it should also be transported into the classrooms itself. Perhaps only if students can reflect on central questions in philosophy of mathematics themselves, they will develop a well-balanced and reflective knowledge about mathematics. However, empirical studies (like François & van Bendegem, 2004) show that there is still a big gap between this claim and classroom practices and even written curricula.

Philosophy as reflecting discipline with respect to mathematics education

Brown (1995) has claimed not to limit the discussion in PhoME to the philosophy *of mathematics*. If we understand philosophy as the reflecting discipline with respect to all aspects of mathematics education, the field becomes much larger, addressing all issues like "how does mathematics relate to society?", "What is learning (mathematics)?", "What is teaching (mathematics)?", and also "What is the status of mathematics education as knowledge field?" (as Ernest, 2004, suggested in much more detail). In all these areas we can apply the idea of making explicit the implicit and can hence do philosophy as a mode of making critical analyses and rigorous interpretation of the questions presented in learning processes.

There was a controversial discussion on the question whether this extensive way of understanding PhoME produces the problem that *all* mathematics education becomes subject of PhoME, hence whether PhoME tends to be reduced to a reflective basic attitude.

Starting from philosophy of education: Impacts on mathematics

A much more focused approach is to reflect on mathematics (education) against the background of a well-founded position in philosophy of education, especially on aims

and rationales of general education and mathematics' contribution to it. One important example for this approach present in the DG was the "Philosophy of Critical Mathematics Education" (Skovsmose, 1994). In this approach, the aim of mathematics education is specified by the ability to critique the uses of mathematics and its "formatting power". For that, students need to engage in mathematics-based projects which focus on its social applications.

Following this pathway to its logical end, this approach to PhoME formulates claims for mathematics itself: If mathematics education aims at reflecting critically on mathematics, the discipline mathematics is responsible for providing mathematics in a way that it can be critiqued by laypersons. For that, it must be presented embedded in its aims and purposes, meanings and senses. This is the core idea of the philosophical program called General Mathematics (cf. Lengnink, Prediger & Siebel, 2001). If this is taken seriously, PhoME can have important impacts on the discipline of mathematics, and not only on the philosophy of mathematics.

Concluding remark: Concurrent or complementary?

In the end, which is the most important perspective? It is an easy (relativist?) first step to emphasize that all perspectives have their important aspects and since they are complementary, they should all be elaborated in future research. On the other hand, we cannot deny that they are clearly concurrent due to restricted time and resources in a research community. That is why, on the one hand, we will have to continue learning from each other and follow the different perspectives, and on the other hand, we cannot stop discussing on priorities of questions to be raised in the community.

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(Contributions for DG 4 can be found at www.icme-organisers.dk/dg04/)

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This report was written by Susanne Prediger, Maria Viggiani-Bicudo and Paul Ernest. The team chairs are happy to be contacted at prediger@math.uni-dortmund.de and mariabicudo@uol.com.br, respectively, for further information on the work of this DG.