How are theoretical approaches expressed in research practices?

A report on an experience in comparing theoretical approaches with respect to the construction of research problems

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Abstract This article explores the idea that theoretical approaches might be usefully compared in terms of the ways in which they lead researchers to construe commonsense classroom problems. It reports an experience when one such problem was posed to a range of researchers with different theoretical backgrounds. They were invited to propose an answer, and to reframe the classroom problem as a research problem. As anticipated, responses adopted particular theoretical perspectives that 'privileged' certain objects of study and modes of explanation. Nevertheless, where responses did appeal to a common theoretical perspective, sometimes used in combination with others, there could be quite sharp differences in conceptualisation, proposed action, and research intention.

How should the scientific community in mathematics education deal with the diversity of theoretical approaches within the field? Rather than the frequent demand for *unifying* theories, an increasing number of researchers plead for the primacy of *comparing and understanding the differences and commonalities of different theories* (e.g. Artigue et al., 2006; Bikner & Prediger, 2006; Cobb, 2007; Lerman, 2006). This process of understanding different theoretical approaches has always been an important part of the disciplinary discourse, e.g. during CERME conferences. Given its complexity and the richness of different theories, it is far from being finished (see Artigue et al.; 2006, Arzarello et al.; in press).

The general plead for comparisons raises the question for concrete criteria, methods and focuses for these comparisons. This article follows a suggestion made by Paul Cobb (2007) in his recent handbook article on "coping with multiple theoretical perspectives", namely to "compare and contrast various perspectives by using as a criterion the manner in which they orient and constrain the types of questions that are asked about the learning and teaching of mathematics, the nature of the phenomena that are investigated, and the forms of knowledge that are produced" (Cobb, 2007: 3). Also Bergsten (2007) raises the question "How does a theoretical basis chosen for a study influence the nature of the purpose, questions, methods, evidence, conclusions, and implications of the study?"

The *focus on the expression of theoretical approaches in research practices* as proposed by these authors can be understood in the light of Charles Sanders Peirce's pragmatic maxim: "In order to ascertain the meaning of an intellectual conception one should consider what *practical consequences* might conceivably result […from it]; the sum of these consequences will constitute the entire meaning of the conception." (Peirce, 1905: CP 5.9).

This article reports on an experience (Prediger & Ruthven, in press) we made while exploring (in line with Cobb's suggestions) one practical way to compare different theoretical approaches

in terms of the ways in which they construe issues of classroom practice (we called it "teaching problems"), and propose in which way these can be researched.
Other authors in this ZDM-issue compare how differences of theoretical approaches influence the ways of analysing given data (such as Gellert, Halverscheid or Maracci). They show striking effects which help to understand the field.
However, all these articles start their comparison at a rather late stage of the research process, although it is already widely recognised that preference for a particular theory will already influence the way in which researchers construe a problem and investigate it (Silver & Herbst, 2007; Schoenfeld, 2007; Lester, 2005; Sriraman & English, 2005/2006). Many decisions have already been taken by the time that we come to the data analysis, including:

- initial identification of a problem in classroom practice, loosely framed;
- conceptualisation of the classroom problem;
- transformation of the problem into more focused research questions;
- development of research design (incl. methodological choices, sample…).

Therefore, we decided to start earlier in the research process by considering how specific theories shape the (re)formulation of an initially loosely framed problem of professional practice. By doing this, we reflect the emphasis given by Schoenfeld (2007: 23ff) on the conceptualisation of a situation as a crucial step in the research process.

To sum up, the following questions guided our experiment:

- How are theoretical approaches expressed in the practice of researchers, especially in the steps from a loosely framed classroom problem to the construction of a research problem and development of a research design?
- What similarities and differences can we find when comparing different theoretical approaches?
- In how far are the research practices determined by theoretical choices?
- What methods can we use to treat these comparing questions?

This article cannot give an extensive and systematic answer to the last question on methods for comparison. But it can report on an interesting experience we (Kenneth Ruthven and myself, see Prediger & Ruthven, in press) made with an experiment which was originally designed to initiate discussion in the ERME working group on theoretical approaches before and while CERME 5 (see also Arzarello et al., in press for the whole group's work). Although the experiment should not be mistaken as a systematic research study, it might give some ideas about how consolidated research methods could evolve from these first experiences.

1. The approach in the experiment

We asked researchers with different theoretical backgrounds to briefly describe, first how they would conceptualise a given teaching problem, and then how they would design an appropriate research study. The initial reference point was a classroom problem, which we chose since we have often heard it expressed along the following lines:

> *How is it that some students can learn to tackle a particular type of mathematical problem successfully (as shown by their performance in the class), but be unable to do so two weeks or months later?*
>
> *What strategies can the teacher use to reduce the likelihood of this occurring?*

The questions themselves presuppose certain assumptions which are not necessarily shared by all researchers, namely the idea that a teacher might be able to reduce the likelihood of the problem occurring, and that research should transcend purely descriptive analytical purposes and include reflections on strategies for changing teaching practices.

In order to see how different approaches frame this classroom problem as a research problem and devise a research design, we asked the following questions:

a) How do you –a priori– answer this question and what are your basic assumptions?
b) How do you transform the raised problem into a research question starting from the question above?
c) What is your research design?
d) What type of results would you expect?

The questions were sent to 34 European researchers who demonstrated an interest in our research questions as they had registered for the working group on theories on CERME 5 (see Arzarello et al.). As a whole, we got eight answers from 14 researchers, which are completely printed in the appendix to Prediger and Ruthven (in press). As some of the responses were too vague for a deeper analysis and for reasons of pragmatic restriction, we chose only 5 of the responses for this article, namely those given by Michèle Artigue and Agnès Lenfant, Ferdinando Arzarello and Ornella Robutti, Marianna Bosch and Josep Gascòn, Tommy Dreyfus and Ivy Kidron, and Helga Jungwirth.

The researchers had a time restriction of 15 days to answer, and they were asked to restrict to 2-3 pages. Hence, the given responses were not completely spontaneous, but should of course not be mistaken as mature research papers. We consider them as self-presentations of snapshots from research practice, in all its tentative form.

As their responses do not strictly follow the questions and have different grades of explicitness in different aspects, the tentative analysis presented in the following three sections is not intended to completely meet quality standards for empirical studies like interrater reliability. Nevertheless, it gives an impression on interesting differences and commonalities.

2. An initial survey of the responses

How do the different authors conceptualise the given classroom problem and reframe it as a research problem? Most of the responses accept to a degree the original terms in which the problem was posed, but suggest that these alone are inadequate to frame it. Equally, while most of the responses recognise that a range of factors may play a part, and that different lines of explanation can be developed, each adopts a particular theoretical perspective which privileges certain objects of study and modes of explanation.

This section gives an initial survey of the responses with longer quotations and orientational comments. By this, we hope to make the researchers speak themselves, although risking that not all arts are easy to understand.

Artigue and Lenfant

Artigue and Lenfant suggest that the original teaching problem is "a rather banal phenomenon": "[W]hat we learn is most often not definitively learnt, and if we do not use what we have learnt, generally, more or less quickly we forget it." However, Artigue and Lenfant argue that this way of thinking about the problem

"does not have a specific didactic flavour and could lead to look for explanations only at the level of the brain functioning or at the level of personal motivation for studying such or such topic, for learning to solve such or such type of task… [whereas a] didactic approach offers alternative or complementary perspectives, and will not necessarily lead to the same suggestions for improving the situation."

Accordingly, Artigue and Lenfant articulate a preference for explanations (and interventions) which frame the teaching problem in distinctively didactical terms. Due to their theoretical framework, these didactical terms should be specifically mathematical:

"There is certainly a lot of literature about such issues in cognitive research. From a didactic perspective, what seems more interesting to us is to transform the raised problem into a research question in such a way that the specificity of mathematics knowledge, of mathematical and didactical organizations could be taken into account, and that a systemic view could be developed, the 'forgetting student' being no longer the exclusive or central object of our attention."

Artigue and Lenfant, proceed, then, within a theoretical framework drawing on the Theory of Didactic Situations [TDS] (see Brousseau, 1997) and the Anthropological Theory of Didactics [ATD] (see Chevallard, 1992), on the basis that "the observed phenomenon [of forgetting], if not created, is highly reinforced by [a wide range of] didactical choices" concerning treatment of the task and organisation of the task environment:

"how this type of task was introduced to the students with what mathematical motivations, how techniques for solving it were developed, how did the respective responsibilities given to the students and the teacher in the solving of this type of task progressively evolved, up to what point some particular techniques were trained and routinized, how the variation around this type of tasks was organized taking into account its didactic variables, up to what point the mathematical knowledge at stake was explicitly pointed out, justified, institutionalized and how the necessary decontextualization of knowledge was worked out, how this type of task was related with other ones in wider mathematical organizations, what opportunities were given to make the

students' relationship with this task evolve beyond the necessarily short period of its official teaching."

Their possible research questions refer to the wide range of questions "orientated towards … understanding … [and] didactical engineering trying to improve the current situation"; and they give examples for general question which would be concretized for specific research projects:

"Q1: Are different types of mathematical tasks equally sensitive to the 'forgetting phenomenon' and what can explain observed differences if any?

Q2: What are the strategies that mathematics teachers tend to use for limiting or controlling the 'forgetting phenomenon'? What is the rationale underlying these and what are their effects?

Q3: Are there characteristics of the usual mathematical organizations which tend to reinforce the 'forgetting phenomenon' and, if so, what are the mechanisms underlying this reinforcement?

Q4: Does an engineering design where specific attention is paid to the balance between the different moments of the study (according to the TAD) and to the completeness of mathematical praxeologies can make a difference?"

To sum up, Artigue and Lenfant present a clear reframing of the classroom problem in didactical terms that refer to the specificity of mathematics.

Arzarello and Robutti

Arzarello and Robutti characterise the original teaching problem as "a natural fact", framing their discussion in terms of the distinction that the Anthropological Theory of Didactics [ATD] (see Chevallard, 1992) makes between technique and theory:

"Maybe that a person many years after she ended the school remembers something about the theories but has forgotten everything concerning the techniques… and so is not able to solve the problem… Maybe a 'feeble' student remembers the technique but not the technology and the theory: so she is not able to solve the problem for different and opposite reasons. It is a question of level at which the knowledge related to the problem must be known to solve it. It is clear that without a continuous training many abilities linked with techniques and technologies become lower. This may cause lower performances and is a natural fact. Of course this depends on the type of performances asked and on the level of assimilation of the techniques, technologies and theories required by the performance itself. Hence to tackle the question the teacher must distinguish carefully at which level the performances of a task are situated."

In this light, Arzarello and Robutti identify three "key variables", namely "different specific mathematical contents", "the level …at which the performances for a specific knowledge …in the task are required, [and] the methodology of teaching".

Their research interest is focused on different methods of teaching which are related to some underlying theorisation:

"A) a traditional approach, based on the sequence: explanation-exercise-repetition-assessment;

B) a more innovative approach, where the knowledge is constructed by students in suitable learning situations, based on the use of laboratory and ICT.

These two approaches can be analysed according to the different ways of teaching-learning they produce from a cognitive point of view. For this, two related types of analysis can be developed,

based on some recent researches, which point out different modalities of learning and of thinking: some researchers distinguish between a perceptuo-motor and a symbolic-reconstructive way; others distinguish between spatio-motoric and analytical thinking …
The methodology A is typically based on a symbolic-reconstructive approach, which may produce analytical thinking while the methodology B can be based on a perceptuo-motor approach, which may trigger spatio-motoric thinking."
Within this conceptual framework (more details in Arzarello, 2006 and 2008), they pose the following research questions:

"RQ1: Does students' specific knowledge that we measure as a performance in some task change according to the level of the task and how does (can) it change? ...
RQ2: Does the knowledge depend on the way the students learn it and how?
RQ3: How can we verify if there is a relationship between the way of learning and the way of thinking?"

With these questions, they take a relatively high distance from the original classroom problem. Their research is then designed as teaching experiments with control groups in order to test three hypotheses on this relationship:

" i) the perceptuo-motor approach produces more spatio-motoric thinking;
ii) the perceptuo-motor learning produces long-term effects;
iii) the symbolic-reconstructive one produces short-term effects."

Bosch and Gascón

Bosch and Gascón refer to the teaching problem as "an aspect of a broader fact" that they describe as follows:
"[A]t school, students are rarely conducted to perform a mathematical activity that goes beyond the resolution of very tightly delimited types of problems, studied in a quite isolated form. They use to work in a narrow 'mathematical space and time', where topics come one after the other only weakly connected. Once the study of a topic is finished, all can be forgotten because a completely new activity is starting. … The identification, description, delimitation, evaluation, connection, etc. of techniques and types of problems is commonly the teacher's responsibility and rarely 'transferred' to the students."
Appealing, like Artigue and Lenfant and Arzarello and Robutti, to the Anthropological Theory of Didactics, Bosch and Gascón are more sceptical about whether the original problem can realistically be treated as a 'teaching problem' at the level of the teacher. They suggest, rather, that it is a manifestation of a more fundamental "dis-articulation of school mathematics" which needs to be addressed at an 'institutional' level:
"[T]he kind of mathematical activity the students carry out (for instance, learning to solve a 'narrowly defined' type of problem for a short period of time and forgetting it afterwards) is mainly a consequence of the kinds of mathematics that exist at school, which are affected by the phenomenon of 'dis-articulation'. …[I]t does not seem that the didactic phenomenon associated with the fact mentioned can be easily modified only by changing teachers' strategies. The kind of solution we can think of is the implementation of new didactic organisations in a system that has strong traditions and imposes many constraints on the way changes can be carried out – at

least if we expect long-term changes, and not only local and temporary modifications. It is thus necessary to study the mechanism and the scope of the phenomenon."

Accordingly, Bosch and Gascón identify their key issues in the following explicitly stated research questions:

"(a) Didactic transposition problem: What are the mechanisms of didactic transposition that can explain the phenomenon of the disarticulation of school mathematics as described above? Why is the current situation as it is? What constraints make things be like this?

(b) Ecology of didactic praxeologies: What kind of didactic praxeologies can be introduced at school, and under what conditions, in order to allow the development of more 'articulated' mathematical activities, that is, to allow the construction of more 'complete' and 'connected' mathematical praxeologies?"

They sketch a research design which consists of the following stages:

1. Curriculum analysis and design of a 'reference epistemological model', "leading to a a priori mathematical design of a Research and Study Course that may articulate different curricular mathematical organisations, linking them through a dynamic of questions/answers".
2. Set up and experimentation of the designed "Research and Study Course" in real classrooms, "observe the study process (data collection), with special attention to the way the different moments of the study process are managed, the share of responsibilities between teacher and students, etc."
3. Analysis of collected data

Their expected results concern the "ecology of mathematical praxeologies" (explained as asking for "new ways of curriculum organisation around powerful generative questions that can give a raison d'être to the mathematical praxeologies to be taught") as well as the "ecology of didactic praxeologies" (i.e. "characterisation of possible didactic devices and strategies to manage the different moments and dynamics of the RSC; description of the didactic constraints … that hinder the experimented study process").

Dreyfus and Kidron

Dreyfus and Kidron also propose to reframe the 'teaching problem', but in a direction quite opposite to that suggested by Bosch and Gascón. Following the concerns of their Theory of Abstraction in Context centred on the RBC-Model (Dreyfus et al., 2001, Hershkowitz et al., 2001, Schwarz et al. in prep.), Dreyfus and Kidron focus on student learning factors, and propose to treat the issue as a 'learning problem':

"Our research would rather start from the perspective of the student. What we want to know is how things are learned, not only how they are taught. What we want to know is whether students' knowledge, their recognition of previously encountered ideas, concepts, processes and strategies, their connections between knowledge elements, explanatory power, and flexibility are excellent, adequate, wrong or lacking. We want to investigate how students reach a state in which, say, their flexibility with respect to a particular cluster of mathematical concepts or processes is excellent or lacking; or what are the learning processes by means of which a student (or a group of students) arrive at excellent (or at only partially correct) connections between knowledge elements; what are the learning processes by means of which a student (or a group of students) acquire (or fail to acquire) explanatory power with respect to a cluster of mathematical concepts or processes."

Dreyfus and Kidron indicate that this focus on individual learning reflects the fact that their research programme is at its foundational stage, and anticipate that it would expand at a later stage to encompass the development of design principles for teaching:

"[O]bviously, such a program of research requires instruction, and instruction needs to be designed. However, in the short term, our choice is not to focus on instructional design as a topic to be researched but to use or adapt an existing design, the choice being based on intuition and past experience of team members… In the long run, we would hope to also derive design principles for constructing and consolidating, derived from experience with RBC analyses."

Equally, Dreyfus and Kidron acknowledge that "as far as teaching approaches are concerned", a role is played by "curriculum design, textbooks and teacher action such as coherently organizing the new material, emphasizing key elements, activating students, etc." For the present, however, Dreyfus and Kidron note that the RBC model only permits the original problem situation to be addressed in terms of individual learning, and their research questions are set accordingly:

"What are the processes of constructing the knowledge under consideration, and what are students' emerging knowledge constructs? In what are these processes of knowledge construction for a given construct different for the learning processes of students who are successful with this specific construct after a year and those who are not? In what are these processes of knowledge construction of the same student different for constructs with which the student is successful after a year and those constructs with which she/he is not?"

Jungwirth

Jungwirth notes likewise that, while there are many ways of elaborating the original teaching problem for purposes of deeper analysis and explanation, she has a preference, and a biographical rationale for it:

"There are so many explanations, and probably more than one will hold in the respective case. They may focus on students, the teacher, their interaction, on contextual events within the classroom, within the school… But I prefer a certain one, anyway. It is due to my interactionist stance towards the world. (Much of my research I have done on this basis). Don't ask me why I favour it. I had an affinity to this stance, from the beginning. It is a viable belief of mine, that is, I have a good rationale for it. In particular, it has proved relevant in initiating steps towards a "better" teaching practice."

Equally however, this interactionist perspective (see e.g. Jungwirth, 1996; Voigt, 1989) is particularly sensitive to certain types of phenomena, and renders one alert to the possibility of their presence:

"My preference in the given case, however, is underpinned by a hint in its description: students did well 'in the class'; which I interpret that I cannot assume that they did well in a test, an exam as well. They performed well in the ongoing process. So their performance can be localized there. If it is sensible to understand the process as an interaction being established by the teacher and the students, which is the case presumably not from my point of view only, interactionism will be on the agenda."

On the base of an interactionist framework for conceptualising the problem, the research question and design is clear: videotape processes of classroom interactions and "analyse them with respect to patterns in the interaction that have been reconstructed by interactionist research before." Hence, the reference to the interactionist framework is far reaching insofar as, in Jung-

wirth's view, the framework (theory and former research) already offers an explanation for the phenomenon. Therefore she can refer to already known results:

"Interaction –everyday, smooth-running interaction– is established by the teacher's and students' adjusting to the acting of each other. So students can successfully participate without an understanding to be located in their 'heads'; for instance, by answering on questions by short, tentative utterances which seem to indicate understanding so that the teacher completes to the desired answer (just 'recalling' what the students already 'know'). As their competence is a phenomenon of the interaction (as I have called that once), an event existing between people, not in people, it is not surprising that some students cannot repeat neither former solutions nor the solution game later without any break-downs."

Taking these aspects as quite well known reasons for difficulties, Jungwirth focuses her response on the next step and describes how to work with the teacher. She proposes to help him or her to recognise and comprehend these phenomena, and develop new communicative and interactive strategies:

"I design a teacher education or individual coaching of the teacher… to make her/him realize the pattern and its routines in order to change her/his part (because if one side does no longer act in the common way the other cannot keep to his; emergence of events in interaction put aside). We develop alternatives for utterances which do not allow students to perform well at the surface. The strategies for the teacher evolve from the concrete, detailed video or transcript reflection together with the teacher."

3. Comparing the conceptualisations

Three of the responses –those from Bosch and Gascón, Dreyfus and Kidron, and Jungwirth– each adopt a particular theoretical perspective that 'privileges' certain objects of study and modes of explanation. While these responses recognise that other factors may play a part, and that other lines of explanation might be developed, they quite consciously restrict themselves to a relatively limited system of factors and pursue particular lines of explanation. This, of course, reflects a logic of scientific enquiry. Equally, Artigue and Lenfant appeal to two cognate theories, TDS and ATD, which have co-evolved in the work of the French school of *didactique*; they too explicitly seek didactic –rather than cognitive or affective– characterisations and explanations of the phenomenon. While Arzarello and Robutti appeal to a more diverse assemblage –ATD, taxonomic hierarchies of educational objectives (such as those developed by Bloom or PISA), and theories of learning and thinking modalities (Antinucci's perceptuo-motor v. symbolic-reconstructive, and Kita's spatio-motoric v. analytical, see Arzarello, 2006) – nevertheless they use these to develop a very specific conceptualisation of the problem and its origins. This gives a hint for the importance of some form of mediation between research activity –with its multiplicity of specific foci and particular theorisations– and the more complex multidimensional world of educational practice.

The different foci of the responses can first be characterised by levels of grain size, reaching from Jungwirth, Dreyfus and Kidron with their fine grained analysis on the micro level to Arzarello and Robutti and Artigue and Lenfant on a meso-level and Bosch and Gascón who work on all levels, up to the most global macro level.

But more informative is a second way of thinking about these responses in terms of the degree to which they appeal to types of factors and corresponding theories, focusing on individual learning, class teaching or on institutional structuring, respectively.

Taking these as three idealised poles, each response can be assigned a location within the resulting space, with proximity to each pole indicating its relative weighting within the response. Some responses provide theorisations which are close to a particular pole because of their emphasis on a particular type of factor and theory: Dreyfus and Kidron focus on processes of individual knowledge construction, and so lie close to the Individual Learning pole; Bosch and Gascón focus on institutional factors which structure treatment of knowledge, and so lie close to the Institutional Structuring pole; Artigue and Lenfant focus on the teacher's management of the development of knowledge in relation to the class as a whole, lying close to the Class Teaching pole, but displaced somewhat towards the Institutional Structuring pole to which they also allude. Other responses occupy a more strongly intermediate position: the micro-interactionist tradition followed by Jungwirth focuses on the fine grain of processes of knowledge construction but locates these within a wider system of classroom communication, and so lies between Individual Learning and Class Teaching. Being the response with the most diverse theoretical background, the response from Arzarello and Robutti appeals to aspects associated with each pole, defining a position which can be located more centrally within the space, but displaced towards the Class Teaching/Individual Learning axis to reflect the emphasis of their proposed research question.

It is notable, then, that while several responses appeal to ATD, they are positioned differently according to whether or how ATD is combined with theories addressing other components of the system. For example, the responses of Artigue and Lenfant, and of Bosch and Gascón display contrasting views as to whether class teaching factors can provide leverage in relation to the original problem. Equally, even where responses share similar types of substantive focus, the underlying theories they adopt to frame these concerns may be different, and also their concrete objects of study.

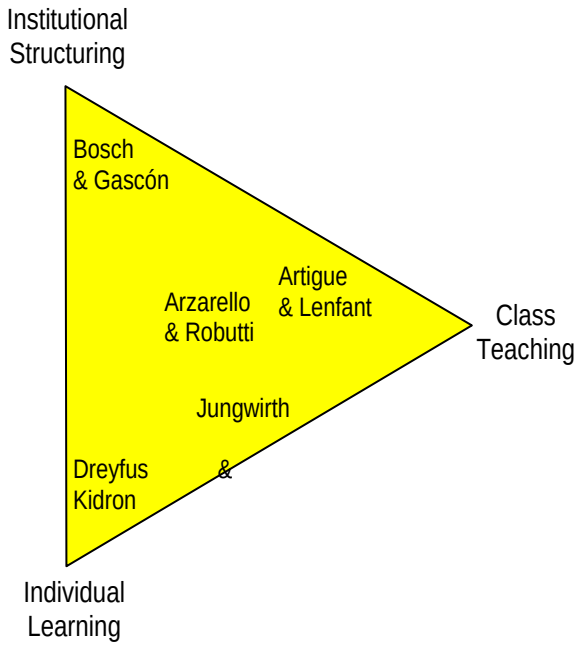

Figure 1: Location of proposed conceptualisations relative to types of substantive focus

For example, the theorisations of Class Teaching to which Artigue and Lenfant appeal and of Individual Learning which Dreyfus and Kidron employ are not really commensurate with the Interactionist theory which Jungwirth brings to bear on similar issues.

4 Comparing the prioritized research intentions

The research questions, strategies and expected aims which follow the conceptualisations in the researchers' responses differ clearly. Artigue and Lenfant point out that research questions and strategies may vary according to whether the aim is one of improved scientific understanding or of improved teaching practice.
They distinguish:
"between research questions orientated towards the understanding of the system functioning and of the influence of its characteristics on the observed phenomenon on the one hand, and research questions associated to the elaboration and evaluation of didactical engineering trying to improve the current situation by playing on one or several levers, on the other hand."
The original request which elicited these five responses explicitly had a dual character in asking both, for explanation of the posited phenomenon and for advice on teaching strategies. Hence it explicitly referred to both major types of intention for research, as distinguished for example by Lester (2005): improved understanding and improved practice. Nevertheless, the responses differed in their relative prioritisation of these aspects.
Whereas some responses emphasise the theory-building purpose of mathematics education research, i.e. to increase understanding of the phenomenon, others stress the theory-applying purpose of developing instructional designs and teaching strategies. We tentatively tried to localise them in Figure 2, although it is not easy on the base of the limited statements given in the responses.
It is easier to locate the papers with clear prioritisation (see below) than to locate the three in the middle. Arzarello and Robutti give more priority to the improved practice as their research design is immediately directed to the evaluation of a teaching approach, not to understanding. Bosch and Gascón might also be located more in the middle, but we preferred this position as they emphasize the complexity of aspects to take into account and research carefully before (and of course while) acting. Taking into account that neither the author nor the reader should overinterpret the tentative localisation of responses, we can still see the interesting effect that the three responses which appeal to the same theoretical framework ATD (Bosch and Gascón, Artigue and Lenfant, and Arzarello and Robutti) do not necessarily have the same placements.

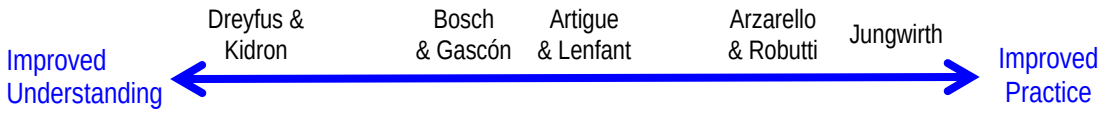

Figure 2: Location of proposed studies relative to prioritized types of research intention

Although the interactionist perspective adopted by Jungwirth is characterized by a strong emphasis on the focus of improved understanding (e.g. Voigt, 1989), Jungwirth's response focuses on using it for improving practice via sensitizing teachers in professional development. This unexpected location might be traced back to her assumption of being able to anticipate the outcome of classroom research and knowing the origin of the teaching problem. Hence, the location of major intention of research seems to depend not only on choices taken by the research groups on their intentions alone but also on the degree to which they consider that adequate explanatory frameworks are already available. This influences the balance of their work between seeking improved explanations and converting available explanations into transformative actions.

Such an effect is visible especially for those responses which appeal to a single theoretical perspective: Dreyfus and Kidron emphasise theory development because they see the RBC model as being at a relatively early stage; Jungwirth, by contrast, sees Interactionist theory as sufficiently well developed for it to guide recommendations for teaching; Bosch and Gascón take a more intermediate position.

5. From an experience to a research approach?

How are theoretical approaches expressed in the practice of researchers, especially in the steps from a loosely framed classroom problem to the construction of a research problem and development of a research design? What similarities and differences can we find when comparing different theoretical approaches? This article reported on a first attempt to answer these questions with an experiment carried out with 14 researchers during the preparation of CERME 5 (see Prediger & Ruthven, in press).

The experiment showed that when the responding researchers adopted their particular perspectives based on different theoretical approaches, they privileged significantly different objects of study and modes of explanation which reflected their theoretical choices. On the other hand, the theoretical base alone did not completely predetermine their conceptualisations. In contrast, where responses did appeal to a common theoretical perspective, sometimes used in combination with others, they also contained quite sharp differences in conceptualisation, proposed action, and research intention. Even the priorities of research intentions were influenced by other factors such as the degree to which the researchers considered that adequate explanatory frameworks are already available. Hence, research practices and theoretical bases of course are strongly connected, but it would be a misleading simplification to propose a direct causal or deterministic connection.

To sum up, already this unsystematic experiment makes evident that it is worth to consider these early steps of research practices when comparing different theoretical approaches with respect to their expression in research practice.

However, the presented experiment has of course serious limitations, being originally not designed for a research study but for a communicative activity at a conference. Especially the quality of the analysed material (the 'data') sets serious limits for the expressiveness of the analysis. The responses were not homogeneous enough in their reference to the four posed questions, they contained gaps and points of vagueness (for example on the basic assumptions). Therefore, the analysis could not be made as thoroughly as desirable.

Nevertheless, the presented approach seems to be a promising first contribution to a slowly evolving methodology of comparing or more generally networking theoretical approaches as described in the introductory paper of this ZDM-issue.
In order to elaborate this experiment and its basic ideas into a consolidated research method for comparing theoretical approaches with respect to the construction of research problems, two key problems must be solved: Firstly, find more systematic and comprehensive ways of data collection (for example oral data from narrative interviews instead of written texts or even ethnographical observations of processes in research groups while developing research designs). The resulting material might than allow, secondly, more systematic and criteria-guided methods of data analysis than we could conduct here.

6. 'Theorizing as bricolage' for research 'in the wild'? – an outlook

In processes of research and theory development, concern to develop and refine a particular theory encourages a tight focus on specific types of phenomena and particular lines of explanation, often through approaches which seek to isolate some single dimension or simple system for analysis. But such a tight focus necessarily means that the theory may struggle to address wider or more holistic problems of practice involving phenomena which are not central to the theory's privileged constructs and lines of explanation.
This raises the question of legitimization of our choice of a theory (cf. Cobb, 2007): do we use a theoretical framework because it has really proved suitable for tackling a particular practical problem, or perhaps only because we are socialised within a research group that has a strong association with one particular theoretical framework? Can we be confident of the appropriateness of the specific theory for a particular problem? And what are the criteria for judging appropriateness?
These questions are less urgent for special projects carefully chosen or expressly created to lend themselves to the development of a particular theory than for problem solving 'in the wild' of ordinary classrooms practices and typical schools.
'Applied problem solving' in these wider circumstances, less amenable to the researcher's control, might be better served by drawing on insights from a range of theories which focus on different aspects of the problem situation and provide multiple lines of explanation (and action); this is what Cobb (2007) calls "theorizing as bricolage". It gives an interesting and promising orientation for the enterprise of connecting theoretical approaches.

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