Networking of theories – an approach for exploiting the diversity of theoretical approaches [1]

Angelika Bikner-Ahsbahs[*] & Susanne Prediger[#]

[*] University of Bremen, Germany, bikner@math.uni-bremen.de

[#] IEEM, University of Dortmund, Germany, prediger@math.uni-dortmund.de

Abstract: Internationally, mathematics education research is shaped by a diversity of theories. This contribution suggests an approach for exploiting this diversity as a resource for richness by the so-called networking of theories. For being able to include different traditions, this approach is based on a tolerant and dynamic understanding of theories that conceptualizes theories in their dual character as frame and as result of research practices. Networking strategies are presented in a landscape, linearly ordered according to their degree of integration. These networking strategies can contribute to the development of theories and their connectivity and, hence, offer an interesting research strategy for the didactics of mathematics as scientific discipline.

In their introductory article to this volume, Lyn English and Bharath Sriraman give an impression of the diversified field of theories in mathematics education. Already in 2005, they emphasized the often repeated criticism of the discipline's lack of focus, its diverging theoretical perspectives, and a continued identity crisis (Steen, 1999), and called for the ambitious project to "take stock of the multiple and widely diverging mathematical theories, and chart possible courses for the future (English & Sriraman 2005, p. 450). With this article, we want to contribute to the "discussion on the critical role of theories for the future of our field" (ibid, p. 451).

Theories in mathematics education evolved independently in different regions of the world and different cultural circumstances, including traditions of typical classroom cultures, values, but also varying institutional settings. This is one important source for the existent diversity of theoretical approaches that frame empirical research.

The (at least equally important) second reason for the existence of different theories and theoretical approaches is the complexity of the topic of research itself. Since mathematics learning and teaching is a multi-faceted phenomenon which cannot be described, understood or explained by one monolithic theory alone, a variety of theories is necessary to do justice to the complexity of the field.

In certain aspects, this article can complement the mostly Anglo-American articles in the monograph with a European perspective, as the European discourse is marked by the idea that although the diversity of theories sometimes is an obstacle and often a challenge, it also offers opportunities that should be exploited more consequently by the community. Its perspective is influenced by the Theory Working Group of the last Congresses of the European Society for Research in Mathematics Education CERME 4-6 (cf. Artigue, Bartolini-Bussi, Dreyfus, Gray, & Prediger, 2006; Arzarello, Bosch, Lenfant, & Prediger, 2008a; Prediger, Bosch, Kidron, Monaghan, & Sensevy, 2009).

In order to substantiate the claim of diversity as a resource for rich scientific progress, this article first tries to clarify the underlying understanding of theories and offers some categories how they can be distinguished. The position towards diversity is then elaborated in section 3, as a base for discussing strategies and strands for connecting different theoretical perspectives, theoretical approaches and theories.

The article mostly refers to theories that frame empirical (mostly qualitative) research in a specific domain, namely mathematics education. All these theories have an empirical component (see below) intended to understand processes of learning and teaching mathematics on different scales.

1. What are theories, and for what are they needed?

Are we sure that we talk about the same thing when we use the terms 'theory' or 'theoretical approach'? Already the sample of contributions in this monograph suggests that there is *no shared unique definition* of theory and theoretical approach among mathematics education researchers (see Assude, Boero, Herbst, Lerman, & Radford, 2008). The large diversity already starts with the heterogeneity of what is called a theoretical framework or a theory by different researchers and different scholarly traditions. Some refer to basic research paradigms (like the interpretative approach within social interactionism), others to comprehensive general theories (like the theory of didactical situations) others to local conceptual tools (like the modelling cycle). Different are not only the ways to conceptualize and question mathematical activities and educational processes and the type of results they can provide, but also their scopes and backgrounds.

Due to this variety of conceptualizations of the notion of 'theory', many authors demand clear distinctions and have tried to offer robust definitions or characterizations of what a theory or a theoretical approach is or is for. Some of them shall be mentioned in the following subsections.

1.1. Static and dynamic views on theories

Mason and Waywood distinguish between different characters of theories: *foreground theories* are local theories *in* mathematics education "about what does and can happen within and without educational institutions." (Mason & Waywood, 1996, p. 1056). In contrast, *background theory* is a (mostly) consistent philosophical stance *of* or *about* mathematics education which "plays an important role in discerning and defining what kind of objects are to be studied, indeed, theoretical constructs act to bring these objects into being." (Mason & Waywood,1996, p. 1058). The background theory can comprise implicit parts that refer to epistemological, ontological or methodological ideas e.g. about the nature and aim of education, the nature of mathematics and the nature of mathematics education. Taking the notions of foreground and background theory as offering *relative*, not absolute *distinctions*, they can help to classify different views on theories.

The diversity of characterizations of 'theory' cannot only be distinguished according to the focus on foreground or background theories, but also according to their general view on 'theory'. For analytical reasons, we distinguish

- a normative *more static view* which regards theory as a human construction to present, organize and systematize a set of results about a piece of the real world, which then becomes a tool to be used. In this sense a theory is given to make sense of something in some kind and some way (for example Bernstein's structuralist perspective, discussed by Gellert, in this volume).
- and a *more dynamic view* which regards a theory as a tool in use rooted in some kind of philosophical background which has to be developed in a suitable way in order to answer a specific question about an object. In this sense the notion of theory is embedded in the practical work of researchers. It is not ready for use, the theory has to be developed in order to answer a given ques-

[1] This article is a preliminary version of an article published in

- Bharath Sriraman / Lyn English (eds.) (2010): Theories of mathematics education: Seeking New Frontiers, Series Advances in Mathematics Education, Springer, New York, 483-505.

It is built upon a synthesis of two articles that formerly appeared in ZDM:

- Bikner-Ahsbahs, A., & Prediger, S. (2006). 'Diversity of Theories in Mathematics Education - How can we deal with it?' *ZDM 38* (1), 52-57. and
- Prediger, S., Bikner-Ahsbahs, A., & Arzarello, F. (2008b). 'Networking strategies and methods for connecting theoretical approaches – First steps towards a conceptual framework', *ZDM 40* (2), 165-178.

tion (for example, most researchers who follow an interpretative approach adhere this dynamic view on theories for example Jungwirth, this volume). In this context, the term 'theoretical approach' is sometimes preferred to 'theory'.

Niss (2007) offers a static view on the notion of theory with his definition of theory as

> "an organized network of concepts (including ideas, notions, distinctions, terms, etc.) and claims about some extensive domain, or a class of domains, consisting of objects, processes, situations, and phenomena. … In a theory, the concepts are linked in a connected hierarchy … [and] the claims are either basic hypotheses, assumptions, or axioms, taken as fundamental (i.e., not subject to discussion within the boundary of the theory itself), or statements obtained from the fundamental claims by means of formal (including deductive) or material (i.e. experiential or experimental with regard to the domain(s) of the theory) derivations." (Niss 2007, p. 1308)

This characterization of theories gives the impression that a theory can only be called a theory when it consists of a hierarchical conceptual structure and when its corpus of knowledge is well defined and deeply clear. However, accepted theories are not always explicitly clear in all these details, they do not always have a hierarchical structure and they may develop through research over time. Even very well developed theories like the theory of didactical situations (Brousseau, 1997) or the Anthropological Theory of the Didactic (Chevallard, 1992) are still in a state of flux and can better be described by a wider and less static consideration of theories.

Also other researchers follow Niss' core idea of a well organized structured system of concepts, for example Mason and Waywood (1996, p. 1055), when they speak of (foreground) theories as "an organised system of accepted knowledge that applies in a variety of circumstances to explain a specific set of phenomena as in 'true in fact and theory'; …" But they continue by focusing on the *function of theories in the practices of mathematics education research*. The purpose of using theories mostly is the human enterprise of making sense, in providing answers to people's questions about why, how, what. "How that sense-making arises is itself the subject of theorizing. …" (ibid., p. 1056).

This wider notion of theory keeps the idea of a structured building of knowledge, but includes also the function of (background) theories as tools which help to produce knowledge *about what, how and why* things happen in a phenomenon of mathematics education.

Whereas Mason and Waywood differentiate between theory as a structured system and theorizing as the way of making sense through using a theory, Maier and Beck (2001) match both aspects by pragmatically taking a dynamic view. They reconstruct the notion of theory by investigating the practices of different researchers. Their understanding of the notion of theory is based upon researchers' views of theory, their practices and how they use and develop theoretical understanding. For them, theory is an individually or socially developed construct which provides understanding or describing a piece of reality in a consistent and systematic way (Maier & Beck, 2001, p. 45). They conclude that theories have an impact on the research interest, assist in raising questions concerning the field of investigation, and provide the language by which questions can be formulated and made more precise.

1.2. Function of theories for research practices

When Maier and Beck point out that the function of using theories is to structure the perception of the research field in a basic way, they meet Mason and Waywood's description of the function of theories for research practices: "To understand the role of theory in a research program is to understand what are taken to be the things that can be questioned and what counts as an answer to that questioning." (Mason & Waywood, 1996, p. 1056)

Silver and Herbst (2007) also approach the notion of theory in mathematics education in a dynamic way. Comparisons of different theories, with respect to their roles as instruments mediating between problems, practices and research, show that *theories in mathematics education are mostly developed for certain purposes*. For example,

- theories which mediate practices and research can be understood as "a language of descriptions of an educational practice" or as "a system of best practices", … (ibid., p. 56)
- theories which mediate problems and practices can be understood as a "proposed solution to a problem" or a "tool which can help design new practices", ... (ibid, p. 59)
- theories which mediate research and problems can be understood as "means to transform a commonsensical problem into a researchable problem" or as a "lens to analyze data and produce results of research on a problem", … (ibid., p. 50)

Some theories are used to investigate facts and phenomena in mathematics education; others provide the tools for design, the language to observe, understand, describe and even explain or predict phenomena.

If we approach the notion of theory in this way from its role in research and from research practices, theories can be understood as guiding research practices and at the same time being influenced by or being the aim of research practices. This dialectic between theory and research has to be taken into account in a discourse about the notion of theory.

Most theories about and within mathematics education share their *research object* up to a certain point: it is about aspects of mathematics teaching and learning. But they differ in the *situations* that are considered and what exactly in these situations is theoretically conceptualized, in the *methods* that are used for generating results for theory building and in their *aims* (cf. Figure 1).

To these four aspects of theory guiding research (aims/goals, objects, methods, situations, see Figure 1) collected by Mason & Waywood (1996), we added the *sort of questions considered to be relevant*, since we know from higher education research and comparative research about scientific cultures that the questions which are considered to be relevant form an important part of the scientific culture of each research group and community (cf. Arnold & Fischer, 2004).

Figure 1: Aspects of theory guiding research

These five aspects of theory guiding research help to describe more precisely how research practice, background theory and its philosophical base are deeply interwoven. For example, the choice of the theoretical perspective on an object and the observed situation influences the aims, the posed questions and the activated methods. Vice versa, the objects, situations, aims, questions, and methods, often suggest a certain theoretical perspective. Furthermore, the different perspectives are deeply connected with epistemological and methodological points of view which shape an integral part of the philosophical base. Adopting the perspective of social constructivism on mathematical knowledge, for example, is usually based on the idea that knowledge is socially constituted (e.g. Bauersfeld, 1988; Voigt, 1994; Jungwirth, 1996). In contrast, adopting a constructivist perspective starts from the philosophical idea that knowledge is mainly individually constructed (e.g. Harel & Lim, 2004).

Whereas the notions foreground theory, background theory and philosophical base try to characterize the function of a theory as a whole, Bigalke (1984) offers notions that allows drawing the lines more locally within one theory and describing its different parts. Starting from the view on theories as structured buildings of concepts, knowledge, claims, values and norms, Bigalke (1984) distinguishes the *core of a theory* from its *empirical component.* The core of a theory comprises central ground rules and norms taken for granted within the theory, the empirical component enlarges aspects, claims and concepts of the core and enriches it by the aspects that refer directly to the empirical field, for example its intended applications. Theories can differ with respect to their lines between empirical component and core, and they can evolve.

The notions of core and empirical component help to characterize theories in line with a dynamic view on theory that includes different research practices with their different ways of characterizing (and using) theory. The idea of exploiting the diversity of theories as a resource for scientific progress is more compatible with referring to a more inclusive, broader working characterization of theory which includes the dialectic of research practices and theory and the dynamic character of theories and their applicability.

To sum up: We can distinguish theories according to the structure of their concepts and relationships, according to the way how theorizing is done in order to deepen insights by the research community and according to their role for determining what kind of insights are gained, what kind of objects are chosen, what counts as research questions and adequate answers, what aims are followed, the view on the research and their methods (cf. Figure 1).

In our understanding, 'theories' are constructions in a state of flux. They are more or less consistent systems of concepts and relationships, based on assumptions and norms. They consist of a core, of empirical components, and their application area. The core includes basic foundations, assumptions and norms which are taken for granted. The empirical components comprise additional concepts and relationships with paradigmatic examples; it determines the empirical content and usefulness through applicability.

Theories guide research practices and are influenced by them. They allow researchers e.g. investigating phenomena in mathematics education or providing the tools for design, the language to observe, understand, describe and even explain or predict phenomena in mathematics education. Research aims, questions, objects, but also units of and methods for investigation are theory laden. At the same time, a theory comprises specific kinds of aims, questions, objects, but also units of and methods for investigation.

2. Diversity as a challenge, a resource, and a starting point for further development

Each discourse about the diversity of theories is shaped by the question, why the marketplace of theories is so diversified.

> "One plausible explanation for the presence of multiple theories of mathematical learning is the diverging, epistemological perspectives about what constitutes mathematical knowledge. Another possible explanation is that mathematics education, unlike 'pure' disciplines in the sciences, is heavily influenced by cultural, social, and political forces." (English & Sriraman 2005, p. 452)

The European example shows that both aspects are interwoven, since it is exactly in the cores of the theories and especially their diverging philosophical bases that the traditions of different national or regional communities differ (see Artigue et al. 2006 and Prediger, Viggiani-Bicudo, & Ernest 2008c). Different regional circumstances concern traditions of typical classroom cultures, values, as well as varying institutional settings (like the location of mathematics educators in the mathematics or education department, their involvement in pre-service or in-service training, the real and the intended curriculum, the books, etc.); they shape conditions for different developments. These regional traditional differences and their institutional backgrounds comprise different priorities in developing foreground theories and background theories, and very different degrees of being explicit about the different components.

Apart from the multiple circumstances under which theories have separately evolved, the (at least equally important) second reason for the existence of different theoretical approaches is the complexity of the topic of research itself. Since mathematics learning and teaching is a multi-faceted phenomenon which cannot be described, understood or explained by one monolithic theoretical approach

alone, a variety of theoretical perspectives and approaches is necessary to give justice to the complexity of the field.

That is why Ernest (1998), Artigue et al. (2006), Bikner-Ahsbahs & Prediger (2006), and many others pleaded for considering the diversity of theoretical approaches as a resource for grasping complexity that is scientifically necessary. However, emphasizing that the diversity of theories is a resource for scientific progress does not mean accepting the co-existence of isolated, arbitrary theoretical approaches which ignore others. Unconnected diversity might cause different difficulties: The first one that comes to mind might be the image of the mathematics education research community as it is regarded from outside, for example, by neighbouring disciplines. The more diverse and unconnected the frameworks, applied to empirical mathematics education research, is the more difficult it seems to be for non-specialists to perceive the community's identity as a coherent research field.

For us, even more important than these exterior difficulties are those *within* the community itself. Independent of image questions, the community often experiences the diversity of theories and theoretical approaches as challenges, but for different reasons:

- challenges for communication:
  "researchers from different theoretical frameworks sometimes have difficulties understanding each other in depth because of their different backgrounds, languages and implicit assumptions" (Arzarello et al., 2008a);
- challenges for the integration of empirical results:
  "researchers with different theoretical perspectives consider empirical phenomena from different perspectives and, hence, come to different results in their empirical studies. How can the results from different studies be integrated or at least understood in their difference?" (ibid.);
- challenges for scientific progress:
  "Improving mathematics classrooms depends in part on the possibility of a joint long term progress in mathematics education research in which studies and conceptions for school *successively build upon empirical research*. But how to do that when each study uses a different theoretical framework that cannot be linked to others? The incommensurability of perspectives produces sometimes incompatible and even contradictory results which not only impede the improvement of teaching and learning practices, but can even discredit a research field that may appear as being unable of discussing, contrasting and evaluating its own productions." (ibid.)

Plurality can only become fruitful, when different approaches and traditions *come into interaction*. In order to meet these challenges, the diversity of theories and theoretical approaches should be exploited actively by searching for connecting strategies. Connecting theories and theoretical approaches can become a fruitful starting point for a further development of the scientific discipline in three ways:

- developing empirical studies which allow connecting theoretical approaches in order to gain an increasing explanatory, descriptive, or prescriptive power;
- developing theories into parts of connected theory ensembles in order to reduce the number of theories as much as possible (but not more!) and to clarify the theories' strength and weaknesses;
- establishing a discourse on theory development, on theories and their quality especially for research in mathematics education that is also open to meta-theoretical and methodological considerations .

A motor for the evolution of theories and their connections in the direction of these aims is a desirable "culture of constructive debate" which need not necessarily lead to a consensus. However, it might clarify whether the theories used in studies are compatible. It might outline the perspectives or situations under which results and teaching proposals could be fruitful. It might also lead to further investigations deepening insights into theories and clarifying their potential for application on the one hand and integration into a new theory on the other.

Theories can be connected for different long-term aims:

- better communication and understanding,
- better collective capitalization of research results,
- more coherence at the global level of the field,
- limiting an exponential inflation of theories,
- gaining a more applicable network of theories to improve teaching and learning in mathematics education.

However, how can we connect theories?

3. Strategies for connecting theories – describing a landscape

The ZDM-issue 40(2) (Prediger, Arzarello, Bosch, & Lenfant, 2008a) assembled papers grown in the Theory Working Group of CERME 5. These papers describe case studies about connecting theories. In an introductory article (Prediger, Bikner-Ahsbahs, & Arzarello, 2008b), we compared the case studies with respect to the adopted strategies for connecting theories and with respect to the way and the purpose for which they were connected. This section presents a landscape of strategies for connecting theories as a first outcome of this comparison. "ZDM-issue" in this section refers to the one mentioned above. The examples given do not only refer to the ZDM issue, but also to two contributions in this volume written by Jungwirth and Gellert who present case studies for connecting strategies.

3.1. Introducing the terms

As the articles in the ZDM-issue show, there is a large variety of different strategies for dealing with the diversity of theoretical and conceptual frameworks and approaches, which we tried to systematize in the landscape shown in Figure 2 and by the following technical terms: We use the notion *connecting strategies* as the overall notion for all strategies that put theories into relation (including the non-relation of *ignoring other theories*).

Ignoring other theories and unifying theories in a global way are poles of a scale that allow distinguishing between different *degrees of integration* of the theories. Whereas *ignoring* is often guided by a pure relativism concerning theories considered as arbitrary and isolated, the call for a *global unification* is led by the idea of having one unique theory (that Dreyfus, 2006, compared to the grand unified theory of which many physicists dream). This idea is possibly inspired by the view on diversity as being an obstacle for scientific progress, but it risks to usurp the richness of theories by one dominant approach (like described by Lester, in this volume), and it is doubtful whether theoretical approaches with contradictory fundamental assumptions in their core (concerning for example their general assumptions on learning) could be globally unified without abandoning the core of one theory. Since we consider the diversity of theoretical lenses as a rich resource for grasping complex realities, this strategy of unifying globally is not further pursued here; it only serves as a virtual extreme position.

On the other hand, the pure relativism of laissez faire starts from the assumption that the diversity of theoretical approaches is a fact that prevents connections at all. Starting from the assumption that diversity is a source of richness, but connections should be drawn for the sake of scientific communication and progress (see Section 2), the authors in the ZDM-issue (and with them many other authors like Cobb, 2007; Lester, in this volume) adopt an intermediate position in repudiating isolationism and emphasizing the gain of different perspectives. The associated strategies for finding *connections as far as possible (but not further)* can be placed in between the two extremes on the scale in Figure 2. We call all intermediate strategies *networking strategies.* Hence, networking strategies are those connecting strategies that respect on the one hand the pluralism and / or modularity of autonomous theoretical approaches but are on the other hand concerned with reducing the unconnected multiplicity of theories and theoretical approaches in the scientific discipline (see Section 2).

Although Figure 2 attempts to order those networking strategies in between the two extremes with respect to the degree of integration, it must be emphasized that it is not easy to specify globally their exact topology, since the degree of integration always depends on the concrete realizations and networking methodologies, as will be elaborated in the next section. Nevertheless, it is worth trying to specify some clear notions as a first step towards a conceptual framework for the discussion on networking theories. The networking strategies are structured in pairs of similar strategies for which gradual distinctions can be made: understanding and making understandable, comparing and contrasting, combining and coordinating, and integrating locally and synthesizing.

Before explaining each of them in more detail, it must be emphasized that most researchers who connect theories apply more than one strategy at once. For analytical reasons, it is nevertheless helpful to reflect on the strategies and their preconditions separately.

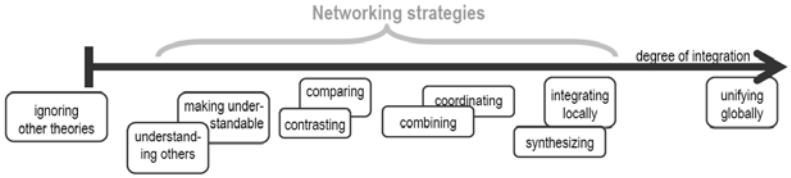

Figure 2: A landscape of strategies for connecting theoretical approaches

3.2. Understanding others and making own theories understandable

Each international conference with researchers from different theoretical and cultural backgrounds provides the practical experience that it is not trivial to *understand* theories that have been developed in unfamiliar research practices. Hence, all inter-theoretical communication and especially all attempts to connect and apply theories and research results must start with the hard work of *understanding others* and, reciprocally, with *making the own theory understandable*. We explicitly included this point into the list of strategies since, in the practical work, this is a laborious task which should not be underestimated: theories cannot be explained by their official terms alone. Understanding a theory means to understand their articulation in research practices which are full of implicit aspects. Already Kuhn (1969) emphasized the importance of – as he called them – *paradigmata* or *exemplars* that are key examples providing access to the empirical content of a theory.

Understanding other theories seems to be a precondition for connecting them, but at the same time, a successively deeper understanding is also a permanent aim of connecting attempts.

3.3. Comparing and contrasting

The mostly used pair of explicit networking strategies is *comparing* and *contrasting* theoretical approaches. Comparing and contrasting only differ gradually, but not in substance. Whereas comparing refers to similarities and differences in a more general way of perceiving theoretical components, contrasting is more focused on extracting typical differences. By contrasting, the specificity of theories

and their possible connections can be made more visible: strong similarities are points for linking and strong differences can make the individual strengths of the theories visible.

A comparison can be driven by different aims. First and most important is the aim to provide a base for *inter-theoretical communication*. As the comparison often leads to make implicit assumptions and priorities in the core of the theories and in their empirical components explicit, a comparison can contribute to a *better understanding* of the foreign and the own theories. Comparing and contrasting can *secondly* be used as a competition strategy on the market of available theories and theoretical approaches. And *thirdly*, comparing and contrasting may offer a *rational base for the choice* of theories demanding to move the debate "to a meta-level at which we are obliged to give good reasons for our theoretical choices" (Cobb, 2007, p. 28), the fact that the choice of theories is always only partially rational notwithstanding (ibid.).

Whereas Cobb (2007) offers an insightful example for contrasting on a rather *general* and global level for four important theoretical approaches, the articles in the ZDM-issue concentrate on more local comparisons which are concretely based each on a common example (like a common phenomenon, a research question or a common piece of data).

Comparisons can never be neutral, since every applicable criterion is already value-laden. That is why Cobb pleads for a discussion on suitable criteria drawn from explicitly stated normative positions. "These criteria reflect commitments and interests … and, for this reason, are eminently debatable and are open to critique and revision." (Cobb, 2007, p. 28). For his competitive comparison that explicitly aimed at an evaluation instead of a neutral comparison, he developed two criteria that focus on

1. "the manner in which they orient and constrain the types of questions that are asked about the learning and teaching of mathematics, the nature of the phenomena that are investigated, and the forms of knowledge that are produced." (Cobb, 2007, p. 3), which is later in his article focused on the way how the individual is conceptualized in the differing perspectives, and
2. "the potential of the perspectives to contribute to mathematics education as a design science", namely to "the enterprise of formulating, testing, and revisiting conjectured designs for supporting envisioned learning processes" (ibid, p. 15).

Even if not all attempts of contrasting are necessarily competitive (see next section), we follow Cobb's agenda to open a debate on suitable criteria for comparing and contrasting theoretical approaches. For this, we specify here the criteria for comparison as used in the different articles of the ZDM-issue and locate Cobb's propositions in between:

Theories can be compared or contrasted with respect to

- the role of well chosen implicit or explicit aspects in the theoretical structures (more general level), e.g.
    - conceptualization and role of individual (Cobb, 2007)
    - conceptualization and role of the social interaction (Kidron, Lenfant, Artigue, Bikner-Ahsbahs, & Dreyfus, 2008)
    - (epistemological) conceptualization of mathematical knowledge and its role in the research (Steinbring, 2008)
- the articulation of the theory in the practices of empirical research (more concrete level), e.g.
    - their enactment in the analysis of a given piece of data (Gellert, 2008; Maracci, 2008),
    - their general approaches to topics (like the students' problems with the limit of functions, Bergsten, 2008, or more generally in Cerulli, Trgalova, Maracci, Psycharis, & Geirget, 2008)
    - their articulation in the conceptualizations formulated for the transition of vague teaching problems into research questions (Prediger, 2008)

    - different conceptualizations of a research problem or phenomenon (Bergsten, 2008; Rodríguez et al., 2008)
- a priori defined criteria for quality of theories, e.g.
    - their potential contribution to design activities and their research background (Cobb, 2007)
    - validity versus relevance (Gellert, 2008)
    - other criteria like degree of maturity, explicitness, empirical scope, connectivity (see Bikner-Ahsbahs & Prediger, 2006)
- ….

3.4. Coordinating and combining

Whereas the strategies of comparing and contrasting are mostly used for a better understanding of typical characteristics of theories and theoretical approaches in view of further developing theories, the strategies of coordinating and combining are mostly used for a networked understanding of an *empirical phenomenon* or a piece of data.

Given that all theoretical approaches have their limitations as a lens for understanding empirical phenomena, the idea of triangulation (see Section 4) suggests looking at the same phenomenon from different theoretical perspectives as a method for deepening insights into the phenomenon. With her distinction between theoretical, practical and conceptual frameworks, Eisenhart (1991) made the point that many practically relevant empirical investigations cannot be drawn with one single theoretical approach alone but rely on various possibly far-ranging sources of appropriate sensitizing concepts and ideas. These sources are then combined in a so-called *conceptual framework*. In this volume, empirical examples for the connecting strategies *coordinating* and *combining* are presented by Gellert and Jungwirth.

The networking strategies of combining and coordinating are typical for conceptual frameworks which do not necessarily aim at a coherent complete theory but at the use of different analytical tools for the sake of a practical problem or the analysis of a concrete empirical phenomenon (see Cerulli et al., 2008; Maracci, 2008). In other projects, more comprehensive theories are combined or even coordinated at least locally (like Anthropological Theory of the Didactic –short ATD– and the APC-Space in Arzarello, Bosch, Gascon, & Sabena, 2008b).

Whereas all theories can of course be compared or contrasted, the combination of (elements of) different theories risks becoming difficult when the theories are not compatible. But different ways of connecting necessitate different degrees of compatibility. We use the word *coordinating* when a conceptual framework is built by well fitting elements from different theories. One example is given by Jungwirth (in this volume) who investigates students' interactive processes in computer-based learning environments. She coordinates an interactionist perspective based on symbolic interactionism and ethnomethodolgy for the micro-analysis with a complementary perspective taken from linguistic activity theory for the more holistic analysis. Both theories refer to students' interaction and activities but in different grain sizes, so Jungwirth can show that the perspectives complement each other in a fruitful way. Especially, she hypothesizes that complementarity with respect to the empirical load of theories is a suitable condition for coordinating theories.

Applying the strategy of coordinating usually should include a careful analysis of the relationship between the different elements and can only be done by theories with compatible cores (and different empirical components, see Section 1). It is especially fruitful when the empirical components (like typical lenses, research questions etc.,) are complementary. For example in the core of ATD and APC we find coherent but complementary theoretical objects (namely the praxeologies in ATD and the semiotic bundle in APC: see Arzarello et al., 2008b): they can support a more complete analysis (sup-

ported by different empirical components, e.g. the semiotic and the didactical analysis, resp. in APC and ATD) of an important didactical phenomenon (e.g. the so called chirographic reduction).
Hence a local coordinated analysis can be developed.

Not in all cases in which theoretical elements are combined, the elements fit in such a way. Sometimes, the theoretical approaches are only juxtaposed according to a specific aspect (like Maracci, 2008). Then we speak of *combining* rather than coordinating. Combining theoretical approaches does not necessitate the complementarity of the theoretical approaches in view. Even theories with conflicting basic assumptions can be combined in order to get a multi-faceted insight into the empirical phenomenon in view, for example Gellert (2008) combines two theories discussing the contradicting concepts of emergence or structure. In this volume, Gellert also presents a networking case of combining two theories with respect to the demand whether teachers should or should not be explicit about "rule use" in mathematics classrooms.

3.5. Synthesizing and integrating

When theoretical approaches are coordinated carefully and in a reflected way, this can be a starting point for a process of theorizing that goes beyond better understanding a special empirical phenomenon and helps to develop a new piece of synthesized or integrated theory (see also Jungwirth in this volume). This is the idea of Gravemeijer's (1994) metaphor of "bricolage" of theories that he uses for the process of theorizing by integrating global and local theories in his practice of design research.

We conceptualized this way of connecting theoretical approaches as the networking strategies *synthesizing* or *integrating (locally)*. Whereas the strategies of combining and coordinating aim at a deeper insight into an empirical phenomenon, the strategies of synthesizing and integrating locally are focused on the development of theories by putting together a small number of theories or theoretical approaches into a new framework.

Again, we make a gradual distinction between the two related strategies which this time refers to the degree of symmetry of the involved theoretical approaches. The notion synthesizing is used when two (or more) equally stable theories are taken and connected in such a way that a new theory evolves. But often, the theories' scope and degree of development is not symmetric, and there are only some concepts or aspects of one theory integrated into an already more elaborate dominant theory. This integration should not be mistaken as *unifying totally*, that is why we emphasized the term "locally" in the strategy's name "integrating locally".

Synthesizing and integrating have stronger preconditions than the other networking strategies. As already emphasized in Bikner-Ahsbahs & Prediger (2006), different parts of incompatible theories should not be synthesized into arbitrary patchwork-theories. Especially when the cores of theories contradict, there is a danger of building inconsistent theoretical parts without a coherent philosophical base. Jungwirth (in this volume) elaborates some conditions for compatibility in more detail.

It is not only accidental that the ZDM-issue comprises more articles applying the networking strategies comparing, contrasting, and coordinating than synthesizing or integrating, for two reasons: First, the last two strategies have stronger preconditions, and secondly, they must usually build upon the less integrative strategies and, hence, need more time to be evolved. This is apparent in the single exception of the ZDM-issue, namely Steinbring's epistemological perspective on social interactions that evolved as a synthesis of social and epistemological approaches (this synthesis is more explicitly explained in Steinbring, 2005).

One interesting example of integrating is given by Gellert (in this volume) who integrates Bernstein's structuralist perspective and Ernest's social semiotics by focussing on the different roles of rules within the two theories. Gellert distinguishes two modes of theorizing within local integrations: *bricolage* and *metaphorical structuring*. Theorizing as bricolage is outlined critically showing that this metaphor should be used carefully in a research context. By his analysis of a data set, he achieves in

detail the meaning of theorizing as metaphorical structuring that enables him to transcend the restrictions of each of the two theories by a paradigmatic change of research question asking "What is an appropriate balance of explicitness and implicitness in mathematics instructions? Is it the same for all groups of students?"

Although it is fruitful for analytical purposes to describe distinct networking strategies, their activation in practice can vary, and often more than one strategy is used at the same time. Besides the different networking strategies, there exist different concrete methods for networking theories. Being far from a complete systematization of networking methods (or even methodologies, respectively), we present some examples from the contributions of the ZDM-issue.

In this volume, Gellert and Jungwirth investigate conditions for building new theory bricks. Jungwirth's example illustrates three suitable conditions for synthesizing and locally integrating: consistent paradigms, neighbouring sites of phenomena, and different empirical loads. Gellert shows what can be meant when stressing that integration only is possible if the theories' principles are close enough. He pleads for an additional view on local integration, namely locally integrating theories from outside mathematics education into the area of mathematics education.

3.6. Strategies and methods for networking

The distinction drawn here between networking strategies and methods for networking can tentatively be illuminated by a metaphor, namely the military distinction between strategy and tactics: A strategy is a set of general guidelines to design and support concrete actions in order to reach a distinct goal. Whereas a strategy is something general and stable, tactics is more specific and flexible. A battle can never be planned by strategies alone, since it involves many actions with open results. These actions that must be decided in real time according to the chosen strategy are then designed by special tactics.
Similarly, the more general networking strategies require specific methods to be developed for their concrete application. This section illustrates different methods for developing or applying a specific networking strategy.

Networking strategies are used to link or relate theories or theoretical approaches. Networking strategies, research methods and techniques are intertwined and can support each other. Different methodical approaches might use the same networking strategies, and, otherwise, one methodical approach might include different networking strategies as well (see below). In the following, some case studies of the ZDM-issue are discussed with respect to their *networking strategies*, *focus*, *methods* and sometimes *methodology*.

Focusing on studies about the concept on limits of functions, Bergsten (2008) presents a meta-analysis comparing theories as mediators among practice, problems and research by using the scholarship triangle of Silver and Herbst (2007). He stresses that it is necessary to develop a network of didactical knowledge and that this is the reason why different theoretical backgrounds have to be considered. His main networking strategy is *contrasting* and *comparing*.

Like Bergsten, Steinbring (2008), too, uses the networking strategy of comparing and contrasting, but in a different way. He focuses on the origin of a new theoretical approach and especially the changing views on mathematical knowledge. Through a historical reconstruction of a pathway of theory development, he shows that the changes of theory viewpoints are rooted in the insight that relevant problems at a specific time could not be investigated by existing traditional theories and therefore demanded a change of paradigm. However, since old traditions may stay alive and develop further, such a situation causes a *branching pattern* of theory evolution, one cause among others for the existing diversity of theories. Hence, Steinbring basically contributes to explaining exemplarily why there are many different theories. In his case, old and new theoretical viewpoints are even incommensurable in some aspects of their core. But even in these cases, networking by contrasting is valuable.

Cerulli et al. (2008) present a very interesting example for a longer-term networking effort, developed by the European research group TELMA. The researchers with different theoretical approaches commonly search for improvements and changes that technology can bring to teaching and learning mathematics. Therefore they want to understand the exact role that the different theoretical approaches play in designing and researching computer environments (so-called interactive learning environments). Realizing the limitations of only reciprocally reading articles, the team developed an interesting methodology for *comparing* and *connecting* theoretical approaches: the so-called *cross-experimentation.* Cross-experimentation means that each team used computer tools developed by another team to develop and study an own learning environment. In this way, the teams could *compare*, *understand* and *make understandable* the role of different theoretical approaches in the research practice of designing learning environments and analyzing teaching experiments. Since these intensive networking efforts were based on collaborative practices and concrete research studies, they directly affected the empirical components of their theories. The teams could reconstruct underlying priorities and assumptions which are not explicit in the different approaches but are characteristics for the core of the theory. In the end, the teams were able to compose their results under a common conceptual framework. They found an interesting contingency for the design practice which turned out to be less predetermined by the theories than supposed.

Prediger (2008) also describes an activity for *comparing* theories. The method is focused on the expression of theories in the interpretation of professional issues for the practice of research. The comparison is based on answers of researchers with different theoretical background. They were asked how they would conceptualize a typical given teaching problem as a research problem. The comparison of the answers indicates another role of theory in the research process: the role to provide an empirical research question. In this way, the theories' empirical components were in the centre of the comparison.

Kidron et al. (2008) *compare* and *contrast* as well, but again their method is different from all the others. They use sensitizing concepts to compare theories according to the question of what the three theories might be able to learn from the others in order to further develop (Bikner-Ahsbahs, 2007). The main method was comparing and contrasting these concepts and their relationships by comparing each pair of theories (*three-by-two-comparison*).

Gellert (2008) *compared* Bernstein's structuralist perspective and the interpretative approach according to the role of the theories and the notion of relevance and validity in the theories. By triangulation, he analyzed the same piece of data in two perspectives. Bernstein's theory can be understood as a tool to make sense of a phenomenon, whereas the interpretative approach prefers to construct a new piece of theory which allows understanding the phenomenon in detail while avoiding subsuming it under a theory. Because of the different roles and grain sizes, the theories' relevance and validity have to be understood in different ways. Referring to the different grain sizes, Gellert proposes a method of dialectical consideration for empirical research to benefit from the strength of both theories which can be regarded as a case of *coordinating*. Different grain sizes seem to offer a possibility to connect theoretical approaches within one conceptual framework. In this volume, Gellert also uses the method of dialectical considerations but this time he aims at locally integrating theories.

Halverscheid (2008) also *coordinates* two tools for empirical research according to their different grain sizes. He uses an epistemic model to analyse the epistemic character of more global actions in experimental learning situations in order to further develop theoretical understanding of modelling processes.

Arzarello et al. (2008b) tried a (local) *coordination* of two theories, the Anthropological Theory of the Didactic and the APC-space, for analysing a specific research object, the ostensives, introduced in the ATD framework. They used them to integrate different time and space grains of analysis, "from the small-scale flying moment of a learning process in a specific classroom as described in the APC-space to the long term and wide events, which produce the praxeologies at regional level described in the ATD". In fact, one of the theories (APC-space) allowed the authors to develop a fine-grained cognitive analysis; on the other hand, the other theory (ATD frame) made it possible to develop an analysis from a cultural and institutional point of view. In this way, the two approaches could be coordinated and benefit from the *merging of different scales*.

Rodriguez, Bosch and Gascon (2008) used a method of *reformulating a problem in a new theoretical framework for comparing theories (and making them understandable)*. They converted the notion of metacognition into the approach of the Anthropological Theory of the Didactic relating it to institutional practices, as the current way of organizing teaching processes and the artificial distinction between 'doing mathematics' and 'studying mathematics'. They show: When a construct like metacognition which originates in a cognitive perspective is studied in a mathematical and institutional perspective, it significantly changes its characteristics. They also show that cognitive approaches on metacognition adopt initial assumptions about the nature of mathematical knowledge which are too close to the educational institution considered. In this way, they were able to compare and contrast cognitive and institutional perspectives.

Converting can also take place on a more general level, when not only an empirical question is converted into another theory, but also theoretical constructs and typical methods are (at least hypothetically) converted from one theory into another, for example: If we take the a-priori-analysis from the Theory of Didactical Situations, what concept within the theory of Abstraction in Context would correspond to it (see Kidron et al., 2008)? If conversing is not only hypothetical, it might also offer a method for further developing a theory.

Although far from providing a complete systematization, this overview on the articles of the ZDM-issue shows that researchers who try to connect theories do not only use *different networking strategies* (like understanding and making understandable, comparing and contrasting, coordinating and combining, integrating and synthesizing), but also *different methods* (like cross-experimentation, dialectical consideration, three-by-two comparison, creating research designs, etc). Furthermore, we see that the articles *focus on different aspects* of theory, for example theory as a mediator among practice, problems and research; theory as a tool; evaluation standards, origin of theories and core concepts.

4. Developing Theories by Networking

English and Sriraman (2005, e.g. p. 453) discuss the claim that theories in mathematics education should be further developed. But what exactly does it mean to develop theories further? This depends on the theory's character, since explanative and descriptive theories develop differently from prescriptive theories.

Empirically grounded theories develop in a spiral process of empirical analysis and theory construction. For example, Bikner-Ahsbahs (2005; 2008) begins the development of the theory about interest-dense situations with a first conceptual component in the context of background theories. Then, the analysis of data leads to a first hypothesis which can again be tested through analysing data. New hypotheses are generated, etc. In this spiral process between theory development and empirical analysis and testing, non-consistent components are systematically sorted out. This sequential chain of construction steps provides losses and gains: Whereas information about the research objects is lost, theoretical propositions are successively gained.

In contrast to such processes of empirically based theory building, the development of prescriptive theories (like Prediger, 2004) is characterized more by argumentative connections to other theory elements and by the successive process of making explicit the philosophical base. Nevertheless, empirical correspondences and relevance for classrooms practices play an important role as criteria of relevance and acceptance.

Theories cannot only develop in different *ways*, but also in different *directions*. In our view, the question of how theories can develop should be discussed in the communities' discourse on theory. We consider at least four directions to be important:

1. *Explicitness*: Starting from the claim that a good theory should make its background theories and its underlying philosophical base (especially its epistemological and methodological foundations) as explicit as possible, the maturity of a theory can be measured by the degree of its explicitness: The more implicit suppositions are explicitly stated and the more parts of the philosophical base shape explicit parts of the background theory, the more we would consider the theory to be *mature*.

2. *Empirical scope*: Formal theories have a large empirical scope. They characterize empirical phenomena in a global way and often cannot exactly be concretized through empirical examples (Lamnek, 1995, p. 123). On the other hand, local and contextualized theories have a limited scope but their statements can more easily be made concrete by the empirical content (see Krummheuer, 2001, p. 199). This proximity to empirical phenomena makes contextualized theories a suitable background to guide practice in schools. However, developing local theories in order to *enlarge their empirical scope* can be an important direction for theory development.

3. *Stability:* A new theory might be a bit fragile because its concepts and the relationships among them are still vague. However, if the theory is substantial it contains a surplus. This surplus is continuously worked out by increasing applications of the theory; its empirical area around its core broadens. If the theory withstands examinations it becomes trustworthy, the concepts become clearer, and the *stability of the theory increases*.

4. *Connectivity:* Science is characterized by argumentation and interconnectedness, as Fischer (e.g. 1993) emphasizes. This can for example be realized by establishing relationships linking theories, by declaring communalities and differences. Hence, establishing *argumentative connectivity* is another important direction for the development of theories.

The growing discourse on theory within the scientific community deals already with the question whether there are more directions of development and whether we can formulate standards for degrees of theory development. Schoenfeld (2002) proposes eight standards for evaluating theories: descriptive power; explanative power; scope; predictive power; rigor and specificity; replicability, generality, and trustworthiness; and multiple sources of evidence (triangulation). Not all these standards fit for every theory, but they can give some orientation in what other directions theories could be further developed.

How to develop theories further is not only an isolated question guiding separate research pathways. It is fundamentally interwoven with the question of how the research community as a whole, with its manifold different theories, can develop further. The answer to the second question is far from being clear. We consider it to be a crucial point for our discipline.

Starting with the assumption that the existing diversity of theoretical approaches is a challenge for the research community as well as a resource for coping with the complexity of the research field, we suggest that there is a scientific need to connect theories, and we propose the networking of theories as a more systematic way of interacting with theories.

This article has discussed some networking strategies and conditions under which connecting theories systematically can help to more consequently exploit the richness of the diversity of theories. The basic frame for this attempt was a dynamic concept of theory whose notion is shaped by its core ideas,

concepts and norms on the one hand and the practices of researchers – and mathematics educators in practice - on the other hand.

As the comparison of case studies has made explicit, the networking of theories can be done in different ways using different networking strategies that focus on different aspects of theory and for different aims, namely

- understanding each other (and ourselves),
- better understanding of a given empirical phenomenon,
- developing a given theory, or
- overall (long-term) aim: improving teaching practices by offering orientational knowledge or design results.

For giving some concluding hints how connecting theories can *contribute to their further development*, we refer to the different directions for theory development named above: 1. Explicitness, 2. Empirical scope, 3. Stability, 4. Connectivity.

Understanding other theories and making one's own theories understandable, comparing and contrasting theories have been conceptualized as important networking strategies because they force the researchers to be more *explicit* on the theories' central implicit assumptions and values, their strengths and weaknesses. This experience is shared by the authors of the ZDM-issue who engaged in this process and started a process of *better communication and understanding* (see Section 3).

The *empirical scope* of theories can be broadened by coordinating and integrating new aspects into its empirical component, seldom changing its core. One instance of this effect is the coordination of theories of different grain sizes as presented by Halverscheid (2008), Gellert (2008) and Arzarello et al. (2008b) who could *better capitalize on the research* of other traditions (see Section 3).

The most difficult aim is that networking should contribute to the *stability of theories.* Isn't the contrary the case, aren't theories questioned by the confrontation with other theoretical approaches? Our first experiences give hope that in the long-term perspective, theories will be further developed, hence, consolidated deep in their core by connecting and questioning them with other theories and by complementing their empirical components. However, the presented case studies are not yet far enough developed to give empirical evidence for this hope.

When emphasizing the tautology that connecting theories can contribute to *connectivity*, it is necessary to recall the arguments why we consider a development into the direction of more connectivity to be indeed a progress. In Section 2, we argued that supporting to develop connectivity of theories means to reduce isolated approaches and gain more connected knowledge within our community. In the long run, we hope that this research direction will contribute to a changed understanding of theories within the scientific discipline. When connectivity becomes more and more established, theories might be seen as parts of a network which frames learning and teaching processes as a whole rather than single and independent knowledge systems. In this way, a new quality of *coherence* might be established giving diversity a structuring frame and offering practice a better guide to *improve teaching and learning mathematics* (see Section 3).

However, so far, we have only made first steps in this direction and should carefully continue to produce solid and applicable knowledge. Since communication of researchers is central for the networking of theories, clarity should be kept at all the levels of work. This can only be achieved through working in a concrete way, using empirical phenomena and with an open minded attitude towards other perspectives and own assumptions, nevertheless let us go *as far as possible, but not further*.

Acknowledgement

This article has grown within fruitful discussions in the Theory Working Group at CERME 4 - 6 and in even more intense discussions with Ferdinando Arzarello, Michèle Artigue, Marianna Bosch, Tommy Dreyfus, Stefan Halverscheid, Agnès Lenfant, Ivy Kidron, Kenneth Ruthven and Cristina Sabena. Many ideas arose in the com-

mon work and are now presented in this article, although the authors are not able to assign them to their original contributors anymore. Hence, all of the above mentioned people have an important part in the development of this article.

References

Assude, T., Boero, P., Herbst, P., Lerman, S., & Radford, L. (2008). *The notions and roles of theory in mathematics education research, Paper presented by the Survey Team at ICME 11*. Mexiko 2008.

Arnold, M., & Fischer, R. (2004) (Eds.). *Disziplinierungen. Kulturen der Wissenschaft im Vergleich [Disciplines. Sciences as cultures in comparison]*. Wien: Turia & Kant.

Artigue, M., Bartolini-Bussi, M., Dreyfus, T., Gray, E., & Prediger, S. (2006). Different theoretical perspectives and approaches in research in mathematics education. In M. Bosch. (Ed.), *Proceedings of the 4th Congress of the European Society for Research in Mathematics Education* (pp. 1239-1244). Barcelona: Fundemi IQS.

Arzarello, F., Bosch, M. Lenfant, A., & Prediger, S. (2008a). Different theoretical perspectives in research from teaching problems to research problems. In D. Pitta-Pantazi, G. Phillipou et al. (Eds.), *Proceedings of the 5th Congress of the European Society for Research in Mathematics Education (CERME 5)* (pp. 1618-1627). Cyprus: ERME.

Arzarello, F., Bosch, M., Gascón, J., & Sabena C. (2008b). The ostensive dimension through the lenses of two didactic approaches. *ZDM - The International Journal on Mathematics Education, 40* (2), 179-188.

Bauersfeld, H. (1988). Interaction, construction, and knowledge - Alternatives for mathematics education. In: Cooney, T. & Grouws, D. (Eds.), *Effective mathematics teaching*. Reston, VA: National Council of Teachers of Mathematics, 27-46.

Bergsten, C. (2008). How do theories influence the research on teaching and learning limits of functions. *ZDM - The International Journal on Mathematics Education*, *40* (2), 189-199.

Bigalke, H.-G. (1984). Thesen zur Theoriediskussion in der Mathematikdidaktik. [Contributions to the discussion about theories in mathematics education.] *Journal für Mathematik-Didaktik*, *5,* 133-165.

Bikner-Ahsbahs, Angelika (2005). *Mathematikinteresse zwischen Subjekt und Situation [Interest in mathematics between subject and situation]*. Hildesheim: Verlag Franzbecker.

Bikner-Ahsbahs, A. (2008). Erkenntnisprozesse. Rekonstruktion ihrer Struktur durch Idealtypenbildung. [Epistemic processes. Reconstruction of its structure by building ideal types.] In Helga Jungwirth & Götz Krummheuer (Eds.), *Der Blick nach innen: Aspekte der alltäglichen Lebenswelt Mathematikunterricht. [The view inside: aspects of the everyday life in mathematics classrooms.]* (pp. 105-144). Münster et al.: Waxmann.

Bikner-Ahsbahs, A. (2007). "Sensitizing concepts" as heuristics to compare and connect different theories. Contribution to the symposium: Networking a variety of theories within a scientific domain - The case of mathematics education. In Ben ö Csapó & Csaba Csikos (Eds.), *Developing Potentials for Learning*, *12th Biennial Conference for Research on Learning and Instruction*. Budapest: University of Szeged.

Bikner-Ahsbahs, A., & Prediger, S. (2006). Diversity of Theories in Mathematics Education - How can we deal with it? (*ZDM*) *Zentralblatt für Didaktik der Mathematik, 38* (1), 52-57.

Brousseau, G. (1997). *The theory of didactical situations in mathematics*. Dordrecht: Kluwer.

Cerulli, M., Trgalova, J., Maracci, M., Psycharis, G., & Geirget, J.-P. (2008). Comparing theoretical frameworks enacted in experimental research: TELMA experience. *ZDM - The International Journal on Mathematics Education, 40* (2), 201-213.

Chevallard, Y. (1992). Concepts fondamentaux de la didactique: Perspectives apportées par une approche anthropologique. *Recherches en Didactique des Mathématiques, 12* (1), 73-112.

Cobb, P. (2007). Putting philosophy to work. Coping with multiple theoretical perspectives, In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 3-38). Reston: NCTM.

Dreyfus, T. (2006). Linking theories in mathematics education. In A. Simpson (Ed.), *Retirement as Process and as Concept* (pp. 77-82). Festschrift for Eddie Gray and David Tall, presented at Charles University, Prague.

Eisenhart, M. (1991). Conceptual Frameworks for Research. Ideas from a Cultural Anthropologist; Implications for Mathematics Education Researchers'. In R. Underhill (Ed.), *Proceedings of the Thirteenth Annual Meeting of Psychology of Mathematics Education – North America*. Blaksburg, VA: Psychology of Mathematics Education.

English, L., & Sriraman, B. (2005). Theories of Mathematics Education: A global survey of theoretical frameworks / trends in mathematics education research. (*ZDM*) *Zentralblatt für Didaktik der Mathematik,* 37 (6), 450-456.

Ernest, P. (1998). A postmodern perspective on research in mathematics education. In A. Sierpinska & J. Kilpatrick (Eds.), *Mathematics Education as a research domain: A search for identity*, *Volume 1* (pp. 71-85). Dordrecht: Kluwer.

Fischer, R. (1993). Wissenschaft, Argumentation und Widerspruch [Science, argumentation and contradiction]. In R. Fischer et al. (Eds.), *Argumentation und Entscheidung. Zur Idee und Organisation von Wissenschaft* (pp. 29-43). Wien-München: Profil.

Gellert, U. (2008). Validity and relevance: Comparing and combining two sociological perspectives on mathematics classroom practice. *ZDM - The International Journal on Mathematics Education*, *40* (2), 215-224.

Gravemeijer, K. (1994). Educational development and developmental research. *Journal for Research in Mathematics Education, 25*, 443-471.

Halverscheid, S. (2008). Building a local conceptual framework for epistemic actions in a modelling environment with experiments. *ZDM - The International Journal on Mathematics Education*, *40* (2), 225-234.

Harel, G., & Lim, K. H. (2004). Mathematics Teacher's Knowledge Base: Prelimary results. In M. Johnsen Høines, A. B. Fuglestad (Eds.), *Proceedings of the 28th Conference on the International Group for the Psychology of Mathematik Education* (pp. 25-32). Bergen: University College.

Jungwirth, H. (1996). Symbolic Interactionism and Ethnomethodology as a Theoretical Framework for the Research on Gender and Mathematics. In G. Hanna (Ed.), *Towards Gender Equity in Mathematics Education. An ICMI Study* (pp. 49-70). Dordrecht: Kluwer.

Kidron, I, Lenfant, A., Artigue, M., Bikner-Ahsbahs, A., & Dreyfus, T. (2008). Toward networking three theoretical approaches: The case of social interaction. *ZDM - The International Journal on Mathematics Education*, *40* (2), 247-267.

Krummheuer, G. (2001). *Paraphrase und Traduktion*. Weinheim: Beltz Verlag.

Kuhn, T. S. (1969). *Die Struktur wissenschaftlicher Revolution [The structure of scientific revolutions]*. Frankfurt a.M.: Suhrkamp.

Lamnek, S. (1995). *Qualitative Sozialforschung [Qualitative Social Research]*. Weinheim, Basel: Beltz Verlag.

Maier, H., & Beck, C. (2001). Zur Theoriebildung in der interpretativen mathematikdidaktischen Forschung. [Towards building theories in the interpretative research of mathematics education]. *Journal für Mathematik-Didaktik*, *22* (1), 29-50.

Maracci, M. (2008). Combining different theoretical perspectives for analyzing students' difficulties in Vector Spaces Theory. *ZDM - The International Journal on Mathematics Education*, 40(2), 265-276.

Mason, J., & Waywood, A. (1996). The role of theory in mathematics education and research. In A. J. Bishop et al. (Eds.), *International Handbook of Mathematics Education.* (pp. 1055-1089), Dordrecht: Kluwer.

Niss, M. (2007). Reflections on the State of and trends in research in mathematics teaching and learning. From here to utopia. In F. K. Lester, Frank K. (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 1293-1312). Reston: NCTM.

Prediger, S., Arzarello, F., Bosch, M., & Lenfant, A. (2008a) (Eds.). Comparing, Combining, Coordinating - Networking strategies for connecting theoretical approaches. *Thematic Issue of ZDM - The International Journal on Mathematics Education*, *40* (2), 163-327.

Prediger, S., Bikner-Ahsbahs, A., & Arzarello, F. (2008b). Networking strategies and methods for connecting theoretical approaches – First steps towards a conceptual framework. *ZDM - The International Journal on Mathematics Education, 40* (2), 165-178.

Prediger, S., Bosch, M., Kidron, I, Monaghan, J., & Sensevy, G. (2009, in preparation). Different theoretical perspectives and approaches in mathematics education research - strategies and difficulties when connecting theories. In *Proceedings of CERME 6*, (in press), Lyon.

Prediger, S., Viggiani-Bicudo, M., & Ernest, P. (2008). Philosophy of Mathematics Education – Strands and Issues. In M. Niss, E. Emborg (Eds.), *Proceedings of the 10th International Congress on Mathematical Education* (pp. 438-442). Kopenhagen 2004; Roskilde University.

Prediger, S. (2008). How are theoretical approaches expressed in research practices? A report on an experience in comparing theoretical approaches with respect to the construction of research problems. *ZDM - The International Journal on Mathematics Education,* 40(2), 277-286.

Prediger, S. (2004). Intercultural Perspectives on Mathematics Learning – Developing a Theoretical Framework. *International Journal of Science and Mathematics Education, 2* (3), 377-406.

Rodriguez, E., Bosch, M., & Gascón, J. (2008). A networking method to compare theories: Metacognition in problem solving reformulated within the Anthropological Theory of the Didactic. *ZDM - The International Journal on Mathematics Education*, *40* (2), 287-301.

Schoenfeld, A. H. (2002). Research methods in (mathematics) education. In L. D. English (Ed.), *Handbook of international research in mathematics education* (pp. 435-487). Mahwah: Lawrence Erlbaum Associates.

Silver, Edward A. & Herbst, Patricio G. (2007). Theory in Mathematics Education Scholarship. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 39-68). Reston: NCTM.

Steen, Lynn A.(1999). Review of Mathematics Education as research domain. *Journal for Research in Mathematics Education, 30* (2), 235-241.

Steinbring, H. (2005). *The Construction of New Mathematical Knowledge in Classroom Interaction - An Epistemological Perspective. Mathematics Education Library, vol. 38.* Berlin, New York: Springer.

Steinbring, H. (2008). Changed Views on Mathematical Knowledge in the Course of Didactical Theory Development. *ZDM - The International Journal on Mathematics Education*, *40* (2), 303-316.

Voigt, J. (1994). Negotiation of mathematical meaning and learning mathematics. *Educational Studies in Mathematics*, *26*, 275-298.