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RELATING REGISTERS FOR FRACTIONS – MULTILINGUAL STUDENTS ON THEIR WAY TO CONCEPTUAL UNDERSTANDING

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Learning situations that concentrate on conceptual understanding are particularly challenging for learners whose first language is not the language of instruction. This article presents an approach that aims to deepen language proficiency and to develop understanding by using integrated activities on several registers. Short snapshots from a case study illustrate how bilingual German-Turkish students deal with challenging tasks on complex relations of fractions by our approach of 'relating registers'.

1. CONTEXT AND AIMS OF THE STUDY

About 20 % of all students in German schools (mostly children of immigrants in first to third generation) have to learn mathematics in another language other than their first language. Turkish is the language of the biggest language minority, but in many classrooms, up to seven different immigrant languages are present. In contrast, most mathematics teachers are monolingual Germans. It is only in the recent past that the German school system has become aware of not sufficiently building upon the cognitive potential of these students. In German classrooms, students with non-German first language are not only less successful than their German speaking classmates, but are also less successful than students in similar countries who do not speak the language of instruction as their first language (OECD 2007, p. 120).

Based on this finding, innovative language classroom practices have been promoted that aim at helping students to deepen their German proficiency in all subjects, including mathematics (e.g. Ahrenholz 2010). Although these general language activities are helpful, our Dortmund project "MuM – Mathematics learning under conditions of multilingualism" follows two complementary teaching strategies for allowing multilingual students to gain a deeper understanding of mathematics: 1. Building upon the *resource of their first languages* (see Meyer & Prediger in press) coupled with 2. '*Relating registers'* as an approach for deepening language proficiency that is *more specific to mathematics* and focuses on conceptual understanding as the central part of mathematical literacy. This article presents the theoretical foundation of the second of the above teaching strategies and some empirical snapshots. It intends to contribute to the development of *topic-specific research-based knowledge* on teaching strategies and learning challenges for multilingual learners in various mathematical topics. We chose fractions as the specific mathematical topic since fractions count as one of the most difficult topics in the middle school curriculum, especially for developing understanding and adequate models. The empirical snapshots we describe will contribute to deepening insights into the complex relation between the strongly intertwined linguistic and mathematical aspects of constructing conceptual understanding in mathematics.

2. THEORETICAL BACKGROUND: REPRESENTATIONAL REGISTERS

2.1 Linguistic registers: BICS, CALP and technical language

For explaining the situation of second or third generation students of Turkish origin, we refer to Cummins' (1979) classical distinction between BICS and CALP: Most of the second generation students have fluent "basic interpersonal communication skills" (BICS) in the *spoken* everyday language. In contrast, their "cognitive academic language proficiency" (CALP) is limited, mostly in German, although some of these students have some such proficiencies in Turkish. The language of instruction in mathematics classrooms consists of a mixture of a highly structured, decontextualized, written language, with more complex grammatical constructs and aspects of technical mathematical language than students will meet outside of the mathematics classroom. This explains the strong statistical connection between CALP and achievement in mathematics found in several studies (e.g. Heinze et al. 2009).

Discussing similar theoretical issues, Clarkson (2009) describes linguistic challenges in mathematics classrooms as the challenge to switch between three linguistic registers (here called BIC-register, CAL-register and technical register, see green levels of Fig. 1) in the first language (abbreviated L1) and the language of instruction (abbreviated L2). Whereas Clarkson mentions even L3 and L4, this study focuses only on a total of six [2 (L1 and L2) x 3 (BIC, CAL, technical)] registers to consider.

2.2 Mathematical representational registers

Whereas linguistic perspectives focus on these 2 x 3 verbal registers, other *non-verbal representations* have relevance for developing understanding in mathematics, including concrete representations, graphical representations, symbolic-numerical and symbolic-algebraic representations. As many researchers have shown (e.g. Goldin & Shteingold 2001; Duval 2006), the cognitive development of mathematical concepts is deeply connected to the ability to relate concepts in different representational modes. That is why Lesh (1979) and others proposed switching representations as a teaching strategy to promote understanding many years ago.

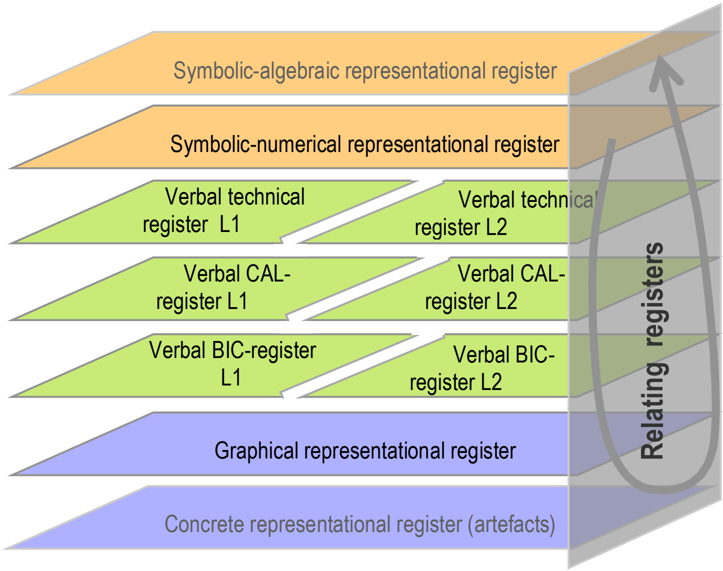


Fig. 1 Integrated model of registers

In this study we synthesized the mathematical modes of representation and the linguistic registers into one integrated model of relevant registers (see Fig. 1, cf. Prediger & Wessel in press). The registers cannot be understood hierarchically in terms of quality, their order indicates an increase of abstraction. By 'relating registers', we mean different cognitive activities like *translating, switching, assigning, or contrasting different registers*. Our teaching strategy especially comprises the idea that the deliberate use of the graphical and symbolic registers can support the development of verbal capacities on the different registers, can help to link L1 and L2, and offers opportunities to construct mathematical meanings and relations of crucial concepts.

2.3 Relating registers for parts of parts and the multiplication of fractions

The mathematical topic of fractions is well known for being one of the most difficult in middle school mathematics. Many (mono-and multilingual) students develop some technical skills, but only a limited conceptual understanding and low capacities for applying fractions in varying contexts (Hasemann 1981; Aksu 1997). One major source of difficulties is the insufficiency of students' mental models (Prediger 2008).

For the example of multiplying fractions, Fig. 2 shows how relating registers require the activation of adequate mental models: the symbolically given multiplication can only be drawn or related to a verbally represented situation if it is *interpreted* by an adequate mental model, here by the part-of-part-interpretation. Vice versa, the *mathematization* of the verbally given situation also requires the part-of-part-interpretation of multiplication (vom Hofe et al. 2006). Recent studies have shown that such activities of relating registers for parts of parts are demanding even for prospective teachers as it necessitates to clarify the relation between the fraction, the represented part, and its referent whole (Prediger & Schink 2009).

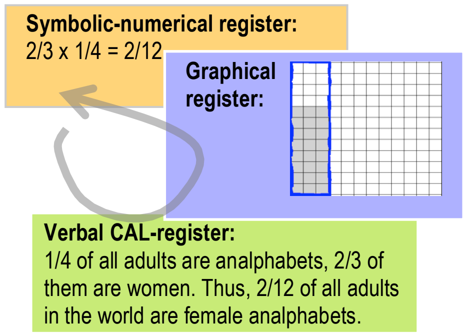


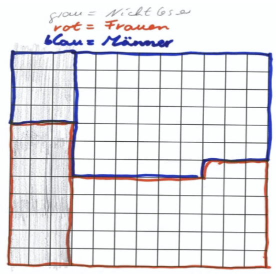
Fig. 2 Parts of parts in three registers

3. CASE STUDY ON THE SITUATIONAL POTENTIAL OF THE RELATING REGISTERS STRATEGY

3.1 Material and design of the interview study

Within the MuM-project, the teaching strategy 'relating registers' is developed and researched within the paradigm of design research by means of design experiments (Gravemeijer & Cobb 2006). The empirical research focus is on the situational potential to initiate substantial mathematical and linguistic activities of students. Twenty-five interviews were conducted with pairs of students in grade 6. These students had Turkish as their first language and good German BICS. The interviews were videotaped, transcribed and analysed qualitatively. For this, the processes of relating registers were coded and then analysed with respect to the chances and challenges for conceptual understanding (more details in Prediger & Wessel in press).

Analphabets in the world
According to a UN report, 1/4 of all adults in this world are analphabets, that means, they cannot read. Due to this, they cannot learn many professions. 2/3 of all analphabets are women.



grey: analphabets
blue: men
red: women

Fig. 3 Interview text and drawing by a fictitious student

The starting point for the interview material was a text in the CAL-register L2 that encodes complex information of a UN-report about rates of analphabets (see Fig. 3). The main conceptual challenge of this text is to refer 2/3 to the adequate referent whole which is neither all adults nor all women but the group of all analphabets (being a quarter of all adults). To give students the opportunity to construct the necessary conceptual relations, five activities of relating registers were initiated:

Step 1: Translate the given CAL-text (in CAL-register L2) into the own words (BIC-register L1/L2).
Step 2: Check if the text matches to another BIC-utterance of a fictitious student Tobias, "Wow, two thirds of all women cannot read? Is that possible?"
Step 3: Translate BIC- or CAL-text into your own drawing.
Step 4: Check if a given drawing (Fig. 3, right side) matches the own text and picture.
Step 5: Assign simpler texts, pictures and fractions (this step is not discussed here).

3.2 The case of Amir and Ekim: Challenges and insights while relating registers

This section presents selected snapshots as first results of an ongoing data analysis. We focus on the case of Amir and Ekim, two 12 year old boys in grade 6. Both were born in Germany and grew up with Turkish as their first language and German as their second language. Their case can illustrate some exemplary phenomena.

Having read the text (in Fig. 3) and contextualized it by talking about analphabets, the boys are asked to find a simpler formulation for it. Amir writes down the following: "1/4 of all adults cannot read. That is why they cannot get many jobs." The first excerpt of the transcript (translated from German) starts in minute 6:36 when Ekim suggests the next reformulations for the phrase "2/3 of all analphabets are women".

59	Ekim	Oh wait, shortly. One quarter [*whispers something not understandable*]
60	Amir	Loud!
61	Ekim	Of one quarter are two thirds
62	Amir	Women
63	Ekim	Who cannot read. Eh, we will write: thereof are two thirds
64	Amir	Shall I comma? Mhm. [*negating his own question*]
65	Ekim	No, don't think so. Thereof are tw-two third [*whispering*], [*louder:*] two thirds women, who cannot read.
66	Amir	[*writes down a slightly changed phrase:* "Thereof are two thirds of women who cannot read*"*]

The big conceptual challenge of rephrasing the text is to refer two thirds to the correct whole. It is difficult because this whole is itself given as a part (the quarter of adults who are analphabets). Ekim comprehends this relationship in line 59 and formulates it explicitly in line 61: The two thirds refer to the quarter. In his second suggestion in line 65, he refers both fractions to the group of humans they stand for. For this, he substitutes “the quarter” by the undetermined adverb “thereof” and constructs an adequate linguistic relation to the first two phrases of their text. The “two thirds” are specified linguistically by juxtaposing “women who cannot read”. But Amir who writes the formulation on the common sheet of paper adds another preposition: Instead of Ekim’s phrase “Thereof are two thirds women who cannot read.” (line 65), he writes “Thereof are two thirds **of** women who cannot read.” (line 66). By the little word “of”, he constructs a formulation that refers two thirds to two different wholes in one phrase. His sentence is thus neither linguistically nor mathematically coherent. How subtle these details are, becomes visible in (the non-printed) line 70: When the interviewer asks Ekim to read the written text aloud, he reads the modified version without seeming to realize the divergence to his suggestion. In sum, this episode shows how the demand to translate a CAL-text into their own BIC-text can initiate a first reflection on structural relations of the given fractions. But although Ekim constructs an adequate mental image of the relation, his understanding and his linguistic precision is not stable enough in this moment to notice Amir’s deviant modification.

In the second step (starting in minute 9:00), the boys are asked to evaluate Tobias’s wrong interpretation (“Wow, 2/3 of all women cannot read? Is that possible?”):

86	I	Mhm. [*agreeing*] I would like to know, what you think about it. Is that possible? What he [*she means the fictitious student Tobias*] has said there?
87	Amir	Yes but it isn’t so [*whispers*]. Because when you drive a car and don’t understand the sign, you just go through.
88	Ekim	Mhm. [*agreeing*]
89	I	And if - ehm – you think about, that he has the same data as you have, Tobias, he has the same data as you. That means, the same text or an easier text. Does the data match the text then? The numbers?
90	Amir	[*break 11sec*] Yes, doesn’t it? [to *Ekim*]
91	Ekim	Well, of one quarter are two thirds of – ehm – of all women cannot read and then [*break 8 sec*]
92	I	Let us just draw it. [….]

Amir argues that Tobias’s phrase does not fit to reality. His argument does not refer to the text but to contextual considerations: These many female analphabets would have difficulties with the traffic (line 87). Although Amir succeeds in making sense of the mathematical meaning by referring to out-of-school considerations, the interviewer guides them back to the question how Tobias’s formulation fits with the text. Ekim first correctly explains the relation between part and whole (line 91). But in the further course of the same utterance, he refers the two thirds to the group of all women as the whole by adopting Tobias’s wording (“of all women” in line 91). He does not yet succeed in identifying Tobias’s mistake. As the interviewer continues to another idea in line 92, it is not discernable from the transcript whether the boys agree with Tobias or not. In sum, the episode shows the challenge of mentally constructing adequate relations between the part and its whole, and of finding adequate verbalizations for it.

In the third step, the boys are asked to draw the situation in a square and to explain the graphical representation. During the next seven minutes, the boys draw the picture (reconstructed in Fig. 4) and add the written explanation "This is 1 of 4 equally big parts." (in line 163). The next excerpt starts in minute 16:30 when the interviewer asks the boys to refer the drawing back to the text and to include the two thirds:

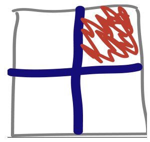


Fig. 4 1st drawing

164	I	Yes, that is very good. And when we refer it back to the text now, can you explain to me what it means for the situation in the text? Again with the information – ehm – that the whole square are all humans in the world?
165	Ekim	Well this [*hints to the whole square*] are all adults and that [*hints to the red quarter*] are all adults who – eh
166	Amir	who cannot read
167	Ekim	exactly. who cannot read. And thereof, now two thirds are women who cannot read.
168	I	Mhm. [*agreeing*] Do you draw that, too? Can you draw that into it?
169	Ekim	Two thirds
170	Amir	Thirds. [*break 4sec*] Yes.
171	Ekim	Shall we do that here? [*hints to the red quarter, but interviewer does not react. Ekim answers himself without any break*] Yes, don't we? We must do that.

After that, they draw the picture as scanned in Fig. 5. Until minute 21:40 (line 219), they subdivide the quarter into three parts (intended to be of equal size) and colour two of them in blue. Ekim's verbalizations in line 165 and line 167 show how the transfer to the graphical register strengthens him to verbalize the structural relations successfully. In line 165, he hints at the graphical elements and uses deictic means ("this" and "that") for expressing his ideas about the part and the whole precisely, although not yet explicitly. On the firm ground of a drawing, he finds a successful explicit verbalization of the structural relation between part and whole in line 167. The scene indicates how the struggle for explicit verbalizations is associated closely with a stabilization of their mathematical insights. Their own drawing helps to transfer the situation to the context of analphabets and to translate the mentally constructed relations into verbal formulations.

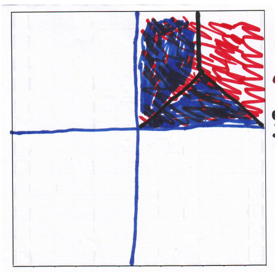


Fig. 5 Amir's and Ekim's drawing

In the fourth step, the boys work on a fictitious drawing (Fig. 3, on the right) and finally confirm that it fits. In step 5, they succeed in assigning all three representational registers and thus solve a cognitively demanding task cooperatively. In minute 43:00, the interviewer comes back to Tobias's utterance in Step 2 and asks the students to compare it to the text. The boys can now explain what Tobias did wrong:

354	Amir	[*nodds*] Mmh, he has not the one quarter
355	Ekim	Exactly
356	Amir	drawn

With his utterance in line 354, Amir indicates that “the quarter” has an important meaning which must not be neglected. Although he still has difficulties in expressing himself in the register of technical language, this shows that he has constructed the structural core of the meaning of the text through the long process of relating the relevant registers.

4. CONCLUSION AND OUTLOOK

The results of the analysed interviews suggest that the teaching strategy of purposefully linking appropriate registers seems to have a high potential to initiate substantial student activities that lead to deeper understanding. The strategy starts with the reformulation of the initial text, which makes the learners reflect on CAL-formulations, questioning the relations and trying to rephrase them in easier ways.

For the group of our 50 interviewees, the main difficulties were not located in unknown vocabulary (the obstacle of the unknown word ‘analphabet’ appeared for many students, but it could always be overcome easily) or complex grammatical constructs. Most children who did identify grammatical difficulties were able to decode and translate them into easier formulations. The common limit of understanding was reached by many students while interpreting the phrase “two thirds of all analphabets are women”. Even if finally successfully interpreted, many students could not initially distinguish that phrase from the phrase “two thirds of all women cannot read”. For this distinction, a high linguistic and mathematic precision is needed. Many students learned to clearly distinguish between these two only during the activities of the interviews. This is not only a linguistic problem of pure decoding, but also a mathematical obstacle when the referent whole is not in the focus (Prediger & Schink 2009). It is interesting to reconstruct how the students gradually developed means to clarify the mathematical relations by using different representational registers. In particular, the graphical register supported the clarification of the deeply interconnected linguistic and mathematical aspects during the processes. One example is given in Fig. 6 that shows an intermediate state of clarification by Imer and Georgius. Both boys drew the two fractions 1/4 and 2/3 in separate pictures. They understood that they belonged to different wholes, but could not nest them into each other. Many examples - like in the transcripts of Emir and Akim - show how deeply interconnected the linguistic and mental mathematical processes are, and how each can develop with each, if there is the opportunity to do so.

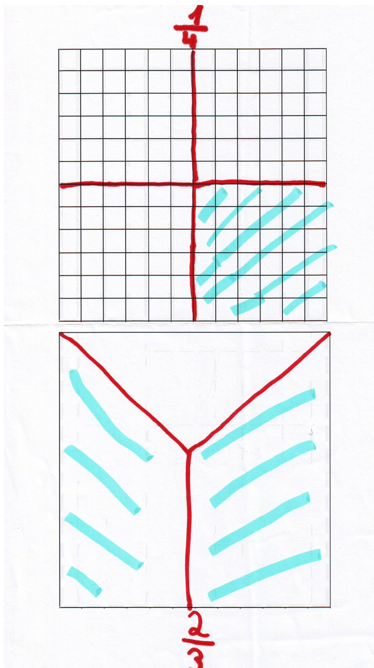


Fig. 6 Intermediate state

Although only a very short insight into the rich data could be given, these snapshots show that the teaching strategy adopted from integrating language and mathematics education experiences seems to be promising as it focuses directly on the conceptual core of linguistic and mathematical challenges. However, further research is needed to find out more on the situational potential and the longer-term effects of this strategy.

NOTE. The study belongs to the project “Understanding fractions for multilingual learners. Development and evaluation of a language- and mathematics-integrated teaching strategy by relating registers” that is funded by the ministry BMBF. This project is part of the larger project MuM – “Mathematics learning under conditions of multilingualism”.

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