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Enhancing language for developing conceptual understanding: A research journey connecting different research approaches

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Typical problems in mathematics education require the pursuit of several research goals: not only explaining the problem and its underlying mechanisms, but also designing and evaluating instructional approaches for overcoming them. When the problem is complex, various research approaches need to be combined. In this plenary paper, I will summarize several steps from a 13-year research journey to serve as an example of how different research approaches can be connected to gain deep insights into language-responsive mathematics instruction. On a meta-level, the paper reflects the affordances and challenges of connecting different research approaches and thereby advertises extension of the networking practices that have been well established in ERME research.

Keywords: Connecting research approaches, conceptual understanding, language-responsive instruction, epistemic role of language

Introduction

With respect to typical problems in mathematics education, researchers can choose to pursue different research goals (Mason & Waywood, 1996; Burkhardt & Schoenfeld, 2003):

- *describing* the problem
- *explaining* the problem and its underlying mechanisms
- *designing* instructional approaches for overcoming the problem
- *evaluating* instructional approaches with respect to their effectiveness
- *describing and explaining* challenges in implementing the instructional approaches
- *designing* teacher professional development (PD) and material support for teachers to overcome the challenges

For each of these potential research goals, the need for different research approaches and different background theories (including theories from outside mathematics education) often resulted in research projects in the early years of the academic discipline making a choice between alternative research goals. However, within the last decades, the academic discipline has successively moved towards connecting several research approaches so that the research goals can increasingly be combined in longer chains of projects. In my view, this trend is highly desirable, as it allows research to really make a difference in mathematics classroom practices (Burkhardt & Schoenfeld, 2003).

My message for this conference paper is that we should continue this *important trend of connecting research approaches* while always keeping our feet on the ground, namely, on sound epistemological analyses of the mathematical topic and the topic-specific learning processes in view. I will try to exemplify what this can mean by reporting from the 13-year research journey of our MuM research group in Dortmund (MuM is an abbreviation for "mathematics learning under conditions of language diversity"). While treating a particular problem in the example (in this case, challenges of language

in school mathematics learning), the aim of the last section will be to draw some more general conclusions beyond the specific problem being treated.

What I present here is grounded in CERME work in two ways: Our research on language has profited enormously from the ideas exchanged in the CERME Thematic Working Group (TWG) on language throughout several years (Planas et al., 2018), and the meta-perspective from which I reflect on the research journey today is a continuation of ideas emerging from the CERME TWG on theories (Artigue et al., 2005; Kidron et al., 2018), valuing different research approaches and striving to network them (Bikner-Ahsbahs, Prediger, & Networking Theories Group, 2014), not only with respect to using different theoretical lenses on the same data, but with respect to combing through and shifting the research questions, research objects, and perspectives across several studies.

Overview of a 13-year research journey

Our research journey started with recognizing a serious problem occurring in many countries: failure to provide equitable access to mathematics for students with low language proficiency. To address this problem, a chain of research questions and design challenges were pursued:

- What exactly is the problem with language in mathematics? (describing and explaining)
- How can language be enhanced in mathematics classrooms? (designing)
- What language demands are crucial for concept development? (designing and explaining)
- Is language-responsive teaching effective for developing conceptual understanding? (providing evidence for effectiveness)
- Who profits from language-responsive instruction? (extending the target group)
- How does teachers' enactment affect the learning gains? (explaining and connecting to high-quality instruction)

In the following subsections, I try to sketch how these questions were iteratively treated from different perspectives in the 13-year research journey, without accounting for all technical or theoretical details. The section is not a typical research report: Its aim is to exemplify the needs that were identified and the decisions that led to identifying the necessity to connect several research approaches in longer research journeys.

What exactly is the problem with language in mathematics? Describing and explaining by large-scale assessments

Usually, educational problems are ill-defined in the beginning (Silver & Herbst, 2007). This was also the case for our problem: Large-scale assessments have repeatedly shown that school systems in many countries fail to provide equitable access to mathematics for all students (Braswell, Dion, Daane, & Jin, 2005; Haag et al., 2013; OECD, 2007). Originally, this failure of school systems was often conceptualized as achievement gaps of students with low socioeconomic status, immigrant status, and/or multilingual background (OECD, 2007; Stanat et al., 2012). This led to problematic deficit-oriented policies and approaches for multilingual students (Gutiérrez, 2008; similarly for other challenges of low-attaining students, Hodgen et al., 2021).

The description of the problem, however, depends on the categories used: When not only immigrant status and multilingual background but also the students' *academic language proficiency* in the language of instruction are captured in the large-scale assessment, this background factor (to be further explained below) turns out to be most relevant for students' mathematics achievement (Prediger et al., 2018) and for students' mathematical learning gains throughout a year of schooling in German schools (Ufer & Bochnik, 2020).

From a sociolinguistic perspective, *academic language* is described as a language register between students' everyday language and the technical formal mathematical language (Schleppegrell, 2004; Snow & Uccelli, 2009) and is functionally characterized as the register "used by teachers and students for the purpose of acquiring new knowledge and skills…, imparting new information, describing abstract ideas, and developing students' conceptual understanding" (Chamot & O'Malley, 1994, p. 40). It is seen as important learning medium, both for classroom communication and as a thinking tool in an important epistemic function for mathematics learning (Schleppegrell, 2004). Although the construct of "register" has sometimes been reduced to lexical and grammatical features, Halliday's (1978) definition also entails language use in larger language units above the sentence level. We refer to interactional discourse analysis to capture these language units above sentence level as discourse practices. Discourse practices are conventionalized genres that are interactively co-constructed relying on patterns available in speech communities' knowledge (Heller & Morek, 2015). Academic discourse practices are those optimized for school purposes, in particular *reporting procedures*, *explaining meanings* to convey or construct knowledge, and *arguing* to negotiate divergent validity claims in classrooms.

From a socioeducational perspective, the critical feature of academic language is that it is an unequally distributed learning prerequisite to which some students get more access at home than others, depending on their family backgrounds (Snow & Uccelli, 2009), which is an example of how social background factors coincide with academic language proficiency.

For education practices, the sociolinguistic and socioeducational theoretical embedding of explanative statistical findings that academic language proficiency is a relevant background factor for mathematics achievement leads to changes in responsibilities: Whereas students' family background factors (immigrant status, socioeconomic status, and/or multilingual background) are considered static and cannot be changed by schools, schools can indeed take responsibility for enhancing students' academic language proficiency in order to reduce language-induced achievement gaps, including those occurring in mathematics (Thürmann, Vollmer, & Pieper, 2010).

Large-scale assessments can also support locating the language-induced achievement gaps within different types of mathematical knowledge: In our own study on a high-stakes test of 1,495 10th graders, we expected the language gaps to be largest for items with high reading demands, following the idea of language biases in the item texts (as found, e.g., by Abedi & Lord, 2001). But a differential item functioning analysis revealed that the language gaps were largest for items with high conceptual demands, even if reading demands were low (Prediger et al., 2018). This finding was strengthened by a recent longitudinal study throughout Grade 2 (Ufer & Bochnik, 2020) showing that students' language proficiency had much less impact on gains in procedural skills ($\beta = 0.16$) than gains in conceptual understanding ($\beta = 0.30$ for using concepts for solving word problems and $\beta = 0.29$ for

representing operations and concepts in visualizations). In both studies, language gaps were largest for *developing conceptual understanding of mathematical concepts*. These quantitative findings resonate with the sociolinguistic characterization of academic language as learning medium.

As an educational consequence of the identified language-induced achievement gaps, enhancing students' academic language is now considered an important step in achieving more equitable access to higher-order thinking across all subjects (Thürmann, Vollmer, & Pieper, 2010; Chamot & O'Malley, 1994). In Germany, this call for enhancing academic language in all subjects has been integrated into educational policy directives and the syllabus, yet without much support for helping teachers to realize this goal (MSW, 1999).

Summing up, *large-scale assessments* detected an ill-defined problem (there are problems with equity) and helped to locate the problem more exactly, namely, in the relevant background factors (the disparities can be traced back to academic language proficiency) and the type of mathematical knowledge (language gaps are largest for items with conceptual understanding). External *theoretical perspectives* (here from sociolinguistics) helped to understand the findings (academic language proficiency has an important epistemic role for developing higher-order thinking, including conceptual understanding). However, neither large-scale assessment nor the external theoretical backgrounds alone can orient instructional approaches to how to overcome the problem (Sullivan, 2015).

How can language be enhanced in mathematics classrooms?
First attempts to learn design principles from good practices

Given the missing support for teachers, it was no surprise that the educational policy directive calling for enhancing students' academic language (MSW, 1999) had only very limited impact in schools. Ten years later, the educational ministry reacted by including language enhancement into the curriculum of initial teacher education of all subject matter teachers. However, this reform risked failure because subject-specific instructional approaches and curriculum materials that could be used in teacher education programs were not available for subject matter teacher educators.

That is why, in 2009, the MuM research group in Dortmund started its work aimed at providing support for mathematics teachers and mathematics teacher educators to enhance students' academic language of school mathematics. The first search for good practices revealed a typical problem that had also been documented in other countries: Many mathematics teachers who start working on enhancing students' language concentrate on simple vocabulary work, mostly concentrated on formal technical mathematical language or grammar (e.g., DFEE, 2000; problematized, e.g., by Moschkovich, 2010), but without an explicit focus on the discourse practices in play. This restriction to vocabulary work was also documented by de Araujo and Smith (2022), who analyzed US algebra textbooks with respect to the hints and materials provided for working with language learners: Reduced cognitive demands and less ambitious learning goals are combined with isolated vocabulary work, but there was no support for engaging students in discourse practices.

Identifying this problem led us to go beyond our attempt to learn from good practices to performing a qualitative classroom video study of 73 mathematics lessons in five Grade 5 classrooms. We studied how teachers engaged their students in discourse practices and what kind of support was given for students with low academic language proficiency (Erath et al., 2018). By coordinating an epistemic

perspective on students' participation in collective knowledge constitution processes with the socio-linguistic perspective of interactional discourse analysis (Quasthoff, 2011), we replicated well-known findings that students' mathematical learning opportunities tend to be very significantly shaped by their ability to participate in the discourse (with various theoretical approaches collected by Herbel-Eisenmann et al., 2011), and "content learning is inseparably bound up with language learning and vice versa" (Barwell, 2005, p. 207). In particular, we showed that students' participation in classroom interaction depends on their proficiency in actively engaging in discourse practices such as explaining and arguing, for which almost no learning opportunities were provided in the observed classroom interaction (Erath et al., 2018).

Summing up, the qualitative classroom video study allowed us to confirm the role of academic language proficiency as learning medium and as unequally distributed learning prerequisite, but we could identify hardly any good practices for enhancing students' discourse competences. Although this study substantially contributed to explaining the problem in the discursive dimension, it was again not able contribute to overcoming it.

How can language be enhanced in mathematics classrooms?
Second attempt through using design principles imported from language education

As our descriptive, non-interventionist video study in regular mathematics classrooms provided hardly any empirical insight into how language can be enhanced in mathematics classrooms, we changed to an interventionist research approach, namely, to design research. Design research is the research approach that combines the research goals of developing instructional approaches with gaining deep empirical insights for explaining mechanisms of the teaching learning processes (Gravemeijer & Cobb, 2006). In our case, we first had to create the language-responsive teaching learning processes we intended to investigate.

A careful literature review in language education (including second-language education) allowed us to import design principles that have proven effective for providing learning opportunities for learning academic language. We imported three generic design principles (DPs) for language learning in language lessons:

- *DP1 Pushed output.* Students learn academic language when consistently being pushed to talk and write (Swain, 1995; Walqui, 2006).
- *DP2 Use multiple representations*. Students can construct meanings for new language when moving between multiple representations and language registers (von Kügelgen, 1994).
- *DP3 Sequence language registers.* Students can acquire language when it is carefully sequenced in a language learning trajectory starting from the everyday language and moving through academic language to the technical language of the subject matter (Gibbons, 2002).

These principles gave us important first ideas and strengthened our confidence that good language-responsive classrooms can also be high-quality mathematics classrooms: DP2 has a long tradition for developing conceptual understanding, even before language registers were considered (Lesh, 1979), and DP3 resonates with a similar sequencing principle for mathematical learning opportunities, the

level principle in realistic mathematics education (Gravemeijer, 1998). DP1, the emphasis on students' active talk, was advocated in mathematics education before language became an explicit focus (as traced by Austin & Howson, 1979; Pimm, 1987).

What academic language demands are crucial for developing conceptual understanding? Design research for overcoming the specification gap

Research approach

Even if the design principles provided first ideas for the design of an instructional approach that can enhance students' language in mathematics classrooms, we detected a crucial *specification gap*, as the generic design principles imported from other academic disciplines do not help to specify the demands of the learning content in detail (Bailey, 2007). What exactly are the academic language demands that are most crucial for developing conceptual understanding in mathematics?

In our topic-specific design research approach (Prediger & Zindel, 2017), specifying and structuring the learning content is a key working area for treating what-questions (what needs to be learned?) that is also informed empirically by design experiments (van den Heuvel-Panhuizen, 2005) as depicted in Figure 1.

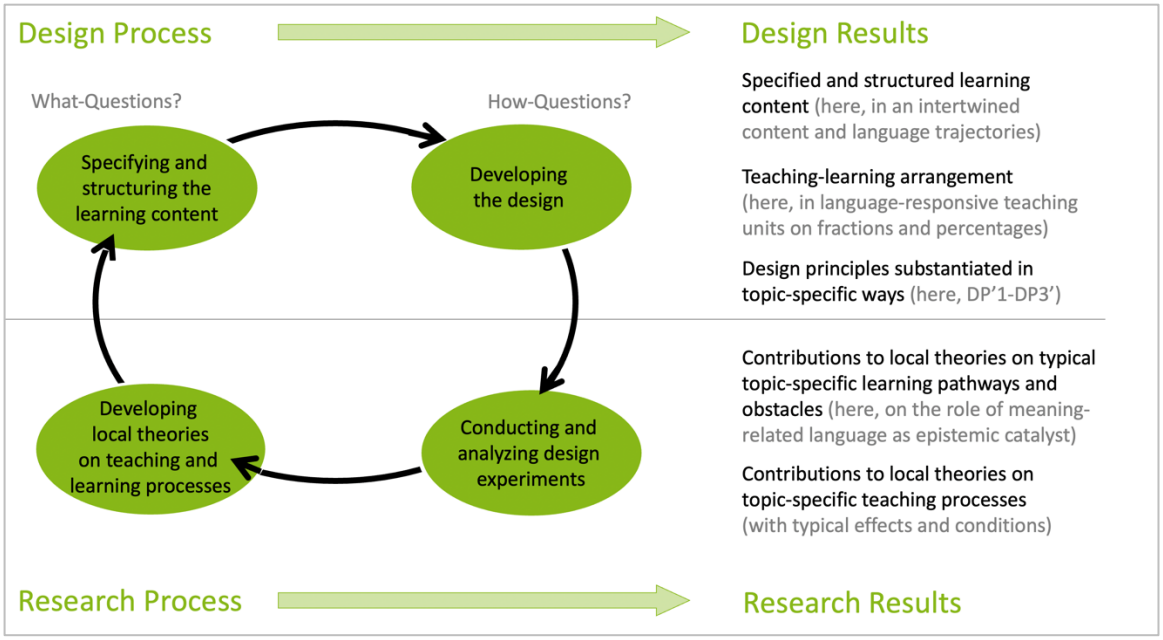


Figure 1. Working areas in topic-specific design research (adapted from Prediger & Zindel, 2017, p. 4168)

In the first design research study (Prediger & Wessel, 2013), we built upon fine-grained existing *specifications* of typical challenges (Behr et al., 1992) and knowledge elements needed for developing conceptual understanding of fractions and overcoming typical mathematical challenges (the part-whole relationship, ordering fractions, and finding equivalent fractions, preferably in a fraction bar model; see Cramer et al., 2009). The language learning content was only vaguely specified in the beginning of our design research study. Developing a first prototype *design of the teaching-learning arrangement* for this learning content is the second working area in the design research cycle (see Figure 1), for which we used the generic design principles DP1, DP2, and DP3.

In the third working area, *design experiments* with monolingual and multilingual seventh graders with low German academic language proficiency were conducted to determine what everyday language resources they bring in and what elements of academic language would support their participation in the process of meaning-making for the mathematical concepts in view.

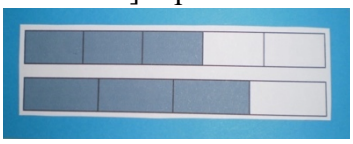
The *qualitative analysis of the design experiments* helped to explain the epistemic role of academic language in more depth. It informed the specification of the crucial academic language demands by acts of empirically grounded theorizing in the fourth working area.

Case study of Cavit and Ismet for specifying the needed academic language

The case of Cavit and Ismet (first analyzed in Prediger, 2013) can illustrate the third and fourth working area, in particular the empirical specification of crucial academic language demands. For this, I provide selected insights into the analysis of one design experiment, conducted in a pair setting with Cavit and Ismet, two sixth graders (12 years old) who were born in Germany and spoke fluent everyday German (and Turkish in their families) but only limited academic language at the beginning of the study. The boys were selected by their teacher as assignments showed incomplete understanding of fractions after their first encounter.

Following the design principle DP2 (using multiple representations), the teacher asks the boys to translate the given fraction bars into symbolic fractions, and they translate easily to 3/4 and 3/5 (see figure in the transcript). Following the design principle DP1 (pushed output) and being aware that students often assign other meanings to graphical representations than the conventional meaning in school arithmetic (Behr et al., 1992), the teacher presses the boys to explain the meaning of the fraction bar.

9 Teacher: What in this graphical representation [*points to the fraction bars*] represents the fraction?



10 Cavit: Eh, the... First, you need to count the fields, that is five, and then, the three colored fields, [*silently*] three fifths.

11 Teacher: Exactly. And which of these two [*points to the fraction bars*] represents the three fifths, actually?

12 Cavit: First, um, you have to count the fields [*hints at the fraction bars*], here it is one, two, three, four, five [*points at every field of the fraction bar*], and then three, colored [*looks at the teacher*].

The brief transcript (author's translation from the original German transcript published in Prediger, 2013) documents a particular variant of speechlessness on both sides, both teachers and students, who speak without any lexical or syntactical errors, but the conversation about the mathematical meanings gets stuck. When asked to explain how symbolic fractions and fraction bars are connected (Turn 9), Cavit limits his answer to the discourse practice of reporting the drawing procedure for the graphical fraction bar instead of explaining the meanings of the four fields, the three colored fields and the relationship between them (Turn 10). From his utterances (following a perhaps too implicit teacher question), it is unclear if he understands the part-whole relationship or not. When the teacher asks again, the teacher's speechlessness is indicated by posing nearly the same implicit question as before

(Turn 11). Cavit understands the interactive demand for a further explication, but explicates his answer only by counting 1, 2, 3, 4, 5 (Turn 12). In this moment, neither the teacher nor the students seem to have the means to talk about the part-whole relationship, and it is still left unclear if the boys understand the relevant part-whole structure in the bar or not.

At this point in most German mathematics classrooms, the teacher would have given up, assuming in a strength-oriented view that the boys have the right idea even if they cannot communicate it explicitly. This teacher, however, continues to engage the students in making connections between the two representations (following DP2 and DP1) by engaging them in comparing the fractions graphically:

17 Teacher: How can you see from this picture [*points to the fraction bars*] which fraction is bigger?

…

24 Ismet: Here is five [fields] though and there four; from this you see that this [*points to the fraction bar of 3/5*] is big and this is small [*points to the fraction bar of 3/4*].

25 Cavit: I agree.

When the boys are asked to compare the fractions in the bars, idiosyncratic ideas about the meanings become evident that were not inferable from their earlier utterances. Ismet adopts a typical comparison strategy (Clarke & Roche, 2009) and argues that 3/5 is larger than 3/4 because its fraction bar has 5 instead of 4 fields (Turn 24), and Cavit agrees (Turn 25). It is only in this discourse practice of describing structures (which is more elaborate than reporting a drawing procedure from Turns 10-12) that their ideas about fractions become explicit and discussable. Rather than focusing on the part-whole relationship, they focus on the digits three and five separately without bundling them into one conceptual entity. By this, they are not able to unfold the meaning of fractions as shares appropriately. This observation resonates with a problem previously identified by Steinbring (2005): the "problem of potentially interpreting the given concrete objects and reference contexts for mathematical signs and symbols … as *relational structures*" (p. 5).

A thorough analysis of the boys' discourse practices and their lexical means to realize them exemplifies typical observations: To articulate procedural aspects in the symbolic representation, technical phrases such as nominator and denominator are sufficient. But in this technical language alone, they cannot express conceptual understanding (see Figure 2). The successful translation between graphical and symbolic representation gives a first indication of some conceptual understanding. However, even if the translation is successful, this does not necessarily mean that the connection between two representations involves a deep understanding.

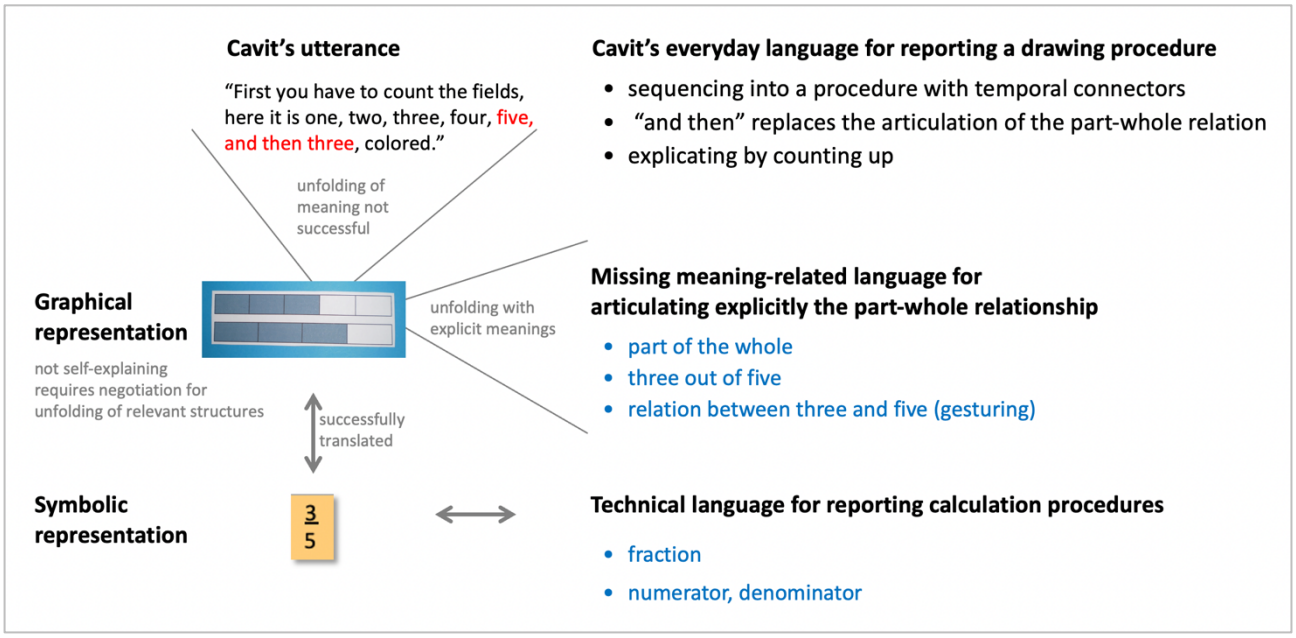


Figure 2. Analysis of involved language registers for the case of Cavit and Ismet

The surface nature of the understanding (sufficient for translating the representation) becomes apparent when the comparison of fractions is to be conducted within the graphical representation (Turn 24), as this requires the unfolding of the fraction concept as a conceptual entity describing the relationship between the whole and the part. From here, it is worth looking back at Turn 12 to analyze Cavit's utterance articulating the surface connection of representations. Rather than explaining the meaning of fractions, Cavit engages in a less demanding discourse practice, namely, reporting a drawing procedure. This proceduralization is an example of sequential reporting also being realized in conceptual contexts, yet without the meaning really being clarified.

Cavit addresses the digits three and five separately; the only connector used between them is "and then," a temporal connector that substitutes the articulation of the part-whole relationship (see Figure 2). The requested further unfolding only leads to counting to five. To articulate the part-whole relationship more explicitly, other meaning-related phrases would have been needed, such as "three out of five" or "part of the same whole." The transcript continues with the teacher offering these meaning-related phrases and filling them with the conventional meaning by heavily gesturing for the relation of five thereof three. By drawing upon gestures for the part-whole relation (indicating the whole, and therein the parts always related to the whole), the teacher attaches the new phrases to students' existing resources and equips them with the meaning-related phrases that enable them to engage in the more demanding discourse practices.

What exactly is the problem with the epistemic role of academic language while developing conceptual understanding? Theorizing by combining epistemic and discursive perspectives

Students' conceptual challenges with fractions as part-whole relationships and comparing fractions have been well documented in the literature (and the existing state of research on student thinking on fractions, e.g., Behr et al., 1992; Clarke & Roche, 2009). The case of Cavit and Ismet's struggle with *articulating* the part-whole relationship adds substantial explanative findings on the role of language

in students' conceptual learning pathways to the existing state of research, as it shows typical phenomena we found in many design experiments, for fractions and other mathematical concepts. These phenomena formed the main object of theorizing in the fourth working area of the design research cycle (Figure 1, systematically reflected on in Prediger, 2019). In our research journey, the grounded theorizing based upon these empirically identified phenomena required the combination of epistemic and discursive perspectives.

From an *epistemic perspective*, mathematical concepts are hard to learn as they are usually abstract and relational in nature (Steinbring, 2005). They compact complex relationships (such as the part-whole relationship or functional relationships) into new conceptual entities (such as fractions or functions) that can then be integrated into higher-order networks and relationships. Technical mathematical language is optimized for addressing these abstract concepts in a highly concise and economic manner and allows experts to communicate and think with the language very efficiently. However, for learning the technical language in its compacted form, it must first be unfolded into its meanings (Prediger & Zindel, 2017).

Early work on language in mathematics assumed that all of the unfolding of the formal technical language can be conducted in students' everyday language (e.g., Gallin & Ruf, 1990). But the analyses of multiple design experiments revealed that there is a mediating language register between the everyday language and the technical language that needs to be equally concise and explicit in expressing relationships yet informal, without technical terms and symbolic representations. This is exactly the part of the academic language register that is most crucial for developing conceptual understanding, and we termed it meaning-related language (Pöhler & Prediger, 2015).

From a *discursive perspective*, meaning-related language can be characterized by the discourse practices of explaining meanings and describing mathematical relationships and structures. These discourse practices are typical for the academic language register (Heller & Morek, 2015), and discursively more demanding than narrating or reporting procedures (as already touched by Setati, 2005). In the lexical dimension, reporting procedures mainly requires the less complicated sequential connectors ("first," "and then," etc.), whereas explaining meanings and describing mathematical relationships requires more complex connectors and meaning-related phrases for articulating the key relationships (such as the examples in Figure 5 below). These phrases can also include grammar features in the syntactical dimension (as elaborated in Prediger & Hein, 2017; Prediger & Şahin-Gür, 2020).

For each mathematical topic, these meaning-related phrases need to be identified by epistemological analysis and ideally by empirical investigations of topic-specific learning processes, drawing, of course, upon the existing states of topic-specific research. Consequentially, the research was extended to percentages (Pöhler & Prediger, 2015), logical structures of proofs (Prediger & Hein, 2017), functions (Prediger & Zindel, 2017), qualitative calculus (Prediger & Şahin-Gür, 2020), and other topics. For each mathematical topic, meaning-related phrases for the rich discourse practices of explaining meanings and describing mathematical relationships were identified as crucial.

Summing up, the four practices of theorizing in the first design experiment cycles (fourth working area in Figure 1) led to building a theoretical foundation for specifying the language-learning content

(first working area in Figure 1): focusing on the rich discourse practices of explaining meanings and describing mathematical relationships and structures, together with the meaning-related phrases used to realize these discourse practices in processes of developing conceptual understanding. Without the strong epistemological focus on conceptual understanding of mathematical concepts and the epistemic perspective on unfolding their highly compacted meanings, the meaning-related language and its importance as intermediate register would not have been identified.

How can meaning-related academic language be enhanced in mathematics classrooms? Substantiating the design principles with respect to specified learning content

Whereas "how-questions" (How can the learning be enhanced?) and "what-questions" (What has to be learned?) are often treated as independent (like in the beginning of our research journey), the research approach of topic-specific design research has the potential to integrate both questions in subsequent design experiment cycles (Prediger, 2019). The sketched theorizing on the role of the discourse practices of explaining meanings and describing mathematical relationships for unfolding the meaning of mathematical concepts also provided the empirical foundation for substantiating the three generic design principles in more mathematics-specific ways, as shown in Figure 3, so that they can be tailored to enhance those academic language demands that are crucial for developing conceptual understanding of mathematical concepts such as fractions, percentages, and functions.

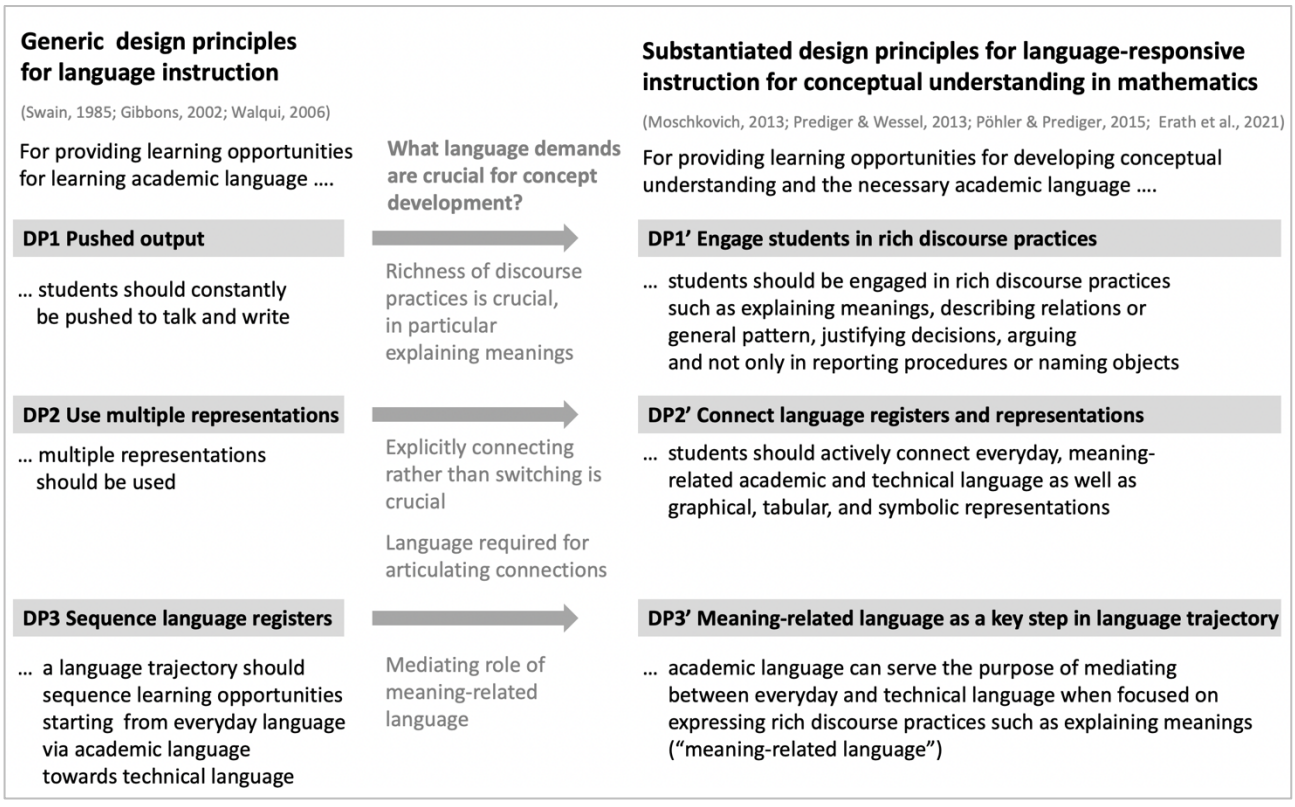
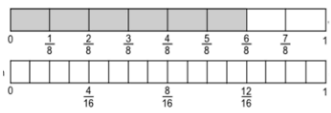
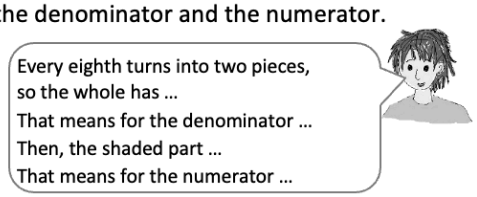


Figure 3. Adapting generic design principles into substantiated design principles for language-responsive instruction of conceptual understanding of mathematical concepts

To refine design principle DP1 (pushed output), we take into account the required *richness* of discourse practices in which students are engaged: Because the discourse practices of explaining meanings or describing mathematical relationships are more relevant for developing conceptual understanding than narrating incidences or reporting procedures, DP1 was refined into "DP1 Engage students in rich discourse practices."

<table>
<tr><th>Design principles</th><th>Example tasks on fractions and fraction bars</th></tr>
<tr><td>DP2 Connect multiple registers / representations
(here, from symbolic to graphical and verbal representation, with meaning-related phrases as scaffolds in the table)</td><td>
2 Understanding the part-whole relationship
a) Tarik shares chocolate bars with his friends.
He produces fifths in the chocolate bars and takes different shares.
Complete the table and report how you decided what to draw.
<table>
<tr><th>Share</th><th>Image of Tarik's part</th><th>How many pieces does the whole have?</th><th>How many pieces are in Tarik's part?</th></tr>
<tr><td>$\frac{1}{5}$</td><td> </td><td>5</td><td>1</td></tr>
<tr><td>$\frac{2}{5}$</td><td> </td><td></td><td>2</td></tr>
<tr><td>...</td><td></td><td></td><td></td></tr>
<tr><td>$\frac{5}{5}$</td><td> </td><td></td><td></td></tr>
</table>
</td></tr>
<tr><td>DP1 Engage students in rich discourse practices
(here, describing the changes in the part-whole relationship, with meaning-related phrases as scaffolds)
DP3 Sequence registers with meaning-related language as key step in language trajectory
(here, connecting everyday language to "part of the whole")</td><td>
b) Examine the table precisely and consider the following:
• What happens with the share that Tarik gets of the fraction bar?
• Why does the share change?
Write down your findings so that another student can understand.
You can use these phrases:
What changes?
- the whole chocolate bar
- pieces of chocolate bar
- part of the whole
How does it change?
- getting more/less
- becoming larger/becoming smaller
- the … the …
</td></tr>
<tr><td>DP1 Engage students in rich discourse practices
(here, explain equivalence and its connection to the procedure)
DP3 Sequence registers with meaning-related language as key step in language trajectory
(here, by connecting meaning-related phrases to technical language)</td><td>
15 Understanding equivalence of fractions
Find a fraction that describes the same share as 6/8 in the other fraction bar.
• Describe how the two fractions and two fraction bars are related.
• Explain what it means that both fractions are equivalent.
• Explain what happens to the denominator and the numerator.



</td></tr>
</table>

Figure 4. Example tasks realizing the refined design principles DP1-3

For design principle DP2 (use multiple representations), the empirical analyses of many learning processes showed that switching between representations is not sufficient. Instead, explicit connections must be drawn and articulated (as for Cavit and Ismet, who switch successfully but cannot explain the connection). Because this applies not only for graphical, tabular, and symbolic representations, but also for everyday language, meaning-related academic language, and technical language, as their connections also contribute to the meaning-making processes (Gibbons, 2002), DP2 was substantiated into "DP2 Connect language registers and representations."

For design principle DP3 (sequence language registers), the findings about the mediating role of meaning-related language as a particular part of academic language resulted in articulating "DP3 Emphasize the meaning-related language as a key step in the language trajectory," because academic language can serve the purpose of mediating between everyday and technical language when focused on expressing rich discourse practices such as explaining meanings ("meaning-related language"). Whereas the refined design principles DP1 and DP2 are present in many instructional approaches (Moschkovich, 2013, and many others; see overview in Erath et al., 2021), the explicit input or elicitation and then consolidation of meaning-related language seems to be the major contribution of the Dortmund research to a vivid field of research and development.

Figure 4 sketches how the design principles were realized in the design of the teaching unit on fractions (Prediger & Wessel, 2013, here cited in the version from Prediger et al., in press). Whereas the tasks have multiple similarities to previous approaches (e.g., Cramer et al., 2003), the consequent demand on explaining meanings is substantially strengthened and supported.

Summing up, the refinement of generic design principles by mathematics-specific and even topic-specific considerations on the epistemic role of meaning-related language in the processes of conceptual understanding led to a substantiation of the design principles for enhancing language not as an end in itself but enhancing *language for mathematics learning*. As observation studies in regular classrooms did not help for this step, design research with its iterative combination of development and qualitative research and of what-questions and how-questions was the research approach of choice in this step.

Within the CERME thematic working group on language in mathematics education, the substantiated design principles found resonance with colleagues and their empirical findings from various countries. Building upon the impressive and complex descriptive and explanative findings on language in mathematics education that have been summarized for the occasion of the ERME birthday book (Planas et al., 2018), the thematic working group has now increasingly also turned to design challenges and their treatment. This extension of common research goals has been expressed in the jointly compiled *ZDM* special issue 53(2) on design principles and teaching practices that enhance students' language for learning mathematics. The review paper introducing the special issue elaborates six design principles, among them the three from Figure 3 (Erath et al., 2021). The other three are "establish various mathematics language routines," "include students' multilingual resources," and "compare language pieces (form, function, etc.) to raise students' language awareness," which are not foregrounded in this paper but have enriched our approach on other occasions. Many working group members have started to combine some of the design principles flexibly, and there is still research to be done on their interdependence.

Is language-responsive teaching effective for developing conceptual understanding? Controlled trials for overcoming the quantitative evidence gap for different target groups

The research review on designs and teaching practices for enhancing language for mathematics learning by Erath et al. (2021) summarizes 26 *qualitative* studies (from many ERME members and beyond) that have provided highly interesting, deep insights into the functioning of some of the language-responsive design principles and conditions of success that need to be considered for their application (and many more studies would have been available for this summary of an impressive body of qualitative research). So far, however, very few studies have provided quantitative evidence for the hypothesized effectiveness of these design principles on students' measurable learning gains. Given the increasing importance of quantitative evidence for policy decisions and research-based professional development, reducing *this quantitative evidence gap* can contribute to two aspects of orienting policies and convincing mathematics teachers and teacher educators: (a) Does it work? (b) For whom does it work? (Cai et al., 2021).

(a) According to methodological standards, quantitative evidence of effectiveness can be gained in controlled trials in pre-post-control-group designs showing that certain instructional approaches result in significantly larger measurable learning gains than mathematics instruction not following these approaches (Styles & Torgerson, 2018). In the MuM research group, we conducted several quasi-experimental and randomized controlled trials to confirm the hypothesis that language-responsive instruction combining the design principles DP1-DP3 can indeed enhance students' conceptual understanding significantly better than in business-as-usual control groups with comparable abilities and teacher backgrounds, first for fractions (Prediger & Wessel, 2013; Prediger et al., in press) then for percentages (Prediger & Neugebauer, 2021).

(b) Beyond confirming hypotheses on effectiveness, controlled trials can also help to deconstruct simplistic ideas about the target group. In the beginning, language-responsive mathematics instruction was mainly planned and offered to a particular target group with a combination of three disadvantaging factors: multilingual background, low academic language proficiency in the language of instruction, and limited school success (see research overview in de Araujo et al., 2018). But from a sociopolitical perspective, the exclusive focus on this particular target group is critical, as it can nurture deficit views on multilingual students, either by the misunderstanding that the three factors always coincide, or by suggesting particularistic approaches rather than inclusive access to good teaching (Barwell, 2005; Moschkovich, 2010). In contrast, the effects of language-responsive instruction on successful multilingual students, on multilingual students with high academic language proficiency, or on monolingual students with diverse language proficiency or school success were hardly investigated, so we decided to conduct three extensions (see Figure 5):

- A first extension of the target group included monolingual students with low academic language proficiency in the language of instruction (measured by standardized cloze tests). They were shown to profit as much as the original target group from language-responsive mathematics instruction (Prediger & Wessel, 2018; Smit & van Eerde, 2013).

- A second extension of the target group involved students with high academic language proficiency: Prediger and Neugebauer (2021) showed in a quasi-experimental field study that monolingual and multilingual students with diverse academic language proficiencies profited equally from a language-responsive intervention on percentages.
- Language-responsive mathematics instruction was, however, still mainly being provided to students with mathematical difficulties at risk of being left behind by the school system. As long as empirical findings about language-responsive teaching concentrate on struggling students, principles of language-responsive mathematics instruction might be misunderstood as being relevant only to overcoming problems. To avoid deficit-oriented policy discourses and practice discourses about language as a problem, it is crucial to provide empirical evidence that not only students with mathematical difficulties profit from language-responsive mathematics instruction, but also successful students. In our recent *JRME* paper, we reported from a randomized controlled trial that confirms exactly this third extension of the target group: Not only students at risk but also successful students in higher tracks can profit significantly more from language-responsive mathematics instruction than from business-as-usual control groups (Prediger et al., in press). Within the language-responsive instruction, low academic language proficiency is less predictive for weaker learning than in the control groups.

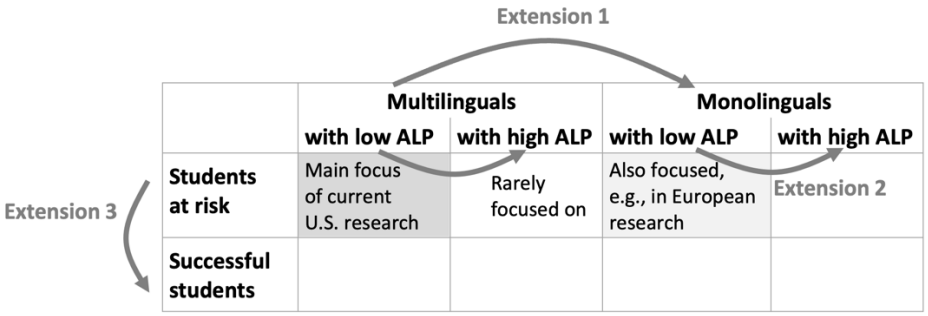


Figure 5. Research base for inclusive language-responsive instruction needs extensions of the target group from multilinguals at risk with low language proficiency to all students (Prediger et al., in press)

Summing up, the quantitative evidence gap can be reduced by findings of a chain of controlled trials showing that students of all backgrounds (monolingual and multilingual backgrounds, students with all degrees of academic language proficiency, and students with the whole range of school success) can profit mathematically from language-responsive mathematics instruction. These indications for effectiveness can strengthen the research base for language-responsive mathematics instruction as being *inclusive for all students* when realized with the strong focus on the design principles from Figure 3. They confirm that enhancing meaning-related language can serve as an epistemic catalyst for developing the conceptual understanding of all students.

This example also shows that all research has a sociopolitical component: As long as our methodological choices (in this case, the choice of our samples) perpetuate deficit orientations or particularistic approaches (in this case, by focusing on a too-specific target group accumulating different chal-

lenges or claiming too-specific needs), instructional approaches meeting the needs of particular students' risk continuing to be positioned in a deficit context outside mainstream classrooms. It is our responsibility as researchers to make wise choices in the research questions, methods, and samples we consider so that they can enter mainstream classrooms. Of course, this emphasis on inclusive use of language-responsive teaching for all students does not question the need to allocate more learning time for students at risk who are slower learners. However, it makes it clear that these students do not necessarily need other kinds of instruction.

At least in Germany, the particularistic character of language-responsive mathematics instruction is also fueled by the separateness of different research approaches: Language-responsive instruction is often investigated completely separately from the general academic discourse on high-quality mathematics instruction, which was something we wanted to overcome, as called for by Cai et al. (2020). This challenge guided decisions for the next step in the research journey.

How does the quality of teachers' enactment affect learning gains?
Video quality study for explaining and connecting the approaches to high-quality instruction

Although the overall effectiveness of our instructional approaches was shown in several controlled trials, we could also identify large differences between classes. In the implementation study for the language-responsive teaching unit on percentages, an intraclass correlation coefficient of 0.32 in the 70 intervention classes indicates that 32% of the variance is explained by which class the students were in. As the class compositions were comparable, we assumed this variance could be traced back to different instructional practices the involved teachers used in enacting the language-responsive approach, even though all worked with the same shared language-responsive curriculum materials (presented in Prediger & Pöhler, 2015) and all participated in the same professional development addressing the design principles and the ideas underlying the approach (four sessions of 3 hours each).

Many qualitative case studies have shown how teachers' enactment of instructional practices can promote or hinder the mathematical and language-related learning of the students (e.g., Barwell, 2020). In two research reviews (Walshaw & Anthony, 2008; Erath et al., 2021), a large number of qualitative case studies were summarized that identified typical teaching practices and established classroom norms and hypothesized whether they were productive or unproductive. However, except for one insightful study (Ing et al., 2015), little quantitative evidence has been provided for validating these qualitatively generated hypotheses on the potential effects of these teaching practices.

Within the research logic of controlled trials, the instruction itself is often treated as black box, so researchers have increasingly called for opening the black box and studying the teaching learning processes that occur during the interventions (Styles & Torgerson, 2018). For our research context, the existing qualitative findings suggested strongly that this trend should be followed. Hence, we decided that we need to change the research approach again and open the black box for a video study in a first approach on the following research question: How does the quality of teachers' enactment affect learning gains?

In our research journey, the research question was pursued by analyzing the video-recorded teaching practices from 18 of our heterogeneous intervention classes in a quantitative video-based approach—a selection that was presumably positive as these 18 teachers volunteered to be videorecorded. Rather

than only capturing the realization of the language-responsive design principles, we decided to take into account the large body of research on instructional quality in mathematics (Charalambous & Praetorius, 2018) in order to overcome the separate treatment of instructional quality in general and language-responsive mathematics instruction in particular. We chose (and adapted slightly) Schoenfeld's (2013) Teaching for Robust Understanding (TRU) framework to capture instructional quality in mathematics classrooms, as this framework shares the same ideas of quality with an emphasis on conceptual focus and was optimized in contexts of equity and underprivileged students. The TRU framework has five interconnected dimensions:

- *Mathematical Richness:* To what extent is the mathematics discussed clear, correct, and well justified (tied to conceptual underpinnings)?
- *Cognitive Demand:* To what extent do classroom interactions create and maintain an environment of intellectual challenge?
- *Equitable Access:* To what extent do activity structures invite and support active engagement from the diverse range of students?
- *Agency:* To what extent do students have opportunities to conjecture, explain, and argue, thus to developing agency and authority?
- *Use of Contributions:* To what extent is student reasoning elicited, challenged, and refined?

With respect to language-responsive instruction, we have slightly adapted the dimensions and added two further dimensions (Prediger & Neugebauer, 2021):

- *Discursive Demand:* To what extent do students engage in rich discourse practices? (aligned to our DP1 from Figure 3)
- *Connecting Registers:* To what extent are language registers and representations systematically and explicitly connected? (aligned to DP 2 and DP3)

Our rating of video-recorded lessons revealed that the assessed instructional quality in 18 intervention classes was on the highest level for 79-92% of the time in four dimensions, namely *Mathematical Richness*, *Cognitive Demand*, *Agency*, and *Connecting Registers*. This can be interpreted as a modest indication that the professional development and language-responsive curriculum material (presented in Prediger & Pöhler, 2015) supported the teachers in enacting an impressive instructional quality. In particular, it shows that the refined design principles DP1-DP3 (see Figure 3) are suitable for increasing the *Mathematical Richness* and *Cognitive Demand*, which are the two most content-related quality dimensions. In contrast, for the quality dimensions *Discursive Demand*, *Equitable Access*, and *Use of Contributions*, the 18 teachers showed a high variance in their enactment. These dimensions therefore seem to be less supported by the curriculum material per se and depended more on teachers' moves and strategies in directing classroom discussions (Ing et al., 2015), as has been indicated by a large body of qualitative research (Walshaw & Anthony, 2008).

To validate the hypothesis that these quality dimensions matter for students' learning, we conducted a multi-level analysis to find out how the seven quality dimensions predicted students' measurable learning gains (while controlling for individual background factors). And, indeed, the three dimensions that varied most with teachers' enactment—*Discursive Demand*, *Equitable Access*, and *Use of Contributions*—had an additional significant impact on students' learning gains, but so did *Agency*, in spite of its high-quality realization (Neugebauer & Prediger, 2022). Although the sample of 18

classes is not yet very large, these findings provide promising (and significant) indications for confirming the hypotheses on productive teaching practices by revealing multilevel quantitative evidence for their impact on measurable learning gains.

Summing up, this video quality study shows that language-responsive mathematics instruction can be realized in ways that achieve high instructional quality for *Mathematical Richness* and *Cognitive Demand.* This again can be interpreted as further evidence that enhancing language is an epistemic catalyst in mathematical classrooms if supported by topic-specific curriculum material. At the same time, teaching practices in the classroom interaction can still heavily vary with respect to *Discursive Demand*, *Equitable Access*, and *Use of Contributions*, and their enactment on a high-quality level has additional impact on students' mathematical learning gains. This means that the more the teacher achieves (in engaging all students in rich discourse practices [DP1] and in building upon students' informal ideas and emerging language for the further learning trajectory [DP3] by connecting these elements [DP2] in their moderation of the classroom discussions), the more the students have developed their conceptual understanding on the mathematical topic in view, in this case, percentages. These findings call for integrating more professional learning opportunities for productive teaching practices to raise and maintain discursive demands, provide equitable access, and make use of students' contributions in the discussion about future professional development offers (as, e.g., Kazemi et al., 2021). We might also assume that teachers need some more time to implement these quality dimensions into their practices than we gave them, but this is only one of the several challenges in implementing promising instructional approaches.

Again, this example shows the strength of combining different research approaches, in this case with the aim of including the insights from qualitative research into quantitative designs and validating the generated hypotheses (Creswell, 2015).

Looking back from a meta-level: What does connecting research approaches mean?

In most ERME research, connecting research approaches has been mainly interpreted in two modes:

- connecting methods for a well-articulated research goal (e.g., in the long and rich tradition of mixed methods approaches; see Creswell, 2015) and
- networking theoretical approaches (most often used for analyzing given data from two theoretical perspectives; see Bikner-Ahsbahs, Prediger, & Networking Theories Group, 2014).

Both modes also occur in our research journey: With respect to combining methods, the chain of our projects is characterized by sequential explanatory and exploratory mixed-methods designs. Explanatory mixed-methods designs are used when quantitative findings on statistical connections are investigated deeper in qualitative methods in order to explain the quantitative findings (as in our case, explaining the epistemic role of academic language in more depth after large-scale assessments showed that language gaps are largest for conceptual understanding). Exploratory mixed methods designs explore the generalizability of qualitatively identified phenomena (in our case, generalizing conditions of success in applying the design principles identified in case studies to a larger number of classes and measurable learning gains).

With respect to theoretical approaches, two characteristics make the coordination and integration of theoretical approaches absolutely necessary for the presented chain of (overlapping and interacting) projects:

- Research on language in mathematics learning always requires both an epistemic perspective and a language-related (here, mainly discursive) perspective and an integration of both for explaining how language learning and mathematics learning are related (Barwell, 2005; Moschkovich, 2010). As these networking practices have not been made explicit in this current paper, I refer to three papers from our work that best document different kinds of networking practices (Erath et al., 2018; Prediger & Şahin-Gür, 2020; Prediger & Hein, 2017).
- Design research is always the bricolage of theoretical elements (Alberto, Bakker, et al., 2019; Prediger, 2019). In our case, the integration of the what-questions into the how-questions led to substantiating design principles based on instructional and epistemological foundations (see Figure 3). As in many other cases, topic-specific design research often requires the integration of instructional theories (underlying the design principles) with epistemological theories (underlying the structuring of the content; see, e.g., Alberto et al., 2019).

Beyond these two most discussed modes of connecting research approaches, the summary of our research journey in Table 1 exemplifies that the connection of research approaches in such a chain of overlapping and interacting projects is substantially fueled by *deriving new research questions* from answers to former research questions, and these new research questions become most productive when involving *shifts in the research objects*.

These shifts in research objects are worth reflecting on more systematically, because the choice and the constitution of the research object can have a substantial impact on what can be achieved as a research goal and how research can provide the foundation for design practices (diSessa & Cobb, 2004). For example, as long as only student achievement gaps are the research object, the research hardly contributes to overcoming the problem (Gutiérrez, 2008; Hodgen et al., 2021), and as long as only students with accumulated challenges are investigated, deficit views might be perpetuated without exploring the commonalities to other target groups. Overcoming the problem requires turning to teaching practices and design principles as major research objects (Sullivan, 2015). Additionally, focused mathematical instruction also requires detailed epistemological and empirical insights into the students' learning pathways of particular mathematical topics: This combination of design principles and topic-specific pathways is typical for didactical design research (Gravemeijer & Cobb, 2006; Prediger, 2019). Beyond these, the teaching practices enacted by teachers while realizing a design approach are important research objects for understanding the challenges in implementation processes (Burkhardt & Schoenfeld, 2003), which is also far beyond what we started to understand modestly in our projects.

Table 1. Summary of the research journey in its several steps: shifts in research questions and objects

Method	Research goal	Research question	Research object
Large-scale assessments	Describing and explaining language gaps	What exactly is the problem with language in mathematics?	Student achievement gap Students' background factors Knowledge types in items
Qualitative video study	Identifying good practices	How can language be enhanced in mathematics classrooms?	Teaching practices in regular classrooms
Import from other discipline	Transferring generic design principles	How can language be enhanced in language classrooms?	Design principles for language instruction Tasks and pedagogies
Design research	Explaining learning pathways and typical obstacles	What is the problem with the epistemic role of academic language while developing conceptual understanding?	Students' learning pathways towards conceptual understanding in a mathematical topic
Design research	Specifying the mathematically relevant language-learning content	What academic language demands are crucial for developing conceptual understanding?	Relevant language in students' mathematical learning pathways towards conceptual understanding
Design research	Substantiating language-responsive design principles for math instruction	How can meaning-related academic language be enhanced in mathematics classrooms?	Design principles for enhancing language for developing understanding of mathematical concepts
Controlled trials	Validating effectiveness of approach for different target groups	For whom is language-responsive instruction effective for developing understanding?	Effects of language-responsive instruction for learning gains of students with different background factors
Quantitative video study	Disentangling conditions of effective language-responsive teaching	How does the quality of teachers' enactment affect the learning gains?	Teaching practices in language-responsive classrooms and their effects on learning gains

Finally, with every shift in research object, new theoretical perspectives also enter the scene (from epistemological backgrounds of the topic, theories of learning, theories of language and learning, theories of instruction, etc.). In the long run, an integrated theory rather than a chain of unconnected research objects should be the overall research aim. However, this aim will require two more decades and the joint communication, cooperation, and collaboration of various researchers from other European and non-European sites to integrate the different empirically grounded theory elements into a coherent theory on enhancing language as epistemic catalyst for developing mathematical understanding.

It is a particular strength of the ERME community not to be afraid of complex connections of theories and research approaches (Artigue et al., 2005; Bikner-Ahsbahs, Prediger, & Networking Theories Group, 2014): This was established as a meta-theoretical position in the TWG on theoretical approaches and has spread widely to other TWGs in ERME. In the same way that the idea of networking theories was spread among the whole ERME community within the last decade (as documented by Artigue et al., 2018), I hope that this paper can fuel the idea that big problems require complex chains of projects within one research group and more complex networks of projects of several research groups to treat them. This is presumably not only true for language as an epistemic catalyst for mathematics learning, but also for other exciting areas in mathematics education research.

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