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Initiating meta-cognitive communication in students' discussions about process steps when solving real-world problems

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Real-world problems have been shown to pose many challenges to students. In particular, students are often not aware of the necessary process steps in the crack the problems for building suitable situation models. Instructional approaches that aim at promoting this awareness often try to elicit, compare and leverage students' initial strategies, but sometimes with limited effects. In our design research project, we identified a possible background of these limited effects, namely students' constraints on the meta-level, as many of the observed students constrained themselves to comparing results and calculation steps rather than strategies for process steps on a meta-level. So, we developed meta-cognitive tasks to support the students in engaging in the necessary meta-cognitive communication. In the case study of this paper, we compare two sixth graders' discussions without and then with meta-cognitive tasks. The qualitative analysis reveals first indications that the tasks might help to engage students in meta-cognitive communication.

Keywords: Real-world problems, metacognition about process steps, comparing strategies

Theoretical background

Missing communication about meta-cognition as obstacle for reflecting process steps

It has been shown that students' competence to tackle real-world problems can be enhanced by raising their *awareness about* and *strategies for* necessary process steps in the modelling process; this applies for the whole range of problems, from authentic open-ended modelling problems (Schukajlow et al., 2015; Hänze & Leiß, 2022) down to less authentic, more narrow word problems (Mevarech et al., 2010, Dröse & Prediger, 2021), even if the particular process steps (such as achieving text comprehension, building a situation model or mathematization, or validation) might be of different nature (Hänze & Leiß, 2022). Effective instructional approaches often focus on raising meta-cognition on necessary process steps, e.g., by worked examples (Hänze & Leiß, 2022) or solution plans (Schulkajlow et al., 2015). In this context, *meta-cognition* is defined as knowledge about cognition and the regulation of cognition (Mevarech et al., 2010; Depaepe et al., 2010).

Establishing meta-cognitive strategies in mathematics classrooms requires explicit communication about cognition (what do we think?) and about the regulation of cognition (how can we come to the right steps and validate their suitability?) (Depaepe et al., 2010), we abbreviate both by *meta-cognitive communication*. The meta-cognitive communication in classrooms can include, e.g., why superficial strategies like direct translation (Hegarty et al., 1995) are not appropriate, or different ways to build a situation model (Geiger et al., 2022). However, engaging students in meta-cognitive communication can be challenging when they are not used to it, this has been briefly observed, e.g., by Blum and Schukajlow (2018), and included briefly in an instructional model (Geiger et al., 2022), but not yet studied in depth how to be realized. Also in our project, we observed that students

constrained themselves to naming numerical results answers or reporting calculation steps rather than discussing their strategies for finding a situation model. This led to the following design research question of this paper:

How can students be engaged in meta-cognitive communication about process steps for tackling real-world problems?

Narrowing down to particular real-world problems in view

We narrow down the research question to word-problem solving because for word problems the existing state of research on meta-cognition is highly informative and can be extended in a next research step to more open modelling problems. The benefit of meta-cognition for successful word-problem solving was identified early (Hegarty et al., 1995; Reusser, 1996), and the effect of meta-cognitive trainings were shown (Mevarech et al., 2010). For implementing meta-cognitive instructional approaches in classrooms, however, teachers proved to be essential to explicitly refer to the process steps and the strategies and to generate a communication about them with their students (Depaepe et al., 2010).

For the case of our word problems in view of this paper, we draw upon Reusser's (1996) process model to identify the most important process steps from a word problem text to the situation model, problem model, and mathematical solution of a problem. We adapted the model (see Figure 2 below) by including validation steps and the strategies needed for situation analysis: Focus on question for finding relevant information, focus in information and its meaning, focus on relations connecting information (Dröse & Prediger, 2021).

Existing task designs for initiating meta-cognitive communication

Various task designs have been developed to initiate students' discussion about strategies. In particular, comparison tasks asking students to discuss given solution pathways have proven effective in many contexts (Alfieri et al., 2013). More particularly, a meta-cognitive task design as suggested by Kaune et al. (2011) seemed promising for our context as it is optimized for eliciting meta-cognitive monitoring and reflection processes, by offering tasks "in which false arguments have to be unmasked and facts have to be corrected [and thereby] go beyond the control of calculations and the detection of errors […] This requires the students […] to train their skills in reasoning and communication." (p. 32). In the case study shown in this paper, we will investigate in how far the discussion of own process steps and a given false solution pathway can indeed engage students into meta-cognitive communication about process steps and the underlying strategies.

Methodological framework

Methods of data collection

The case study presented in this paper stems from the third design experiment cycle of a larger design research study. In Cycle 1 and 2, the instructional approach for enhancing students' real-world problem-solving abilities with a solution plan was iteratively developed, experimented and the initiated teaching-learning processes were qualitatively investigated, with the finding of limited meta-cognitive communication, the focus of Cycle 3. To investigate the potential effects of meta-cognitive task designs for students' meta-cognitive communication, we compare two design options by

contrasting two discussions of the focus pair with the design experiment leader (DE leader) as their teacher. In Episode 1 they work on a comparison task without meta-cognitive prompt, in Episode 2 on a meta-cognitive comparison task (tasks printed in Figure 1) that was developed after observations in Cycle 2 that the comparison task alone is not sufficient.

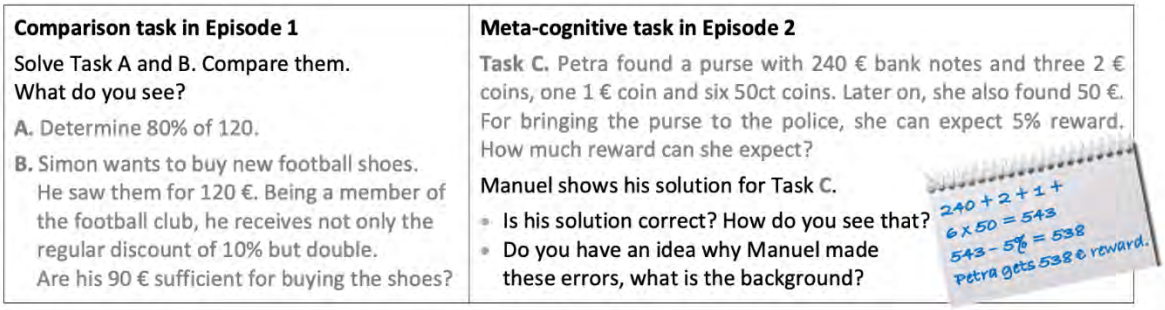

Figure 1. Comparison task and meta-cognitive comparison discussed in the two analyzed episodes

As Cycle 3 took place during pandemic school closures, the design experiments were organized in recorded video conferences. We conducted seven design experiments with 2 pairs of students (of 12–14 years), with 3–4 sessions of 90 minutes each, that were partially transcribed afterwards. The empirical part presents the case of one focus pair, Ophelia and Ralf, selected according to their ability to speak in detail about their solving processes.

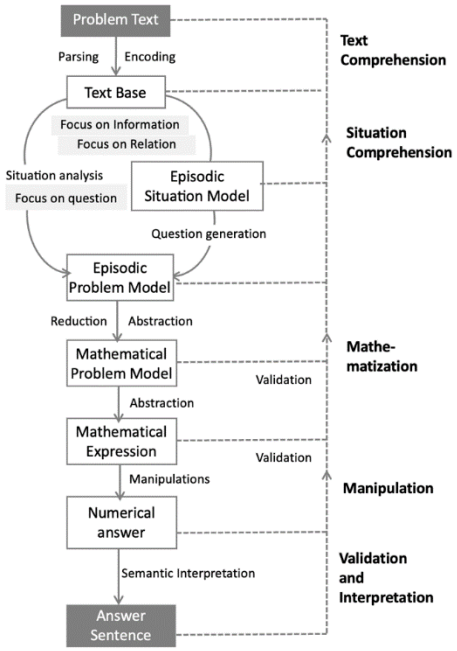

Figure 2. Process steps for word problem solving (adapted from Reusser, 1996, p. 86)

Methods of data analysis

Potential effects on students' meta-cognitive communication are investigated in a qualitative procedure by coding all process steps that are explicitly addressed by the two students in their discussion. The coding scheme was deductively derived from the adapted process model by Reusser (1996) and Dröse & Prediger (2021) as printed in Figure 2. For the research in view, an utterance was

coded as *addressing a process step or underlying strategy*, when a student talks about it, even in other words. Examples for operationalizations are printed below, all codes were assigned in consensus of both authors.

Empirical insights

Episode 1: Ophelia's and Ralf's constraint meta-cognitive communication

The transcript of Episode 1 starts after Ophelia and Ralf have solved Task A and B (from Figure 1 left) individually. Students immediately start to report on their solution pathways:

323 a	Ralf:	Okay, so the first thing I did with number A was to divide 120 by 100, and I
b		got 1.2. Then I multiplied that by 80. Then I calculated one times 80, which is
c		of course 80, and 0.2 times 80 is 16. And I got that by simply calculating 8
d, e, f		times 2. […]
324	Ophelia:	Yes, should I- perfect. I got a bit confused by your way of calculating because there are quite a lot of steps.
325	DE leader:	Yes, should we share it? ... Otherwise?
326	Ophelia:	But it sounds coherent. I got the same results. But should I still present mine?
327	Ralf:	Well, it was easy for me. Yes, please. Absolutely.
328	Ophelia	Well, yeah. So, for A, I calculated 120 x 80 = 9600. And 9600 : 100 are 96, so 96%. […] His savings aren't enough. He's missing 6 €.
329	Ralf:	Hm. Okay. Yeah, I found that out, too.
330	DE leader:	Ralf, did you also think of the well- of the answer?
331	Ralf:	Well, no. Well, I actually wrote down an answer, but not in the same way as Ophelia understood the task. But I just thought what about- what did we notice about our calculation. And there I wrote [reads out loud], "I overlooked something in the task because I only read up to the last number." I thought this was the way we were supposed to write down what we noticed. In other words, considering our calculations, our approach.
334 a	Ophelia	[...] So what I noticed is that once you have an overview of the tasks, and you
b		pick up exactly the information and maybe write down again, what do I actu-
c		ally have to calculate now. Then the procedure is actually not that difficult or
d		as difficult as you thought it would be. You just have to get the important
e		information out of this text.

In his long utterance, Ralf reports on his calculation procedures, referring in detail to every step of inner-mathematical manipulation, yet without articulating the process steps from the problem text to the mathematical model (Turn 323: *calculate* is the only addressed process step). Ophelia provides the feedback not to follow his detailed manipulations (Turn 324: *calculate*). She does not ask for how he decided to use these operations, but explicitly evaluates the *mathematical correctness* (Turn 326).

Although Ophelia presents another solution pathway (Turn 328: *calculate*), the two students do not start a productive comparison at this point, neither of their two diverging solution pathways, nor on the process steps for building the mathematical model, nor on the difference between Task A and B.

Instead, Ralf simply confirms that her result corresponds to his (Turn 329: *mathematical correctness*), and the solution was "easy" for him, yet without alluding to any process steps (Turn 327).

When the DE leader invites Ralf to complete his solution with an answer sentence to the last question, Ralf finally starts to compare with Ophelia's writing, stating that she has written an *answer sentence* (Turn 331) whereas he has written first meta-cognitive thoughts, by diagnosing for himself hasty reading and not taking into account the last question (Turn 331: *construct text base*). So, Ralf reports to have interpreted the question "what do you see" as a meta-cognitive prompt. This also prompts Ophelia to explicitly address process steps *construct text base* (Turn 334a), *focus on information* (Turn 334b), *mathematizing into the mathematical expression* (Turn 334c).

Before the shift to the meta-cognitive prompt cited by Ralf in Turn 331, Episode 1 exemplifies the observation mentioned in the theory section that students often constrain their communication to (a very detailed) report of manipulations (Blum & Schukajlow, 2018) and our design experiment Cycle1/2. And it shows how this hinders to exploit the learning opportunities that were intended to be provided for increasing students' awareness of process steps.

Episode 2: Engaging students in meta-cognitive communication about other process steps

In the end of Session 1, Ophelia and Ralf have solved Task C (from Figure 1, right side) individually, and briefly compared their solutions. Episode 2 starts in the next session, with considering the solution pathway of the imaginary student Manuel (in Figure 1).

71		DE leader: How could you decide "no there is something wrong", so quickly? [...]
72 a	Ophelia:	Well, you can already tell by the approach, if you break down the task.
b		He simply didn't read the text carefully and didn't actually think about what
c		the values stand for and how one would calculate which values in which way.
d		So, for example, he didn't look closely: Do I now have to subtract 5%?
e		If he read it carefully, he would've read- you have 5% finder's reward,
f		he would know you have to subtract 95 and not 5%.
g		He would know that for example.
h		Or here where he multiplied by 50, he would exactly know that he would
i		have to multiply by 5 here and that this generally makes no sense at all.
j		His approach is just completely wrong, and he didn't read it properly.

The contrast to the start of Episode 1 is immediately visible: Already in her first answer, Ophelia engages in long and detailed analysis, not only about Manuel's inner-mathematical manipulations, but also about the process steps that might have brought him to his wrong calculations. Ophelia hypothesizes on many different backgrounds for Manuel's erroneous calculation, including superficial reading (Turns 72b, 72e: *text base*), an incomplete situation model and problem model as Manuel did not *focus on the given information* and their *mathematization* (Turn 72c) or *focus on the question* (Turn 72d). She even articulates consequences of these missing process steps which result, in her eyes, necessarily on *wrong mathematizations and wrong numerical answers* (Turn 72e-g). She holds the superficial reading also as responsible for missing validation steps (Turn 72e/f). When explaining why Manuel's solution is wrong, she first refers to the context (Turn 72h), but then takes Manuel's

wrong mathematization over to subtract 5% (which she previously completed correctly), so her correction stays false.

73	DE leader:	Mh. We'll take a closer look at this again later on, you already pointed out a lot of aspects that are important for later. Ralf also raised his hand.
74 a	Ralf:	[…] Because he obviously did read the text. Maybe he did actually read it,
b		but he mistook- he had to seriously make a mistake because he multiplied by
c		50, where he should have added 50.
d		And then he calculated 543, where the number is already wrong, minus 5%.
e		Because you would get exactly 5% and not 5% minus 100%.

Ralf, in contrast, remains his main focus on calculations, as shown already in Episode 1, by pointing to calculation errors and trying to correct them (Turn 74b, 74d: *calculate*). However, he also traces these errors back to a deficient reading step, possibly inspired by Ophelia's analysis (Turn 74a: *construct text base*). Most interestingly, he refers to the consequences of deficient reading for transferring a misunderstood relation to the mathematization (Turn 74b, c: *mathematize relations*; 74d: *mathematize expressions*). In contrast to Ophelia, his analysis does not go deeper into the critical steps of situation comprehension.

Comparing the addressed process steps

Table 1 summarizes all coded utterances in Episode 1 and 2, separately for Ralf and Ophelia. It allows to compare three steps: In Episode 1, the meta-cognitive prompt occurred only from Turn 331 on, they are marked in brackets. Episode 2 is guided by the meta-cognitive task. In line with the design intentions, more Turns are listed in Episode 2 in which process steps of situation comprehension and mathematization were explicitly addressed in the meta-cognitive communication.

Process steps explicitly discussed by		**Ralf**		**Ophelia**	
		Episode 1 → (after meta-cognitive prompt)	Episode 2 with meta-cognitive task	Episode 1 → (after meta-cogn. prompt)	Episode 2 with meta-cognitive task
Text comprehension	Construct text base	(331)	74a	(334)	72b, e, j
Situation comprehension	Focus on question				72d
	Focus on given information			(334)	72b, c
	Focus on relations				68
Mathematization	Mathematize relations		74b, c		68
	Mathematical expressions		74b, c	(334)	70, 72c
Manipulation	Calculate	323a–f	74d	324, 328	72d
Validation / Interpretation	Mathematical correctness	329	74d	326	68, 70, 72e, h false
	Fit of the model / answer		74b, c, e		58, 70, 72f, i
Answer	Answer sentence	(331)			

Table 1: Addressed process steps: meta-cognitive communication without / with meta-cognitive tasks

The analysis of Table 1 reveals that prior to Turn 331, both students only talk about inner-mathematical process steps such as calculation and mathematical correctness. Due to the rather superficial comparison of the two solutions, they miss to deal with other relevant process steps. Unintendedly, the DE leader succeeds in initiating the meta-cognitive communication because both students

misunderstood one subtask as a meta-cognitive prompt (as articulated in Turn 331 and 334). That also engages Ophelia in talking even about steps of situation comprehension and mathematization. This tendency is strengthened in Episode 2. The meta-cognitive task (and the DE leaders' questions) engages our focus pair of students in articulating something about substantially more and other process steps. In Ophelia's case, for example, the focus is more on situational understanding and its impact on the validation process. Interestingly for both of them, the mathematization, which the imaginary student Manuel has done, now plays an essential role to justify his mistakes, whereas previously they only contrasted their calculations without going into detail. They also reflect more on the fit of the model or answer for validating the proceeding instead of mathematical correctness, which is a deeper reflection.

Conclusion and outlook

Tentative conclusions from the analysis of the first focus pairs

The case study of Ophelia and Ralf provides first insights into typical challenges for learning strategies. Like the focus pair in Episode 1, many students constrain their communication to comparing their own solutions, with hardly addressing other process steps than calculation and validating correctness (Depaepe et al., 2010).

In our design research study (in the long run, aiming at enhancing students' strategies), we treat this challenge of constrained meta-cognitive communication by designing and investigating meta-cognitive tasks that ask students to analyze imaginary students' strategies, following Kaune et al. (2011). At least for these two students, this task leads to a broader discussion about process steps and too superficial strategies, as identified as critical in earlier studies (Depaepe et al., 2010). The comparison of Episode 1 and 2 reveals that the analysis of an incorrect solution pathway can lead students' communication towards process steps such as situation comprehension and mathematization which were identified as essential (Hegarty et al., 1995).The meta-cognitive potential of the task (together with the DE leaders' questions) even extended from students' search for errors in process steps towards underlying causes in the process, so that students also discuss possible superficial strategies. In other studies, this kind of meta-discussion could lead to strategies (Hegarty et al., 1995; Mevarech et al., 2010). Furthermore, the meta-cognitive task seemed to have initiated more substantial validation steps, not only with respect to mathematical correctness, but also to "fit of the model" or "fit of the answer".

These observations can be interpreted as first promising insights that meta-cognitive can promote students' talk about different strategy uses and the importance of becoming aware of different strategies when solving real-world problems (Mevarech et al., 2010).

Limitations and outlook on future design research steps

Of course, the current case study that compares only two tasks for one single focus pair of students in a laboratory setting with narrow problems. So it is only a very first, modest indicator that must be extended in future studies. It remains to be seen, for example, if the design elements can reveal similar effects for other (less talkative) students and in communication without an adult scaffolding it. Additionally, it remains to be investigated to what extent talking about the process steps also has consequences for one's own task processing. But it seems promising to explore, in the future, how

the design approach can be adapted also to more open modelling problems for which other strategy-focused interventions only proved effective in modelling competency for older students, so far (e.g., Hänze & Leiß, 2022).

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