Unlocking minds by conceptually focused, student-centred explicit mathematics teaching and quantitative evidence for effectiveness

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Unlocking formerly underserved students' minds requires conceptually focused mathematics teaching approaches, with student-centred and explicit teaching practices for explicating meanings and mathematical structures. Convincing policymakers requires unlocking false methodological dichotomies and providing hard quantitative evidence for effectiveness of these approaches and practices. The contribution aims at convincing about these claims by giving example evidences from the authors' work.

The conference theme *Unlocking minds* offers an interesting metaphor for goals of mathematics education: all minds can grow, but some have been locked (e.g., by a low-expectation education, Boaler, 2002). The theme holds an interesting ambiguity of *whose* minds we want to unlock. First, mathematics education can develop and investigate instructional approaches that contribute to *unlock all students' minds*, with an equity emphasis on underserved students (from underprivileged family backgrounds or with mathematical difficulties). Second, this can go along with *unlocking some teachers' minds* for overcoming low ambitions for underserved students (Beswick, 2007; Wilhelm et al., 2017) and empowering them for ambitious teaching practices. Third, mathematics education research should aim at advising policymakers about promising instructional approaches and teaching practices (I write as a member of state and national policy advisory boards). As we cannot easily change policymakers' minds, this contribution suggests *unlocking our researchers' minds* by providing not only deep qualitative insights but also hard quantitative evidence, which policymakers often look for.

State of academic and policy discourse

Unlocking students' minds requires overcoming false instructional dichotomies

One old false dichotomy was historically referred to as *conceptually versus procedurally focused* instructional approaches, prioritizing either conceptual understanding or procedural fluency (Hiebert & Carpenter, 1992). In most countries, consensus emerged that both forms of knowledge need to be intertwined as they can only grow together (Kilpatrick et al., 2001). Even students with difficulties in mathematics cannot be restricted to basic skills for procedures, as conceptual understanding of basic concepts and procedures is pivotal for further learning (Gersten et al., 2009). For example, multi-digit multiplication requires place value understanding and the meaning of multiplication, and expanding fractions requires an understanding of equivalence through refining fraction bars and justifying why refining fraction bars corresponds to multiplying numerator and denominator (Prediger et al., 2022). Training symbolic procedures is thereby only sustainable after students explain actively how symbolic and graphical representations are connected.

The controversy about the second dichotomy is still ongoing: Scholars in mathematics education have often promoted *student-centred approaches* in which students are invited to discover strategies and connections themselves, ideally in independent seat or group work (de Jong et al., 2023; Scherer et al., 2016). For students with difficulties, in contrast, empirical evidence was collected through randomized controlled trials that *explicit direct instruction* is an effective approach, in which teachers state a goal, show the steps of what to do, and provide opportunities for practicing them (Zhang et al., 2022). Lambert and Tan (2020) problematized this dichotomy from a disability study perspective, as students with difficulties risk to be systematically denied

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access to high-quality instructional approaches when controlled trials restrict to direct instruction. On the other hand, completely open inquiry approaches without guidance have been shown to disadvantage students from marginalized family backgrounds (Lubienski, 2000).

Clarke (2005) already pleaded for “deconstructing the teacher-centred/student-centred dichotomy” (p. 8) and showed in various qualitative analyses that in teacher-moderated whole-class discussions, the responsibility of knowledge construction can be shared with students. Thus, the deeper structure of shared responsibility in knowledge construction, not just the surface level of activity structures, needs to be considered. Hence, conducting whole-class discussions must not be confused with direct instruction.

Regarding deeper structures of instructional approaches, empirical studies have shown that approaches are most efficient when they combine two steps: initial inquiry with student-centred work for eliciting students’ prior knowledge, followed by explicit instruction, e.g., in two-step approaches such as problem-solving before instruction (Loibl et al., 2017), other kinds of guidance such as scaffolding (NAP, 2008), and subsequent training (de Jong et al., 2023).

To sum up, there seems to be an emerging international academic consensus in mathematics education research that overcoming two false dichotomies (procedural vs. conceptual focus and explicit instruction vs. inquiry approaches) is required for unlocking *all* students’ minds, i.e., empowering students to develop full mathematical competence. However, this academic consensus has not yet reached all mathematics teachers and policymakers.

Unlocking teachers’ minds to offer rich learning opportunities

Academic consensus about instructional approaches is only a first step in a long way to implement them in classrooms, as not all mathematics teachers might already be prepared for it. 20 years ago, Boaler’s (2002) qualitative study revealed that teachers’ orientations toward mathematics and their underserved students’ learning influenced the richness of their teaching practices. Wilhelm et al. (2017) showed quantitatively that teachers who assign the source of students’ difficulties in mathematics to their marginalized family backgrounds offer less cognitively demanding and conceptually focused teaching than teachers who acknowledge their own teaching contribution to the difficulties. Hence, teachers’ low expectations align with creating restricted learning opportunities, which justifies the term “underserved students”. Beswick (2007) identified significant differences in how 21 teachers’ thought about students with mathematical difficulties, and provided first quantitative evidence that these teachers’ minds could be unlocked in a nine-hour professional development program, after which the teachers articulated more equitable orientations with higher ambitions. Other authors have also shown that teachers’ minds can indeed be unlocked so that they are willing to offer rich learning opportunities to *all* students, in particular those formerly underserved.

Policymakers’ minds about research on mathematics teaching and learning

Implementing research-based instructional approaches depends not only on individual teachers, but also on policy decisions regarding standards, syllabi, textbook regulations, exams, and teacher education standards. But not all research findings about the learning needs of students and teachers receive the same attention in the policymakers’ decision-making processes. This is partially because policymakers seek advice aligned with their own positions and partially because the idea of empirical evidence is narrowed down to hard quantitative evidence from randomized controlled trials (e.g., No Child left behind act, 2001 in the U.S.; see Attard, 2025 for the Australian policy discourse). The entire spectrum of design research, qualitative classroom research, and qualitative professional development research, from which many highly important insights into the complexities of classroom ecologies were obtained, are then at risk of being sacrificed to the so-called gold standard of quantitative research (Attard, 2025). I emphasize that quantitative research can be a promising last step *after qualitative deep dives.*

Unlocking our researchers' minds and seeking for quantitative evidence for deep phenomena and subtle effects found in qualitative research

Clarke (2005) pleaded for the complementarity of research approaches by showing how qualitative video studies reveal deeper insights than classical black-box controlled trials and quantitative video studies if they assess mainly procedural skills (without understanding) and capture mainly surface structures (such as talk time, without deeper structures).

Twenty years later, I suggest that the *time has come* to transcend this complementarity and seek for *quantitative evidence for complex and deep phenomena and subtle effects* that were identified in qualitative research. The suggestion stems from a parallel to medicine: In German medical systems, only medical treatments are paid for which hard empirical evidence of effects is provided in randomized controlled trials. Alternative treatments from traditional healers are rarely supported, not because studies have shown they are not effective, but because no controlled trial has shown their effects, so far. However, in recent years, more randomized controlled trials are conducted for alternative treatments, and increasingly, they are accepted in the medical system (or excluded due to proven non-effectiveness).

Similarly, future educational research might aim to gain more hard evidence for complex or deep phenomena and subtle effects of teaching practices that have not yet been in focus of quantitative research on mathematics teaching and learning. For this, we need more refined quantitative instruments. Below, I will substantiate my suggestion with first (humble) examples from our Dortmund research group by which we reacted to limitations in our advisory powers.

Examples from Dortmund research on conceptually focused, student-centred approaches: Successive qualitative–quantitative research strategies

Example 1: Relevance of meaning-related language for explicating structures for students' conceptual understanding: Do students really have to learn that?

For students from marginalized family backgrounds, *academic language proficiency* has long been identified as background factor that strongly correlates with mathematics achievement (Ellerton & Clarkson, 1996). These large-scale findings on overall correlations mainly led to teaching approaches for simplifying language in curriculum materials (as a recent study on textbooks still reveals, de Araujo & Smith, 2022). In contrast, qualitative studies led to *nuanced insights into what language exactly is required* for enhancing all students' conceptual understanding (Adler, 2001; Erath et al., 2021). The part of school academic language that was identified as most relevant has later been termed *meaning-related language* (Prediger, 2019) and has been shown to include discourse practices and (informal yet explicit) phrases to articulate mathematical structures, such as "three groups of five" to explain the meaning of 3×5 (Anghileri, 1991) or "the part 128 of the whole 240" to explain fractions (Prediger et al., 2022). Design research studies identified those content-specific meaning-related phrases and provided cases of students who adopted the language for explicating structures and better developed the targeted conceptual understanding (overview in Erath et al., 2021).

However, when discussing syllabus changes in our educational contexts, we were confronted with sceptical questions: "Do students really have to learn these phrases?" Thus, we realized that no quantitative evidence existed that students who actively use structure-explicating meaning-related phrases have a better conceptual understanding than those who articulate ideas only implicitly or with gestures. Thus, we conducted an assessment study with 414 fifth and sixth graders with this very particular in-depth focus in order to convince German syllabus writers. Indeed, we could show that students who actively used the structure-explicating meaning-related phrases (e.g., "three groups of fives") to explain the connection between symbolic and graphical representations scored higher on a comprehensive assessment of multiplication and division understanding (Hankeln & Prediger, 2025). Conversely, students who tended to provide surface-level translations – identifying only the numbers but not the multiplicative unit structure (e.g., "this dot array matches 3 × 5 because here is three and here is five") – received lower total assessment scores (ibid.). This quantitative evidence convinced the syllabus writers.

The underlying research strategy of conducting *focused assessment studies to test hypotheses on the relevance of specific aspects of mathematical competence* (generated in qualitative studies) rather than wide-range assessments (e.g., PISA covering mathematical competence in total) is not new. However, it could be exploited more systematically in sequential mixed-methods designs for preparing hard evidence arguments for policy decisions.

Example 2: Effects of engaging students in conceptually focused discourse practices in interactions: Do teachers really need to do more than make students talk?

Early quantitative video studies already revealed substantial differences in students' talk time across classrooms (Flanders, 1970; Stigler et al., 1999). Yet students' talk time alone was shown not to be quantitatively predictive for learning gains (Inagaki et al., 1998; Pauli & Lipowsky, 2007). So quantitative observation protocols and video analysis tools have been criticized for covering primarily on surface structures of students' talk (Clarke, 2005; Pauli & Lipowsky, 2007). Does this mean that students' engagement in mathematical conversations is not as important as commonly promoted in practice-based teacher professional development?

Qualitative case studies have specified conditions of richness of talk (Lampert & Cobb, 2003; Walshaw & Anthony, 2008), suggesting to engage students in conceptually deep and discursively rich discourse practices such as explaining meanings, connecting representations, and arguing for enhancing students' deep conceptual understanding (Moschkovich, 2013). Yet, even in professional development programs aiming at these conceptually focused discourse practices, it is not always easy for teachers to realize the necessary richness (Post & Prediger, 2024), so researchers called for long-term PD programs with a clear focus on initiating and maintaining rich discourse practices (Moschkovich, 2013; Post & Prediger, 2024).

However, policymakers' decision-making processes regarding the focus and intensity of professional development and teacher support still seem to doubt (at least in my German context) that rich discourse practices are necessary. Instead, some state departments of education spread professional development on generic pedagogies to make students talk, while others spread curriculum materials for personalized trainings to schools, promising quick gains in procedural skills for underserved students. Again, more hard empirical evidence is required to convince policymakers (a) that conceptual understanding can be achieved by nearly all students, even those formerly underserved, (b) that engaging all student in rich discourse practices can be achieved by targeted scaffolding, and (c) that the richness of discourse practices makes a difference for students' learning gains. Our instructional theories, developed in qualitative design research projects, generated exactly these hypotheses, but were not heard by policymakers. Hence, we felt the need to *provide harder quantitative evidence for their validity*.

We started with a *cluster-randomized controlled trial* with 589 middle school students for testing Hypotheses (a) and (b) for a language-responsive intervention on conceptual understanding for fractions and compared three conceptually focused treatments: A business-as-usual control group in which fraction understanding was taught with regular textbooks (using multiple representations without much language support; engaging students in talk, but rarely explicitly in rich discourse practices of explaining meanings) was compared to a *discursive intervention* in teacher-led small groups of 4-6 students guided by three design principles (DP1. Engage students in rich discourse practices. DP2. Connect multiple languages and representations. DP3. Macro-scaffolding to sequence content- and language-learning opportunities in combined trajectories). The *discursive-lexical intervention* followed the same three design principles in teacher-led small groups and additionally a fourth design principle (DP4. Build up shared meaning-related vocabulary before formal vocabulary). The analysis revealed that the discursive and discursive-lexical intervention groups had significantly higher conceptual learning gains than the control group. However, contrary to our expectations in Hypothesis (c), the slight advantages of the discursive-lexical intervention over the discursive intervention were not significant (Prediger et al., 2022). So, this black-box research strategy, which compared treatments without investigating the actual enacted interaction, could confirm the hypothesized effects of language-responsive designs to promote conceptually focused learning in rich discourse practices, but not the hypothesized effects of additional lexical scaffolding.

However, engaging students in rich discourse practices and lexical scaffolding are not determined by the curriculum material alone, but heavily depend on teachers' moderation of the interaction (Walshaw & Anthony, 2008). Thus, we decided to *open the black box of the teaching learning processes* by analyzing the videos of 49 teacher-led small groups (210 students from the intervention) to *scrutinize the role of the interaction quality for students' learning gains*. By this, we were able to validate Hypotheses (b) and (c). In a huge work package, interaction quality in the intervention small groups was coded with respect to (i) students' talk time, (ii) the conceptual richness of discourse practices into which students were engaged (e.g., explaining meanings and explicitly connecting representations), and (iii) the kind of lexical scaffolding provided by the teachers' moves (not only the curriculum material). These quality degrees were then related to the measurable learning gains. A regression analysis yielded predictors for students' learning gains, showing that (i) students' talk time was not predictive for high conceptual learning gains. However, (ii) the degree to which small groups engaged in conceptually rich discourse practices (not each student's individual discursive participation) predicted conceptual learning gains for formerly underserved students. Finally, (iii) the degree to which the teachers' moves invited students to engage with the lexical scaffolds was predictive for all students' learning gains (Prediger et al., 2024).

Both study examples invite reflection on underlying research strategies: In the first study (Prediger et al., 2022), we conducted a regular controlled trial on a conceptually focused student-centred approach. Interestingly, our reviewers initially criticized that the complex fraction understanding was not *comprehensively* assessed with the simple pre-test and post-test. Fortunately, we could convince the journal editor that a study should be allowed to focus on selected conceptual aspects without suffering from not comprehensively covering *all* aspects of a complex concept. Otherwise, the fear against reducing complexity would restrict all randomized controlled trials one-sidedly to procedural skills or factual knowledge. I firmly belief that conducting more controlled trials on ambitious learning goals is strategically essential in order to provide hard evidence for at least selected conceptual aspects. A larger challenge was that our subtle design differences did not lead to the hypothesized significantly different learning gains. This challenges repeatedly occurred as the design of curriculum material alone does not determine classroom interactions, even if teachers are carefully trained to implement comparable interventions. So, it was again a research strategical decision to disentangle the interaction quality among the 49 groups who shared the curriculum material but still realized quite different interaction qualities (Prediger et al., 2024). This research strategy allowed to provide hard empirical evidence for qualitatively generated hypotheses. The coding work has been immense, yet future AI-assisted coding might lighten the coding work.

With the second study (Prediger et al., 2024), we have now first quantitative evidence for the important effects of interaction quality. This combination of qualitative deep dives and quantitative evidence helps to weaken policymakers' hope that teachers might simply be informed to make students talk with generic pedagogies. It also shows that careful teacher preparation substantially matters, as differences in the surface structure of talk time are not predictive, but conceptual richness of discourse practices is.

Example 3: Effects of explicating structures in student-centred interventions: When teachers don't explicate structures, perhaps it is not necessary?

In the last 10 years, several research groups (not only in Dortmund) have developed various student-centred language-responsive interventions in design research studies on different mathematical contents (see Erath et al., 2021) and disseminated them in teacher professional development programs (Prediger, 2019). During this time, we have more clearly identified the key challenges that teachers face when implementing language-responsive instruction.

Many teachers in our context have overcome mindsets of low expectations described above (Boaler, 2002; Beswick, 2007; Wilhelm et al., 2017) and developed strong ambitions to create integrated concept- and language-learning opportunities for their students (especially those formerly underserved) through student-centred conceptually focused approaches. Yet, video-based qualitative analysis of their teaching practice documented a persisting challenge: a lack of explicitness in the knowledge organization phase after students' independent inquiry (Post & Prediger, 2024). Following Selling (2016), we do not equate explicit instruction with direct instruction. Instead, we follow the two-step approach in which students first explore new ideas independently and are then supported to organize their first experiences into consolidated content and language knowledge (Loibl et al., 2017) by explicating the main aspects collectively (e.g., in whole-class discussions). For language-responsive teaching, this involves engaging students in explicitly articulating structures (e.g., part-whole-structure of fractions) while connecting multiple representations and explaining meanings. In our qualitative case studies the teachers successfully launched discussions about meanings, but they gave limited support for explicating mathematical structures in discussions (Post & Prediger, 2024).

In the implementation contexts in which we work, these case studies did not sufficiently convince the policymakers and PD program planners that the identified non-explicit teaching practices were not only single cases. Thus, we decided to gather also quantitative evidence for the non-explicitness. At the start of several parallel PD courses, we collected data from 102 teachers through a vignette-based activity, in which they were asked to explain meanings. Only 10% of the teachers explicated the multiplicative structures involved (Prediger & Wischgoll, 2023). When discussing these findings with PD facilitators, scepticism raised the following question: When teachers don't explicate structures, perhaps it is not necessary for students? And is it perhaps only important for multiplication, or for every topic?

To provide hard evidence for explicitness as necessary for students' learning, we developed an intervention on another topic: justifying the conversion procedure for measurement units (Why do we multiply by 1000 when converting kilograms to grams?). Two intervention variations both followed the design principles DP1–DP3 (from above), but differed in a fourth principle (DP4. Provide structure-explicating scaffolds). The analysis revealed that the two intervention groups yielded significantly higher conceptual learning gains than a control group. Additionally, the intervention group with structure-explication scaffolds had significantly higher learning gains than the intervention group without (Bielinski et al., submitted).

In this way, the new randomized controlled trial provided empirical evidence for the relevance of structure-explicating scaffolding. From a research strategical perspective, it is important to emphasize that we were only able to develop these two targeted interventions and the focused comparative research design based on many qualitative studies conducted before, this this is again a plead for successive mixed methods research, not for randomized controlled trials alone. We decided to focus on this subtle difference of structure-explication, as it was the aspect mostly doubted in our practical contexts and policy contexts.

In total, these sketches of our research journey provide examples of how we established qualitative insights and based upon them, hard empirical evidence for the effectiveness of conceptually focused, student-centred, explicit mathematics teaching.

Conclusion: Unlocking qualitative researchers' minds to provide hard evidence for complex or deep phenomena and subtle design elements

As a qualitative researcher, I have promoted design research approaches for many years as an ideal research approach for developing instructional approaches, curriculum materials, and for generating instructional theories about design principles and relevant learning goals. Design research approaches are powerful as they can appropriately combine constructive and reconstructive research goals and capture the complexity of learning ecologies (Gravemeijer & Cobb, 2006). However, we must acknowledge that any design principles and conditions of success we identify are only articulated in generated hypotheses.

On the other side of the assumed dichotomy of research approaches are randomized controlled trials, often criticized as too narrow in their learning goals (as standardized tests are often limited to procedural skills and factual knowledge), and in their short-term and black-box nature, which does not allow us to learn much about the complexity of student-centred instructional approaches. Nevertheless, we must not leave the field of providing hard quantitative evidence to those researchers who promote simplified learning goals and simple instructional approaches (Attard, 2025). Unlocking my mind as a qualitative researcher led me to overcome the fear of reducing complexity in quantitative designs, while still maintaining the ambition to capture complex and deep phenomena and subtle differences qualitatively identified.

Of course, the examples of successive mixed methods research briefly sketched in this paper need further elaboration (to be found in the cited papers). More importantly, they need further research as they can only give first humble examples of how deep insights from qualitative research can fuel quantitative studies of different kinds, and this is urgently needed to convince policymakers. In the future, AI-assisted data analysis will support the coding of complex phenomena, but not avoid the need to start with deep qualitative analysis, before quantification.

Finally, let me emphasize that most practicing teachers cannot be convinced by quantitative evidence. For them, we should provide opportunities for good teaching experiences that allow them to experience the power of new approaches or subtle differences themselves. These experiences can unlock minds more effectively than effect sizes and betas. Also for this field of action, there is still much more research and development needed in the future.

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