ESTABLISHING PRACTICES OF UNPACKING MEANINGS OF MATHEMATICAL CONCEPTS IN MULTIPLE LANGUAGES

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Unpacking and comparing the literal meanings of phrases for mathematical concepts in multiple languages can unfold productive discussions to deepen students' conceptual understanding. However, as students often translate only by overall sense, productive practices of language comparison must first be established. The design research study explores a multilanguage-responsive approach with prepared phrases and literal meanings from multiple languages, for the case of delineating area and perimeter. Qualitative analysis of two episodes of whole-class discussions revealed four practices of language comparison with varying degrees of contribution to meaning-making. The analysis indicates that the prepared phrases can support to establish productive practices that students can later transfer to their own home languages.

INTRODUCTION: PRACTICES OF LANGUAGE COMPARISON AND THEIR (PARTIALLY UNEXPLOITED) EPISTEMIC POTENTIAL

Comparative linguistic analyses revealed that different languages can express mathematical concepts with slightly different conceptualizations (Barton, 2008). For example, the English expression "three fifths" for fractions emphasizes the counting of new units "fifths". Meanwhile, the Turkish expression "beşte üç" begins with the whole "beş" (five), followed by the suffix "te" (therein) and the part "üç," carrying the whole-part structure ("five-therein three") more explicitly than English (Prediger et al., 2019).

In mathematics classrooms, the practice of comparing phrases from multiple languages for the same mathematical concept can therefore be productive to deepen students' conceptual understanding. This can be achieved when students realize and connect different concept elements for the concept in view, which together form its holistic meaning (Prediger et al., 2019). While this epistemic potential has been exemplified in some case studies (Barwell, 2018; Prediger et al., 2019), further classroom observations revealed that the epistemic potential is often left unexploited, resulting in missed learning opportunities (Prediger & Uribe, 2021; Ferrari et al., 2023). To address this, we developed a multilanguage-responsive instructional approach with prepared phrases from multiple languages and their literal translations. This approach aims to establish language comparison practices that can later be spontaneously extended to other languages (Prediger & Uribe 2021; Schüler-Meyer et al., 2023). The paper presents a sub-

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study within a design research project that aims to substantiate this approach for the case of delineating the measurement concepts of area and perimeter. We qualitatively analyze classroom discussions with respect to the following research question:

What practices of language-induced reflection do students engage in when interpreting phrases for area and perimeter from multiple languages, including their home languages, and how do these practices contribute to meaning making?

THEORETICAL BACKGROUND

Delineating area and perimeter as a typical conceptual challenge

Perimeter and area are measurement concepts for two-dimensional figures. Perimeter refers to the *length around* the outer boundaries of a figure (e.g., measured in standard units of 1 cm), and the area refers to the *space inside* (measured by covering or tiling it with two-dimensional standard units, e.g., squares of 1 cm^2). For students, "one of the most persistent findings related to learning to measure area is students' struggle to distinguish area from [perimeter]" (Smith & Barrett, 2017, p. 364).

In the empirical part of this paper, we explore to what extent unpacking the phrases used to express the concepts of area and perimeter can enhance students' ability to sustain a clear delineation of the meanings of two distinct concepts: For example, in German language, "Umfang" (perimeter) includes the prefix "um" meaning "around" in "around-catch", while "Flächeninhalt" (area) the prefix "in", referring to "inside", as "area-content-inside"). Figure 1provides additional examples from other languages.

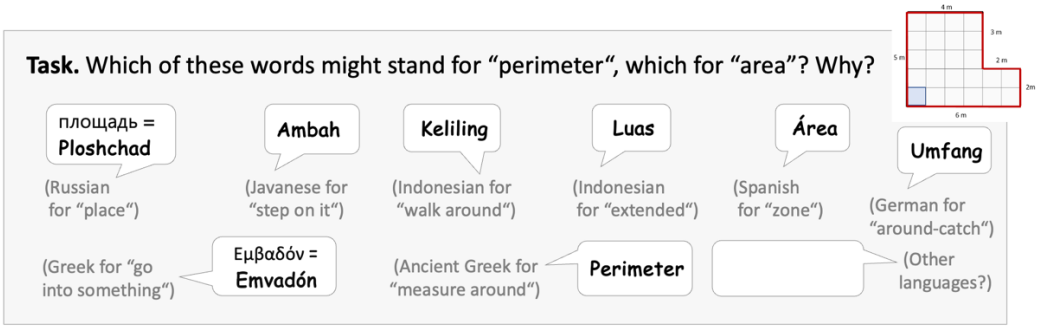

Figure 1. Language reflection for deepening understanding of area and perimeter

Documented challenges in exploiting epistemic potentials of language reflection

From some classrooms, compelling cases were presented how multilingual students drew upon their home languages to unpack the literal meanings of other mathematical concepts and compare the literal translations to connect and delineate different meanings (Barwell, 2018; Prediger et al., 2019). However, in many other classrooms, students' language awareness was less developed, for instance a Greek-speaking student might translate "Emvadón" simply as "area", conducting translations by overall sense rather than a word-for-word literal translation. Teachers who do not speak the language in view cannot unpack the more subtle nuances of the literal translation of "go into

something" (Prediger & Uribe, 2021). So, the productivity of language comparison practices seems to depend on a) the phrase chosen from the foreign language, b) the kind of references made for interpreting the foreign phrase into German, c) the explanation of what the German translation means, and d) the addressed concept elements, which might enrich the meaning of the underlying concept.

Design of prepared phrases with literal translations from multiple languages

In order to overcome these well-documented challenges, we have developed tasks to compare and delineate *prepared* phrases from multiple languages together with their literal translations (Schüler-Meyer et al., 2023). These tasks were intended to support teachers to *establish productive language comparison practices* and thereby provide learning opportunities for students to experience what the epistemic potential could be.

This design approach is theoretically based on a definition of (mathematical or discursive) *practice* as recurrent ways of acting mathematically or discursively, which are co-constructed in the classroom interaction (Cobb et al., 2001), and thereby might create certain conceptual learning opportunities, in our case for meaning making on area and perimeter. While tasks can pose certain demands, new practices only emerge in the interaction, which is nurtured by students' contributions and supported by teachers' prompts and scaffolding. Within this theoretical perspective on learning, students' participation in collectively enacted (and supported) practices can lead to more independent accomplishments. Yet, empirical investigations must be conducted to concisely characterize productive and unproductive practices, their factual contributions to meaning-making and teachers' means to establish and support students' engagement in these productive practices.

METHODS

Methods of data gathering in design experiments. The research question that guides our current study is pursued within the research context of the larger design research project ML^2, which develops and investigates learning environments for exploiting multilingual resources in Grade 5–8 mathematics classrooms (Schüler-Meyer et al., 2023). Typical for the design research methodology is the dual aim of (a) developing and optimizing learning opportunities (in this paper, for meaning making through multiple language comparison), and (b) generating deep insights into the initiated learning processes (in this paper, the language comparison practices and their potentials for meaning making) (Cobb et al., 2001). Design experiments were conducted in three Grade 5–7 classes (with 10-13-year-old students) by research-near teachers new to the classes. The experiments lasted 90 minutes each, of which 30-40 minutes treated the concepts of area and perimeter in view of this paper. In the empirical part, we present the analysis of two whole-class episodes on the task in Figure 1 and the transfer to students' own home languages. The videorecorded episodes were fully transcribed in German.

Methods of qualitative data analysis. The qualitative analysis for the two episodes was conducted using the original German transcript. Afterward, the transcript was translated into English, taking care of preserving the language subtleties in view from the original transcript. For each student contribution, we coded with the analytic scheme in Figure 2, a) which phrase in a foreign language students reflected on, b) to what they referred in its interpretation, c) into what phrase in German they translated it and how they explained it (if they did), and from this, we derived interpretatively d) what concept elements they addressed through the language reflection, which might contribute to enriching the meaning (see Figure 3 for examples). By comparing different moments in the data, we identified different practices of language reflection and their potential for meaning making.

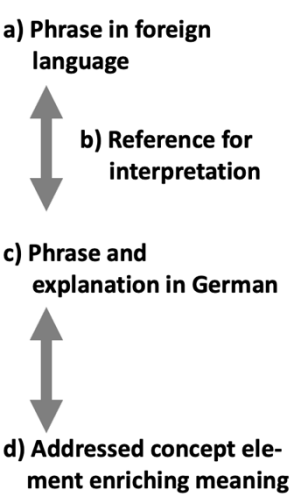

Figure 2. Analytic scheme

EMPIRICAL INSIGHTS: TWO EPISODES OF LANGUAGE REFLECTION

Episode 1: From associating world knowledge to unpacking given literal translations

The first episode stems from a Grade 5 class. The students discuss which phrases refer to perimeter and which to area, based on the task from Figure 1.

10 Paul:	I think that [*the Spanish phrase*] "Área" comes [*translates to* "*Fläche*"] because of the [*US*] word "Area 51".
11 Teacher:	Ok, yes, you would have derived it from English.
12 Sina:	Yes, it [*the literal translation*] says "zone", and the area is such a zone?
13 Teacher:	Yes, exactly, great, great explanation.
14 Lisa:	I have one more word. In Indonesian ["*keliling*"] I have "walk around" at the bottom, that's why I thought of perimeter? Because it's also like in the figure, like this [*gestures around the figure*].

Paul (in T. 10) refers to the Spanish phrase "Área", associating it with the US term "Area 51" by drawing on world knowledge and cultural references related to the secret military space. The teacher (in T. 11) explicitly acknowledges that this derivation comes from English, reinforcing the linguistic connection. While this world knowledge association is productive for exploring cross-linguistic similarities, it does not necessarily contribute to a deeper understanding of the mathematical concept of area itself.

Sina (in T.12) builds on this discussion by referencing the literal translation "zona" (zone), suggesting that area can be understood as a type of zone, so her contribution becomes more mathematically relevant. While the teacher reinforces her practice of referring to the literal translations given below the speech bubble in the worksheet (T.13), the conceptual implications of "zone" remain unexplored.

Lisa refers to the Indonesian phrase "keliling" for perimeter and immediately connects it to its literal translation (T.14). Lisa's focus on literal translation may have been encouraged by the teacher's earlier reinforcement of Sina's contribution (T. 12/13). In this way, language reflection by unpacking literal meanings is introduced and practiced. However, it is Lisa's second sentence by which this moment becomes particularly productive for her meaning making: She seems to realize that the term closely aligns with her mental representation of perimeter, which she visually explains by gesturing around the figure. It is this connection from the literal translation "walk around" to this gesture that bears most epistemic potential.

Episode 2: From unpacking given literal translations to own home languages

The second episode stems from a Grade 7 whole-class discussion summarizing what students have individually examined in the given provided phrases for area and perimeter (Figure 1). The teacher prompts students to identify phrases they consider inaccurate for these concepts.

31 Cornelius:	[*Refers to* "*Ambah*"*, Javanese for* "*step* ***on*** *it*"*]* So, I find it's [*referring to "on"*] … confusing. Perhaps one should try to understand it more as entering **into** an area or simply being inside.
32 Teacher:	… In #Greek ["*Emvadón*"*, Greek for* "*go into something*"*]* that is included. Would it be enough then to simply enter the area?
35 Rafael:	No, because one still needs to move within the area, so that's just too small.
37 Rafael:	Walk around
38 Teacher:	Walk around. But walk around was perimeter, right?
39 Rafael:	Yes, but you can walk around within a surface. […] Inside
44 Teacher:	How does that look in Ukrainian? [*Teacher asks Svenja*].
45 Svenja:	In Ukranian, area is called "площа" (ploscha), which … means "place".
48 Teacher:	What does that have to do with determining area? [*3 sec. break*] Place, area?
52 Denise:	Mm, so, area in Bosnian is "površina", and it means "area".
58 Lena:	[*referring to question in T. 48*] The area occupies a place.

Cornelius (in T. 31) articulates his concerns that the phrase "Ambah" (Javanese for "area" meaning "step on it") is confusing, not in delineation to perimeter, but in distinguishing ideas of "on" and "in". Instead, Cornelius suggests a more fitting interpretation, such as "entering into" or "being inside". The teacher (T. 32) highlights that this idea of "going into something" aligns with the Greek phrase and asks whether it sufficiently conveys the meaning of area. *Rafael* (in T. 35-39) disagrees, referring to a more dynamic idea of moving within the area. However, he does not explicitly express the idea of moving to cover the entire area, leaving it implicit. The teacher (T. 38) uses Rafael's phrase, "walk around", to initiate a conceptual contrast, asking whether this description might better fit the perimeter rather than the area. Cornelius and Rafael

refer to the given phrases in different languages and their literal translations. In both cases, this contributes to their meaning making as they distinguish between "on" and "in", i.e., covering with standard units versus tiling with standard units.

To further mobilize the multilingual resources of the students, the teacher asks *Svenja* for an ad hoc translation into Ukrainian. Although such spontaneous practices of semantic unpacking do not always succeed, Svenja, who recently arrived from Ukraine, is able to provide one, as she has attended math classes in Ukrainian in her home country. She translates the literal meaning as "place", possibly inspired by the similar translation provided in Russian. In order to exploit this phrase for students' meaning making, the teacher engages also other students in discussions about meanings of the literal translation by asking: "What does that have to do with determining area? Place, area?" (T. 48). At this point, an interesting discussion over 13 turns occurs (non-printed due to space restrictions), which finally ends in the highly condensed and deep explanation by Lena (a student from a monolingual family), summarizing: "The area occupies a place". (T. 58), in which "occupy" offers a third element of meaning for determining areas, "stepping on" and "going inside".

During the discussion, *Denise* (T. 52) introduces her home language, Bosnian, yet only by translating by overall sense in which the Bosnian word is assigned to the German label without a one-to-one word equivalent or further unpacking. Neither Denise nor the teacher were able to quickly structurally unpack the word: According to our inquiry with Bosnian-speaking adults, po-vrš-ina seems to be composed of the prefix "po" for "on" or "above", the component "vrš" stands for "upper layer" and the suffix "ina" is often used for constructing nouns describing properties or physical expanse, together "the upper expanse". This chance for deeper conceptual exploration was missed.

DISCUSSION

Outcome 1: Replication that language comparison can be applied to initiate meaning-making processes, also for delineating area and perimeter. The multilanguage-responsive approach to unpack and compare phrases from multiple languages has been shown to bear epistemic potential for enhancing students' understanding of fractions (Bartolini-Bussi et al., 2014), proportional reasoning (Prediger & Uribe, 2021) and algebra (Schüler-Meyer et al., 2023; Ferrari et al., 2023). Our qualitative analysis revealed insights how the potential can also be exploited for delineating area and perimeter, an often-documented student challenge (Smith & Barrett, 2007), with multilingual and monolingual students who do not share all their home languages.

Outcome 2: Disentangle four language comparison practices with increasing potential to deepen understanding. Beyond these replications, the main contribution of the current study is to disentangle how productive practices can be characterized, through the components of the analytic scheme (Figure 2). In the analysis of two short episodes, we identified four practices (Figure 3): *translating by overall sense* was shown not to contribute to deepening conceptual understanding, whereas *associating*

world knowledge (such as "Area 51") could have contributed if the teacher had leveraged this idea by unfolding the "secret inside" into a good mnemonic bridge. The *intended practice of semantic unpacking* was found in four moments, each of which made additional contributions to deepening conceptual understanding. Three of these moments emerged with prepared phrases (Sina T. 12, Lisa T. 14, Cornelius & Rafael T. 31-35), indicating that the design approach seems to bear the intended effects. The fourth moment (Svenja in T. 45-58) provides first indications that students' transfer from prepared phrases in given languages to their own language is possible, but becomes richer for deepening conceptual understanding when the teacher supports the explicit connection to the mathematical meaning.

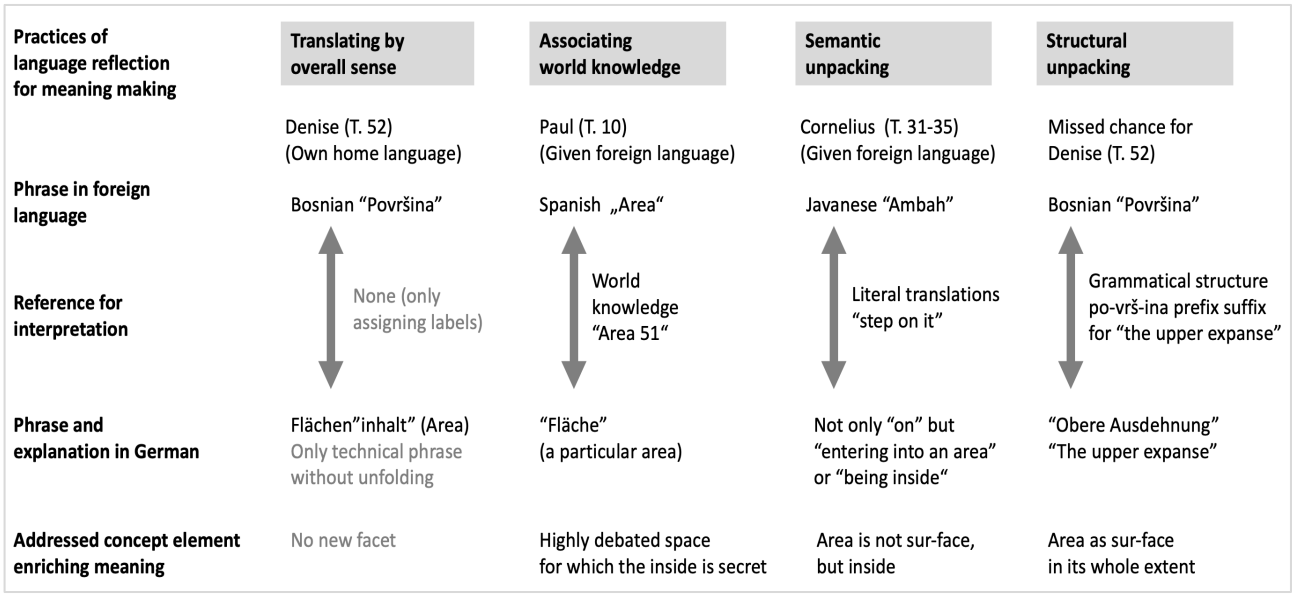

Figure 3. Analytic outcome: Four language comparison practices with increasing potential to meaning-making (3 more moments of semantic unpacking unprinted)

The *fourth practice of structurally unpacking phrases*, however, was still on the list of missed chances. While a structural unpacking of the self-introduced Bosnian construction with prefix and suffix would have been highly interesting to reflect on, Denise only offered a translation by overall sense. In other transcripts, we found the first seeds of structural unpacking, but also the need to provide more scaffolding for it.

Limitations. Two short episodes provide only a small empirical base, our findings are further substantiated by analyses of additional classroom videos from the larger project, where we found these four practices as repeated pattern. As the contribution to meaning making can only be inferred from students' explicit contributions, we miss insights into silent meaning making thoughts of students who do not participate verbally. To address this limitation, future studies should combine the analysis of whole-class discussions with more individualized data gathering, e.g., in pair work.

Conclusion and design implications. In summary, we disentangled the differences between *semantic unpacking* (semantically drawing on the components of the words) and *structural unpacking* (breaking down the phrases into prefixes and suffixes and

analyzing their structural, word-forming grammatical functions). While semantic unpacking was well initiated, structural unpacking rarely occurred. Therefore, we suggest that in a next project, we should also support teachers in the process of structural unpacking. While the current material has successfully encouraged teachers and students to discuss semantic aspects of different literal translations, future designs should also sensitize students to the possibility of structural unpacking the grammar in prefixes and suffixes, conjunctions, prepositions, as these are the most interesting connectors through which different mathematical relations are expressed.

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References

Bartolini Bussi, M.G., Baccaglini-Frank, A., & Ramploud A. (2014). Intercultural dialogue and the history and geography of thought. *For the Learning of Mathematics, 34*(1), 31−33.

Barton, B. (2008). *The language of mathematics: Telling mathematical tales*. Springer

Barwell, R. (2018). From language as a resource to sources of meaning in multilingual mathematics classrooms. *Journal of Mathematical Behavior, 50*, 155–168.

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in classroom mathematical practices. *The Journal of the Learning Science*, *10*, 113–163.

Ferrari, E., Meaney, T., & Lekaus, S. (2023). The complexity of task design for utilising the epistemic potential of multiple languages in developing pattern understandings. In P. Drijvers et al. (Eds.), *Proceedings of CERME 13* (pp. 1585-1592). ERME. /hal-04393040.

Prediger, S., & Uribe, Á. (2021). Exploiting the epistemic role of multilingual resources in superdiverse mathematics classrooms. In A. Fritz, E. Gürsoy, & M. Herzog (Eds.), *Diversity Dimensions in Mathematics and Language Learning* (pp. 80–97). De Gruyter.

Prediger, S., Kuzu, T., Schüler-Meyer, A., & Wagner, J. (2019). One mind, two languages–separate conceptualisations? *Research in Mathematics Education*, *21*(2), 188–207.

Schüler-Meyer, A., Meaney, T., Uribe, Á., & Prediger, S. (2023). Design heuristic for generating conceptual learning opportunities through multiple languages. In A. Schüler-Meyer, J. Ingram & K. Erath (Eds.), *Proceedings of ETC 12 on Language in the Mathematics Classroom* (pp. 102–110). ERME. hal.science/hal-03992500

Smith, J. P., & Barrett, J. E. (2017). Learning and teaching measurement: Coordinating quantity and number. In J. Cai (Ed.), *Compendium for Research in Mathematics Education* (pp. 355–385). National Council of Teachers of Mathematics.