EXPLAIN WHY “TIMES 1000” FROM KILOGRAM TO GRAM: CONTROLLED TRIAL ON LEARNING TO EXPLAIN IN TWO DIGITAL ENVIRONMENTS WITH DIFFERENT TOPIC FOCUS

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Learning to explain procedures (e.g., convert kilogram into gram) has topic-independent discursive components (general readiness to articulate ideas) and topic-specific conceptual components (understanding this procedure). To study if topic-specific components need to be explicitly treated, we conduct a controlled trial with n = 282 fifth graders and compare the effects of two digital teaching-learning environments on topics of mass measurement: The topic-specific environment on the mass unit conversion procedure yielded a medium intra-group effect size (d = 0.73) on explaining why the conversion works. Meanwhile, the environment on applying and explaining estimation strategies yields a small transfer effect (d = 0.14) on explaining non-treated conversion procedures. Thus, explaining can only slightly be fostered topic-independently.

INTRODUCTION

Whereas nearly all students learn to engage in discourses practices of *reporting how to conduct a procedure*, only few students get productive learning opportunities for *explaining why a certain procedure works*, by drawing upon the conceptual foundations needed for this justification, e.g., in a visual model and its underlying structures (Moschkovich, 2015; Fuson et al., 1997). Design research studies and qualitative classroom observation studies contributed to identify design principles and teachers’ moderation practices for enhancing students’ competence of explaining (Erath et al., 2021; Moschkovich, 2015), often independent from the content in view (Erath et al., 2018; Mercer & Sams, 2006; Walshaw & Anthony, 2008). However, there is still a lack of interventions with *quantitative evidence* for effects on students’ learning gains in explaining (see survey Erath et al., 2021), and no research on how topic-specific the learning opportunities for explaining need to be. Thus, the current study aims at providing quantitative evidence that students *can* indeed learn to explain why a procedure works, and that topic-independent interventions can contribute much less than topic-specific interventions that focus students attention to the conceptual components underlying the procedure in view. For this, we have chosen the procedure of mass unit conversion (e.g., from kilograms to grams and back) as the topic in view.

We first present the theoretical background of our controlled trial before we articulate the research question and hypothesis to be tested, present the methods and the findings.

Bielinski, S., & Prediger, S. (2025). Explain why "times 1000" from kilogram to gram: Controlled trial on learning to explain in two digital environments with different topic focus. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.), *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 115–122). PME.

THEORETICAL BACKGROUND

Mass measurement and its different topics, among them explain unit conversion

Students' competences for dealing with measurement units comprise several topics, among them the big idea of measuring through iterating standard units, benchmarks for typical measures, estimation strategies, and the *procedure of converting between units* which is focused in this paper (Smith & Barrett, 2017). For the strategies and procedures in view, students do not only need the procedural skill to conduct them, but also to *explain them and justify why they work* (Freudenthal, 1981, for unit conversion: Smith & Barrett, 2017). For the *procedure of converting mass units*, students are offered the visual model of fine-grained and coarsely grained base-ten blocks on the balance scale (Figure 1). To explain why 4.3kg is converted into 4300g by "times 1000", students can visualize the 4.3kg by four 1kg-blocks and three 0.1kg-flats (which is grounded in place-value structures and bundle structures, Fuson et al., 1997). They have the same weight on the balance scale as the finer-grained four 1000g-blocks and the three 100g-flats. When refining a kg-block into a 1000-g-block, each block is splitted into 1000 finer 1g-cubes, so four blocks into four thousands, thus we calculate 4×1000, and 0.3×100. This explanation is grounded in the refinement structures (Bielinski & Prediger, submitted).

Explaining why procedures work as discourse practice with topic-independent discursive and topic-specific conceptual components

Explaining is one of the most important discourse practices that is relevant for many mathematical topics (Moschkovich, 2015). Students' competence to explain procedures and their conceptual foundation has *topic-independent discursive components* (general readiness for articulating mathematics ideas and their conceptual foundations) and *topic-specific conceptual components* (understanding the procedure in view by connecting it to the underlying structures). The general readiness refers to the overall willingness to articulate mathematical ideas (students often start with only naming facts rather than elaborating and skip writing explanation tasks) and topic-independent sociomathematical norms for good discourse practices (e.g., when asked to explain, my teacher wants me to refer to visual models, not only symbols, Erath et al., 2018).

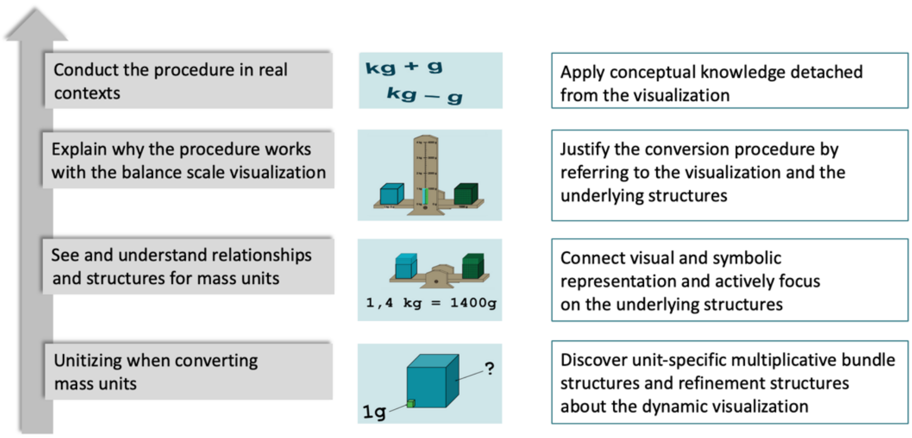

Figure 1: Learning trajectory of the digital teaching-learning environment for IG-CP

Furthermore, topic-specific conceptual components are relevant as students need to learn about the meanings of the visual and symbolic representations and the underlying structures of the topic in view (Fuson et al., 1997). For converting mass units, the place-value structures, bundle structures and refinement structures were identified as relevant to focus for students (Bielinski & Prediger, submitted).

Design principles for learning to explain in digital teaching-learning environments

Students can learn to explain strategies or procedures in teaching-learning environments with two design principles: engaging students in rich discourse practices and connecting multiple representations, as to be explained in the following. The principle of *connecting multiple representations* (Lesh, 1979; Fuson et al., 1997) suggests to flexibly work with visual, material, symbolic, verbal and other representations, in our case the visual representation of finely and coarsely grained blocks on the balance scale and their symbolic translation. Digital tools and *digital learning environments* have been shown to bear potential for connecting representations by multi-representation tools with dynamic links between representations (Kaput, 1986; Drijvers et al., 2016). A condition is that not uncommented (or only automated) translations between representations are conducted because enhancing students' meaning making requires the explicit articulation of how these representations are connected (Renkl et al., 2013). The essential role of teachers' moderation for these explicit articulations is emphasized by the term *teaching-learning environment*. Effects on students understanding have been shown to be larger when the tool is embedded in more structured teaching-learning environments, which sequence the sub learning goals and tasks towards conceptual understanding for procedures in carefully designed learning trajectories (Sacristán et al., 2010), as for our environment (Figure 1). These trajectories follow approaches of progressive schematization, starting from informal experiences in material and visual representations over concept-based strategies and progressively develop towards justifiable symbolic procedures (Freudenthal, 1981; Bielinski & Prediger, submitted).

The principle of *engaging students in rich discourse practices* is realized by tasks and teacher prompts that elicit students' contributions for collective or monological discourse practices, but also by (written and oral) scaffolds that support students' engagement in the collective practices, and by language models that demonstrate what is expected (Erath et al., 2021; Walshaw & Anthony, 2008). Digital teaching-learning environments have been shown to bear potential for fostering students' discursive readiness by digitally assisted communication (Geiger et al., 2023), e.g., with appropriate visual representations as scaffolds, and with strategy conferences on different approaches into which students are introduced and then have to explain to each other.

Empirical evidences for the efficacy of digital environments with these two principles were provided for various mathematical topics (Drijvers, 2018; Sacristan et al., 2010), including explaining measurement formulas such as for volume (Huang & Wu, 2019). But so far, no empirical evidence exists for explaining unit conversion procedures, so Smith and Barrett (2017) suggest to "harness technological capacity to support learning" (p. 379) and to explore how multi-representation tools "can be used as productive components of … measurement teaching" (p. 379).

RESEARCH QUESTION AND HYPOTHESIS

In studies on enhancing students' discursive readiness, learning to explain has sometimes been implicitly treated as if requiring mainly topic-independent learning opportunities and being transferable from one topic to the next (Erath et al., 2018; Geiger et al., 2023; Mercer & Sams, 2006, Walshaw & Anthony, 2008). However, research about explaining arithmetical procedures substantiates the counter hypothesis that learning to explain is topic-specific in that explaining the conceptual foundation of a particular procedure includes specific conceptual components that need to be understood (Fuson et al., 1997). So, we ask the following research question:

RQ. *To what extent do students' explanations of the mass unit conversion procedure improve in two digital teaching-learning environments that both engage students in rich discourse practices about mass measurements, but with different topic focus?*

In a classwise randomized controlled trial, we test the following counter hypothesis:

H. *Students working in the topic-specific digital environment with topic focus on conceptual foundations of the conversion procedure improve their explanations more than students working in a digital environment with topic focus on estimation strategies.*

METHODOLOGICAL FRAMEWORK

Research design of the controlled trial. To test the hypothesis, we conducted a classwise randomized controlled trial. As the *independent variable*, we compared the effects of two digital teaching-learning environments about mass measurement which lasted 90 to 135 minutes, each. As summarized in Figure 2, both environments shared the same design principles and the visual representation of the balance scale, but had different topic foci: the topic-specific intervention IG-CP focused on conducting and explaining conversions of mass units as introduced in the theory section, whereas the topic-deviating intervention IG-ES focused on conducting and explaining estimation strategies. With explanatory videos and strategy conferences, explaining to peers was introduced and trained more intensively in IG-ES than in IG-CP, as documented in the example tasks in Figure 2. The conceptual components underlying unit conversions were not addressed, but benchmark knowledge and strategies.

As the *dependent variable*, students' explanations on the topic of unit conversion were assessed before and after the interventions, all within the digital teaching-learning environments. No significant difference occurred in the pretests. As *control variables*, students' self-reported gender, immigrant background (student or one parent born outside the country) and multilingual background (other languages at home).

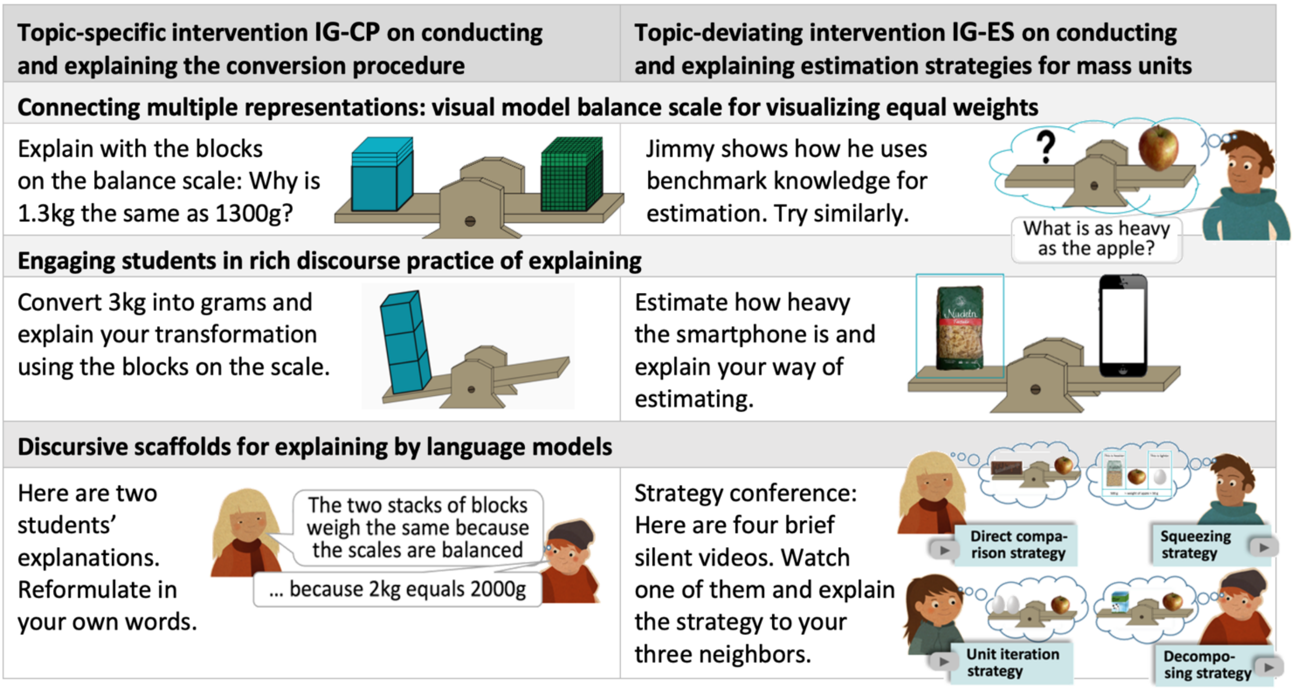

Figure 2: Same principles, different topic foci: Example tasks on two interventions

For the *sampling*, 11 classes were randomly assigned to the two intervention groups. In total, n = 282 fifth graders (aged 10-12 years) completed pretest and posttest, they form the intervention whole sample, (n = 124 in IG-CP and n = 158 in IG-ES). Both intervention groups were comparable in gender, multilingual background and immigrant background (Chi^2 tests with $p > .05$).

Methods of data analysis. Students' explanations were scored according to explicitly addressed structures, the precise/imprecise articulation of representations, and the richness of the discourse practices, with a maximum score of 11.5 and satisfactory interreliabilities of Cohen's κ between 0.71 and 0.77. Statistical analysis determined descriptive data and intra group effect sizes *d* (counting as small < 0.50, medium between 0.50 and 0.80, large > 0.80). An ANOVA with repeated measures was conducted to test the hypothesis on a 5%-level.

EMPIRICAL FINDINGS

Figure 3 shows the learning gains from pretest to posttest in explaining the conversion procedure for both intervention groups. In the *topic-deviating intervention group IG-ES* (working on estimation strategies for mass measures) students started with an average explanation score of $m_{pre} = 1.06$ (and standard deviation $SD_{pre} = 1.62$) and ended with $m_{post} = 1.34$ ($SD_{post} = 1.90$), so indeed, learning to explain on another topic can still reveal a small intra-group effect (with Cohen's $d = 0.14$) on explaining the non-treated conversion procedures, even if it is not significant in the ANOVA. Meanwhile, the topic-specific intervention group IG-CP (working on mass unit conversion) started with a higher average explanation score of $m_{pre} = 1.61$ ($SD_{pre} = 2.23$) and ended with

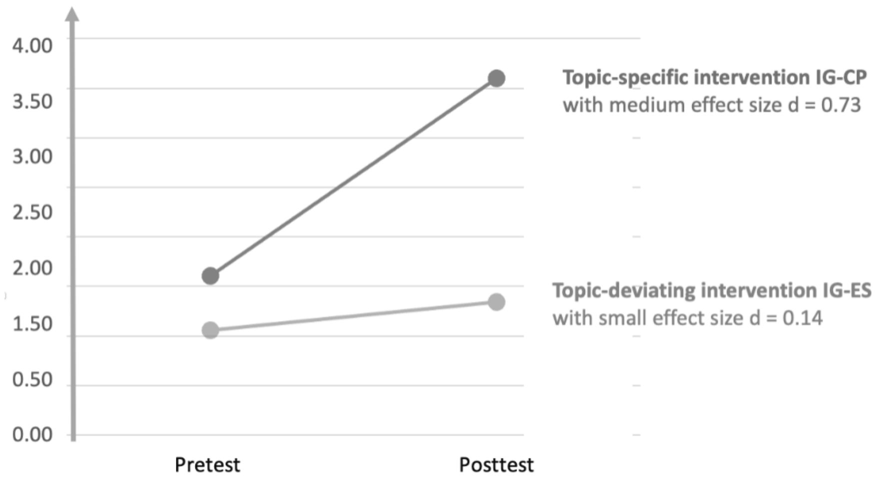

Figure 3: Higher learning gains in explanation scores in topic-specific intervention

significant gains to m_{post} = 3.60 (SD_{post} = 2.25), with a medium intra-group effect size (d = 0.73). According to the ANOVA with repeated measures, the intervention group IG-CP had significantly higher learning gains than the IG-ES (F_{time} (1, 547) = 13.27, $p < 0.001$; $F_{time\ x\ group}$ (1, 547) = 5.67, $p < 0.02$). Thus, hypothesis H can be validated: Students working in the topic-specific IG-CG improved their explanations more than those working in IG-ES.

DISCUSSION AND OUTLOOK

Learning to explain is not at all easy, as many qualitative studies have shown (Moschkovich, 2015; Erath et al., 2018), nor is understanding why a procedure works (Freudenthal, 1981; Huang & Wu, 2019). Our digital teaching-learning environments for mass measurement have improved students' competence to explain, so we can provide further empirical evidence for the efficacy of digitally assisted interventions for measurement, as called for by Smith & Barrett (2017). By this, we have replicated findings from other mathematical areas that the design principles of connecting multiple representations and engaging students in rich discourse practices can be effectively realized in digital teaching-learning environments (Drijvers, 2018; Sacristán et al., 2010) by transferring them to the under-researched area of explaining why the procedure for mass unit conversion works. When comparing existing review studies, Drijvers (2018) identified various aspects that might contribute to the efficacy of digital tools for learning: younger students (primary and lower grades of secondary school) seem to benefit more, just as interventions that seem to focus more on higher-order learning goals and short interventions (p. 173). All of these aspects apply to our interventions.

Beyond these replications, the novel contribution of our study is the validation of the topic-specificity hypothesis: Students working in the topic-specific digital environment with a topic focus on conceptual foundations of the conversion procedure improved their explanations more than students with a topic focus on estimation strategies. Although it may sound obvious, this finding is not trivial, neither in the literature, where explanation learning processes are often studied independent of the topic under consideration (e.g., by Erath et al., 2018; Mercer & Sams, 2006), nor obvious in our data: The topic-deviating intervention also yielded a small (yet not significant) effect (of $d = 0.14$), presumably achieved by familiarizing with mass measures, using the same visual representation of the balance scale and establishing the sociomathematical norm that explanations should refer to the visual representation and any attempt is better than no answer, these are topic-independent learnings. However, the topic-specificity hypothesis was validated as students in the topic-specific intervention group had significantly higher gains in their explanation scores (with a medium effect size of $d = 0.73$). This indicates that treating topic-specific conceptual components leads to much higher gains, which in our study were the visual representation of place-value structures and refinement structures underlying a concept-based justification of the conversion procedure (Bielinski & Prediger, submitted).

Due to *methodological limitations*, the results must be interpreted with caution. The intervention was taught by relatively unexperienced teachers who were not yet familiar with the digitally-assisted approach nor with the classes, thus they might have not fully exploited the potentials of the digital teaching-learning environments. In the future, we should analyze the teachers' enactment in terms of teacher moves and provided oral scaffolds. The class-wise random assignment did not result in fully comparable prior explanation scores. Although this was controlled for in the ANOVA with repeated measures, future studies should examine whether more comparable intervention groups lead to similar results. In future analysis, we will investigate also procedural knowledge and language knowledge and the impact of background variables.

But already now, the findings are so promising that a transfer to digital teaching-learning environments for other topics with the same design principles should be sought in order to further investigate the transferability. Future studies should also examine longer interventions and verify sustainability through retention tests.

Funding. *divomath* is funded by State Ministries of Education North Rhine-Westphalia and Brandenburg (grant to S. Prediger / C. Selter), the controlled trial conducted and analyzed for Startchancen (BMBF grant 01PL2401C/G to S. Prediger).

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